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Fatigue failure of disordered materials

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Abstract. We present an experimental and theoretical study of the fatigue failure of heterogeneous materials under cyclic compression considering asphalt as a specific example. Varying the load amplitude, experiments reveal a finite fatigue limit below which the specimen does not break, while approaching the tensile strength of the material a rapid failure occurs. In the intermediate load range, the lifetime decreases with the load as a power law. We introduce two novel theoretical approaches, namely, a fibre bundle model and a fuse model, and show that both capture the major microscopic mechanisms of the fatigue failure of heterogeneous materials, providing an excellent agreement with the experimental findings.

Keywords: fracture (theory), fracture (experiment)

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Contents

1. Introduction

The fracture of disordered media represents an important applied problem, with intriguing theoretical aspects. Statistical models have been successfully applied in the past to analyse fracture under quasistatic conditions, but the effect of cyclic loading is less explored [1]. Laboratory experiments reveal that fatigue failure under repeated loading is due to a combination of several mechanisms, among which damage growth, relaxation due to viscoelasticity, and healing of microcracks play an essential role [2]–[4]. Theoretical approaches have serious difficulties in capturing all of these mechanisms [2]–[5] and fatigue life prediction is still very much an empirical science. Understanding this problem has crucial implications even for everyday applications. For example, fatigue failure occurring in roads due to repeated traffic loading causes the main problem, limiting the lifetime of asphalt pavements.

In this paper we present a detailed experimental and theoretical study of the fatigue failure of heterogeneous materials considering the fatigue performance of hot mix asphalt (HMA) as a specific example. We carried out fatigue life tests on asphalt specimens measuring the accumulation of deformation with the number of loading cycles and the lifetime of specimens varying the load amplitude. Experiments revealed a monotonic increase of the deformation of the specimen with the number of loading cycles which accelerates when approaching macroscopic failure. The lifetime (the number of cycles to failure) of the specimen strongly depends on the external load, i.e. when the load approaches the tensile strength of the specimen rapid failure occurs, while at the other extreme there exists a threshold load below which the specimen suffers only partial failure and has an infinite lifetime. For intermediate loads the lifetime exhibits a power law decrease (Basquin law). To obtain a theoretical understanding of the experimental findings, we worked out two novel modelling approaches for fatigue failure, namely, a fibre bundle model [6]–[9] and a fuse model [10]. We show that both descriptions capture the stochastic nature of the fracture process, the immediate breaking of material elements and the cumulative effect of the loading history. Two physical mechanisms are considered which limit the accumulation of damage: a finite activation threshold of crack nucleation below which the local load does not contribute to the ageing of the material and healing

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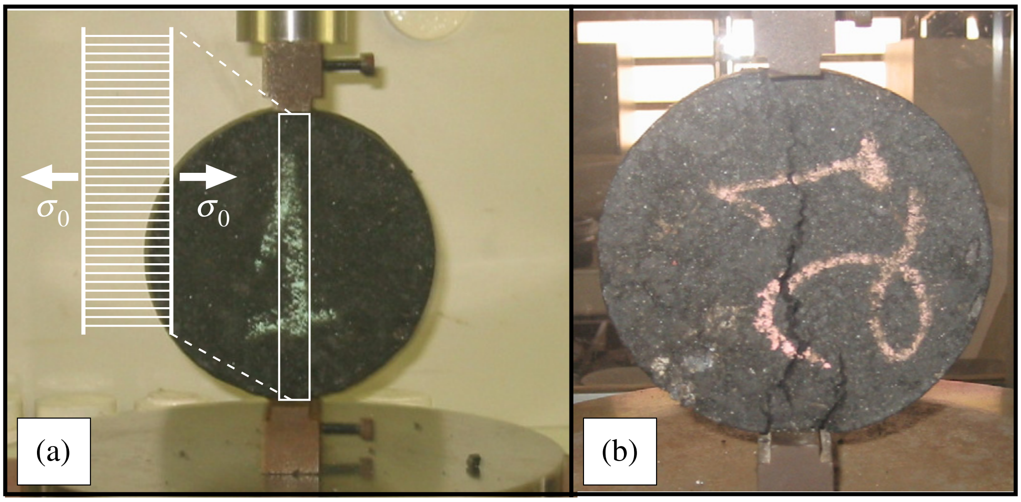

Figure 1. Set-up of the experiments. (a) A cylindrical asphalt sample is subjected to diametrical compression applied periodically. FBM discretizes the region where tensile stress emerges (white rectangle) in terms of fibres. (b) At complete failure a crack spans the cylinder along the load direction.

of microcracks under compression, which leads to damage recovery. The analytical and numerical results of the model calculations provide a good quantitative agreement with the experimental findings.

