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Mass-velocity correlation in impact fragmentation

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Abstract. We study the impact fragmentation of two- and three-dimensional disordered solids in a discrete element model of heterogeneous brittle materials focusing on the spatial distribution and mass-velocity correlation of fragments. Our calculations revealed that depending on the energy of impact the breakup process can have two different outcomes: at low impact energy the sample gets damaged, however, to achieve fragmentation the imparted energy has to surpass a critical value. Based on large scale computer simulations we show that the position of fragments inside the original body with respect to the impact site determines their mass and velocity in the final state. A novel relation of the mass and velocity of fragments is revealed: In the damage phase the mass and velocity of fragments are strongly correlated, however, in the fragmented phase correlation emerge solely for large fragments. The correlation function decays as a power law with a universal exponent in an excellent agreement with recent experimental findings.

Introduction

Fragmentation, i.e. breakup into a large number of pieces occurs when a large amount of energy is imparted to a solid within a short time. Impact induced fragmentation of heterogeneous brittle materials is abundant in nature having also a high industrial importance especially in mining and ore processing. During the past decades research on fragmentation mainly focused on the statistics of fragment sizes/masses which revealed that these quantities are power law distributed with universal exponents. Recently, impact fragmentation of glass plates was investigated experimentally by shooting a projectile parallel to the plane of the plate. High speed camera observation made it possible to determine even the velocity of fragments with a high precision. No correlation of the mass and velocity of fragments was pointed out [1].

In the present paper we study the impact fragmentation of disordered solids based on a three-dimensional discrete element model with the aim to analyze the spatial distribution of fragments inside the original body and the correlation of the mass and velocity of the generated