2. Experiments

In order to obtain a quantitative characterization of the process of fatigue failure, we carried out fatigue life tests of asphalt under cyclic diametric compression of cylindrical specimens at a constant external load σ_0 (see figures 1(a), (b)). HMA is the primary material used to construct and maintain pavements and roadways due to its good mechanical performance and high durability. From the structural point of view asphalt is a combination of aggregates (usually crushed stone and sand), filler (cement, hydrated lime or stone dust) and a bituminous binder. Cylindrical samples of HMA were produced using the Marshall method and then loaded using a hydraulic device.

Under repeated loading at a constant amplitude σ_0, the deformation ε was monitored as a function of the number of cycles N_{cycle}. Furthermore, the total number of cycles to complete failure N_{f} was measured varying σ_0. Figure 2(a) presents representative examples of $\varepsilon(N_{\mathrm{cycle}})$ recorded at loads 30% and 40% of the tensile strength σ_{c} of the specimen. It can be observed that due to the gradual accumulation of damage, the deformation ε caused by the same load σ_0 monotonically increases until catastrophic failure occurs after a finite number of cycles N_{f}. The derivative of deformation also increases which indicates the acceleration of the accumulation process when approaching the point of macroscopic failure. Increasing the external load the functional form of $\varepsilon(N_{\mathrm{cycle}})$ remains the same; however, the lifetime of the specimen N_{f} gets shorter. The fatigue lifetime N_{f} measured at different fractions of the tensile strength σ_{c} (figure 3(a)) reveals the existence of three distinct regimes. First, approaching the tensile strength of the material $\sigma_0/\sigma_{\mathrm{c}} \to 1$ the

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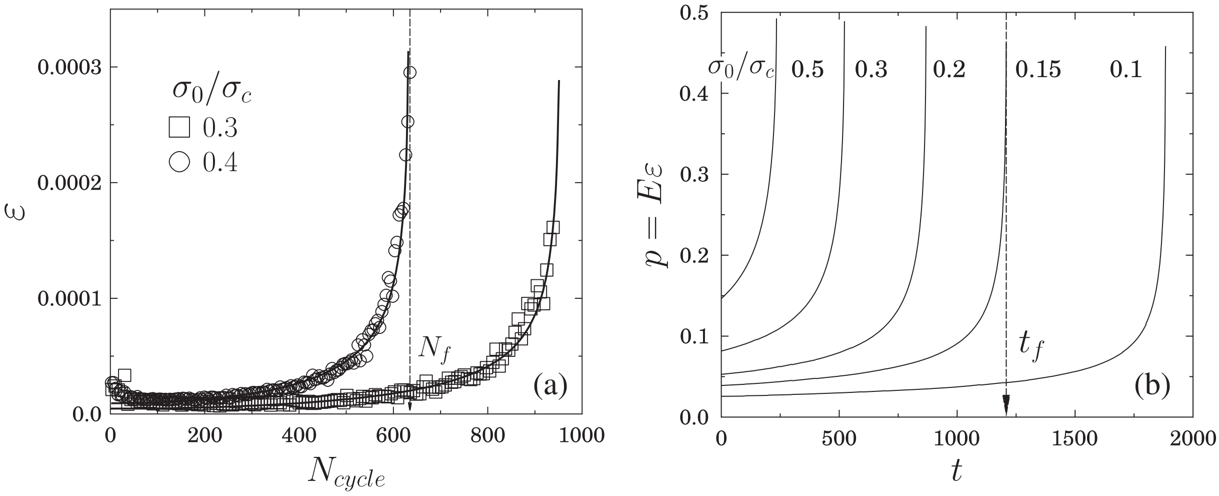

Figure 2. (a) Deformation as a function of the number of loading cycles. The continuous lines were obtained by fitting the experimental results with our theory. The vertical dashed line indicates the occurrence of macroscopic failure. (b) Load on the fibres in FBM as function of time at different values of σ_0/σ_c for uniformly distributed threshold values setting $\tau = \infty$ in equation (5).

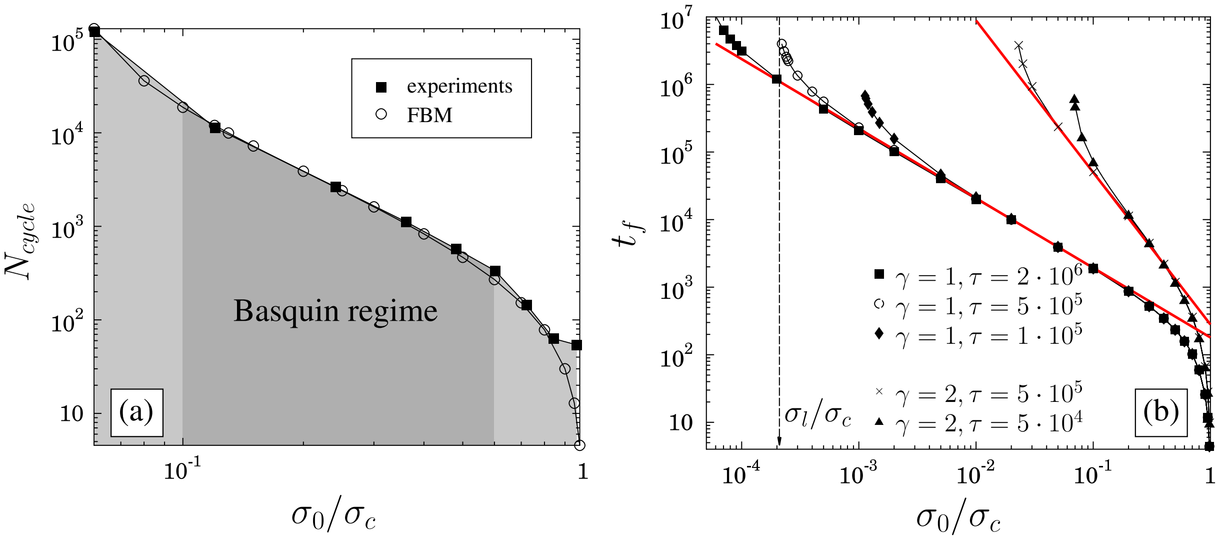

Figure 3. (a) N_{f} as a function of load $\sigma_0/\sigma_{\mathrm{c}}$ for FBM varying the value of τ. FBM provides an excellent fit of the experimental results with $\gamma = 2.0$, $\tau = 15\,000$, and $a = 0.01$ using Weibull distributed failure thresholds. (b) Lifetime t_{f} of FBM as function of $\sigma_0/\sigma_{\mathrm{c}}$ obtained numerically for different values of γ and τ. The vertical dashed line indicates an example of the fatigue limit.

lifetime N_{f} rapidly decreases, indicating an immediate failure of the specimen. At the other extreme, a lower threshold value of the external load σ_l can be identified below which the specimen suffers only partial damage giving rise to an infinite lifetime (fatigue limit). In the intermediate regime the experimental results follow a power law known as

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the Basquin law [2]–[4]

$$N_{\rm f} \sim \left(\frac{\sigma_0}{\sigma_{\rm c}}\right)^{-\alpha}, \tag{1}$$

where $\alpha = 2.2 \pm 0.1$ was obtained by fitting, as shown in figure 3(a).

3. Fibre bundle model with damage accumulation and healing

The experiments show that the fatigue crack growth is localized to a narrow region between the loading plates (see figure 1(b)) where locally a tensile stress emerges perpendicular to the external load. To give a theoretical description of the failure process, we focus on this region and discretize it by a fibre bundle model (FBM) as illustrated in figure 1(a) [6]. We consider a bundle of parallel linear elastic fibres with the same Young modulus E. Under diametrical compression of the disc-shaped specimen, the fibres experience a tensile loading and gradually fail due to immediate breaking or to the ageing of material elements [2]. More precisely, the following two mechanisms are considered:

(I) Fibre i ($i = 1, \ldots, N$) breaks instantaneously at time t when its local load $p_i(t)$ exceeds the tensile strength $p_{\rm th}^i$ of the fibre.

(II) All intact fibres undergo a damage accumulation process due to the load they have experienced.

The amount of damage Δc_i that occurred under the load $p_i(t)$ in a time interval Δt is assumed to have the form $\Delta c_i = a p_i(t)^\gamma \Delta t$; hence, the total accumulated damage $c_i(t)$ up to time t can be obtained by integrating over the entire loading history of fibres:

$$c_i(t) = a \int_0^t p_i^\gamma(t')\,{\rm d}t'. \tag{2}$$

The exponent $\gamma > 0$ controls the damage accumulation rate and $a > 0$ is a scale parameter. The fibres can only tolerate a finite amount of damage and break when $c_i(t)$ exceeds a threshold value $c_{\rm th}^i$. Each fibre is characterized by two breaking thresholds $p_{\rm th}^i$ and $c_{\rm th}^i$ which are random variables with a joint probability density function $h(p_{\rm th}, c_{\rm th})$. Assuming independence of the two breaking modes, the joint density function h can be factorized into a product

$$h(p_{\rm th}, c_{\rm th}) = f(c_{\rm th}) g(p_{\rm th}), \tag{3}$$

where $f(c_{\rm th})$ and $g(p_{\rm th})$ are the probability densities and $F(c_{\rm th})$ and $G(p_{\rm th})$ the cumulative distributions of the breaking thresholds $p_{\rm th}$ and $c_{\rm th}$, respectively. For simplicity, we assume that after each breaking event the load of the broken fibre is equally redistributed over the intact ones in the bundle irrespective of their distance from the failure point (global load sharing) [6]–[9].

Under a constant tensile load σ_0, the load on a single fibre p_0 is initially determined by the quasistatic constitutive equation of FBM [6]–[9]

$$\sigma_0 = [1 - G(p_0)]\, p_0, \tag{4}$$

which means that fibres with breaking thresholds $p_{\rm th}^i < p_0$ immediately break. It follows that the external load σ_0 must fall below the tensile strength of the bundle $\sigma_0 < \sigma_{\rm c}$;

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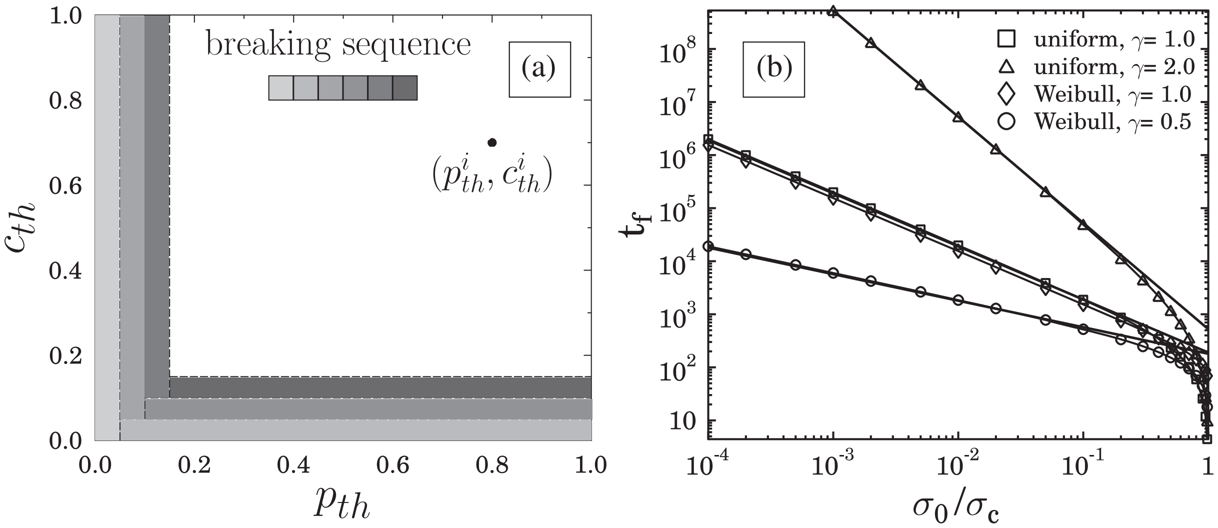

Figure 4. (a) Plane of breaking thresholds $p_{\rm th}$ and $c_{\rm th}$ using uniform distributions between 0 and 1. Each point of the plane represents a single fibre with threshold values $(p^i_{\rm th}, c^i_{\rm th})$. Subjecting the specimen to a constant load, fibres with breaking threshold $p^i_{\rm th} < p_0$ break immediately (lightest grey region). Then the damage accumulation $c(\Delta t) = ap_0^\gamma \Delta t$ gives rise to additional breakings $c^i_{\rm th} < c(\Delta t)$ (second lightest grey region), which increase the load on the remaining intact elements resulting again in immediate breakings. The greyscale indicates the first six steps of the breaking sequence. (b) $t_{\rm f}$ as a function of $\sigma_0/\sigma_{\rm c}$ for uniform and Weibull distributions ($m = 2.0$) varying the value of γ. The slope of the straight lines is equal to the value of the corresponding exponent γ. For clarity, $\tau \rightarrow \infty$ was set for the range of memory; hence, no fatigue limit emerges.

otherwise the entire bundle will fail immediately. As time elapses, the fibres accumulate damage $c(\Delta t) = ap_0^\gamma \Delta t$ and break due to their finite damage tolerance $c^i_{\rm th} < c(\Delta t)$. These breakings, however, increase the load on the remaining intact fibres which in turn induce again immediate breakings. This way, in spite of the independence of the threshold values $p_{\rm th}$ and $c_{\rm th}$, the two breaking modes are dynamically coupled, gradually driving the system to macroscopic failure in a finite time $t_{\rm f}$ at any load values σ_0. This sequence of immediate breaking and failure due to ageing is illustrated in figure 4(a). Experiments have shown that healing of microcracks plays an important role in the time evolution of the system especially at low load levels $\sigma_0 \ll \sigma_{\rm c}$. Healing of microcracks can be captured in the model by introducing a finite range τ for the memory, over which the loading history contributes to the accumulated damage [3, 11]. In polymeric materials like the asphalt binder the rebinding of polymer molecules typically leads to an exponential form of the memory term. Finally, the evolution equation of the system can be cast in the form

$$\sigma_0 = \left[1 - F\left(a \int_0^t {\rm e}^{-(t-t')/\tau} p(t')^\gamma \, {\rm d}t'\right)\right] [1 - G(p(t))] \, p(t), \tag{5}$$

where the integral in the argument of F provides the accumulated damage at time t taking into account the finite range of memory by the exponential term [11]. In principle, the range of memory τ can take any positive value $\tau > 0$ such that during the time

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evolution of the bundle the damage accumulated during the time interval $t' < t - \tau$ heals. Equation (5) is an integral equation which has to be solved for the load $p(t)$ on the intact fibres at a given external load σ_0 with the initial condition $p(t = 0) = p_0$ obtained from equation (4). The product in equation (5) arises due to the independence of the two breaking thresholds. We note that equation (5) recovers the usual constitutive behaviour of FBM [6] when damage accumulation is suppressed either by increasing the exponent γ or decreasing the range of memory $\tau \rightarrow 0$.

Figure 2(b) presents examples of the solution $p(t)$ of equation (5) obtained for breaking thresholds uniformly distributed in the interval $[0, 1]$ at different ratios σ_0/σ_c setting $\tau \rightarrow \infty$ (no healing is considered). Since $p(t)$ is simply related to the macroscopic deformation ε of the bundle $p(t) = E\varepsilon(t)$, these results can directly be compared to the experimental findings. In can be seen that the results are in nice qualitative agreement with the experimental findings, i.e. the deformation is a monotonically increasing function of time with an increasing derivative when the point of macroscopic failure is approached. Lowering the external load σ_0 the lifetime t_f of the bundle increases. For the quantitative comparison we considered a Weibull distribution for the breaking thresholds

$$P(x) = 1 - \exp\left[-\left(x/\lambda_b\right)_b^m\right], \tag{6}$$

where the index b denotes p and c for immediate breaking and damage, respectively. The excellent quantitative agreement of the experimental and theoretical results presented in figure 2(a) was obtained by varying solely three parameters a, γ, and τ.

One of the most important outcomes of our work is that the Basquin law equation (1) can be deduced from equation (5), i.e. it can be shown analytically that for $\sigma_0/\sigma_c \ll 1$ and $\tau \rightarrow \infty$ the lifetime of the system has a power law dependence on the external load:

$$t_f \sim \left(\frac{\sigma_0}{\sigma_c}\right)^{-\gamma}, \tag{7}$$

where γ is the damage accumulation exponent, independent on the type of disorder. Figure 4(b) shows that the numerical results are in excellent agreement with the above analytic prediction. Calculations were carried out for uniform and Weibull distributions setting $\tau \rightarrow \infty$ for the range of memory. It can be seen in the figure that the Basquin exponent is solely determined by the exponent γ of the damage accumulation rate.

Due to equation (5), without healing ($\tau \rightarrow \infty$) the cumulative effect of the loading history gives rise to a macroscopic failure of the system at any load. However, our experiments revealed that damage recovery caused by healing of microcracks results in a finite fatigue limit σ_l, below which the sample does not break. Since healing takes place in the polymer binder, it can be controlled by changing the temperature [3]. In our FBM the healing of microcracks is captured by the finite range of memory τ limiting the time interval over which microcracks contribute to the total damage. Of course, healing becomes dominating at a given load σ_0 when the range of memory τ is comparable to the lifetime t_f of the sample measured without healing $\tau \rightarrow \infty$. Varying the external load σ_0 at a fixed value of τ, the competition of the nucleation of new microcracks and healing of the existing ones results in a finite fatigue limit (a threshold load) σ_l below which the specimen suffers only partial failure and has an infinite lifetime $t_f \rightarrow \infty$. Figure 3(b) shows that the value of σ_l is controlled by τ such that on increasing the range of memory the fatigue limit gets smaller. It can be seen in figure 3(b) that when the external load

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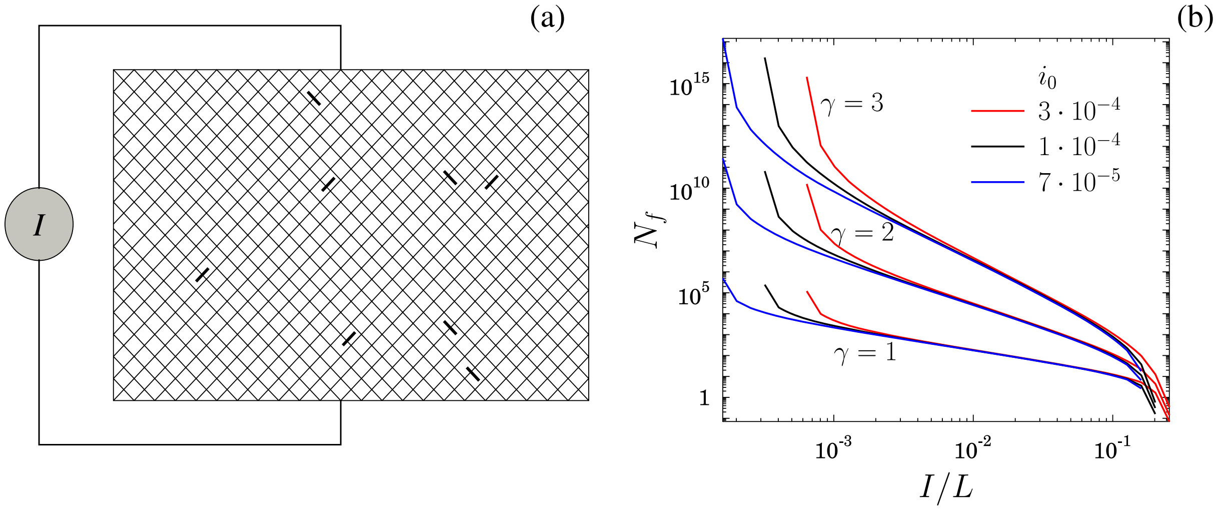

Figure 5. (a) Illustration of the fuse model. Initially, a tilted square lattice of intact fuses is considered. Lattice bonds crossed by a bold line indicate failed fuses. (b) Lifetime of the system as a function of the current I normalized by the lattice size L for different values of the threshold current i_0 and damage accumulation exponent γ.

approaches the fatigue limit σ_l from above at a fixed value of τ, the lifetime t_{f} of the sample rapidly increases. It follows from the Basquin law equation (7) that the divergence has the functional form $t_{\mathrm{f}} \sim (\sigma_0 - \sigma_l)^{-\gamma}$ for $\sigma_0 \to \sigma_l^+$. It is important to note that the value of τ does not have any influence on the Basquin regime of $t_{\mathrm{f}}(\sigma_0)$; the Basquin exponent is solely determined by the exponent of the rate of damage accumulation γ.

4. Fuse model with ageing fuses

Besides healing, another important mechanism which limits damage accumulation is a finite activation threshold of microcrack nucleation. In order to study this effect we consider the random fuse model (RFM) [10] of fracture and extend it by introducing a history dependent ageing variable of fuses. We construct an $L \times L$ tilted square lattice of initially fully intact bonds with identical conductance but random failure thresholds i_{c} (see figure 5(a)). The threshold values i_{c} are uniformly distributed between a small current value i_0 and 1 ($i_0 \ll 1$). For a given value of current I applied between two bus bars of the lattice, the local current through each bond is determined by solving numerically the Kirchhoff equations. Fuses burn out irreversibly when the current exceeds the local failure thresholds. This process is then followed by the recalculation of the current values. In order to capture fatigue cracking in the model, intact fuses are assumed to undergo an ageing process, modelled by a variable

$$A(t) = \sum_{t'=1}^{t} a(i(t') - i_0)^{\gamma} H(i(t') - i_0), \tag{8}$$

where the Heaviside function H expresses that only currents above the threshold value $i > i_0$ contribute to the ageing variable. A fuse fails due to fatigue when $A(t) > A_{\max}$,

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where $A_{\max}$ is a failure threshold uniformly distributed between $1-b$ and $1+b$ with $b=0.1$. Comparing the two modelling approaches, the ageing variable $A(t)$ of RFM is analogous to the accumulated damage $c(t)$ of FBM; however, only current values above i_0 contribute to $A(t)$, which captures the finite activation threshold of microcrack nucleation. We carried out computer simulations of the fatigue process varying the threshold current i_0 over a broad range and the value of the ageing exponent γ. Figure 5(b) demonstrates that RFM for ageing fuses provides qualitatively the same behaviour as FBM, i.e. rapid failure at high current values I, a Basquin regime equation (1) at intermediate currents with an exponent equal to γ and a finite fatigue limit σ_l determined by the threshold current i_0.

5. Summary

We carried out an experimental and theoretical study of fatigue failure of asphalt occurring under cyclic compression. Our experiments revealed three regimes of the failure process depending on the load amplitude: instantaneous breaking, a Basquin regime of a power law decrease of lifetime and the existence of a fatigue limit below which no failure occurs. We introduced two novel modelling approaches, namely, a fibre bundle model and a fuse model, which both capture the essential ingredients of the fatigue failure of bituminous materials. The models proved to provide a comprehensive description of the experimental findings. Computer simulations and analytic calculations showed that the Basquin exponent coincides with the exponent of damage accumulation rate. We showed that healing of microcracks controls the failure process at low load levels determining the fatigue limit of the material below which the specimen suffers only a partial failure and has an infinite lifetime. In the framework of the random fuse model we demonstrated that a finite activation threshold of microcrack nucleation has a similar effect to healing on the failure process.

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References

[1] Herrmann H J and Roux S (ed), 1990 *Statistical Models for the Fracture of Disordered Media* (Amsterdam: North-Holland)
Alava M J, Nukala P and Zapperi S, 2006 *Adv. Phys.* **55** 349
[2] Li Y and Metcalf J B, 2002 *J. Mat. Civ. Eng.* **14** 303
[3] Si Z, Little D N and Lytton R L, 2002 *J. Mat. Civ. Eng.* **14** 461
[4] Sornette D, Magnin T and Brechet Y, 1992 *Europhys. Lett.* **20** 433
[5] Sornette D and Vanneste C, 1992 *Phys. Rev. Lett.* **68** 612
[6] Hidalgo R C, Kun F and Herrmann H J, 2002 *Phys. Rev. Lett.* **89** 205501
[7] Kun F, Zapperi S and Herrmann H J, 2000 *Eur. Phys. J.* B **17** 269
[8] Hidalgo R C, Kun F and Herrmann H J, 2001 *Phys. Rev.* E **64** 066122
[9] Kun F, Hidalgo R C, Raischel F and Herrmann H J, *Extensions of fibre bundle models*, 2006 *Modelling Critical and Catastrophic Phenomena in Geoscience: A Statistical Physics Approach* (*Springer Lecture Notes in Physics* vol 705) (Berlin: Springer) pp 57–92
[10] de Arcangelis L, Redner S and Herrmann H J, 1985 *J. Physique Lett.* **46** 585
[11] Herrmann H J, Kertész J and de Arcangelis L, 1989 *Europhys. Lett.* **10** 147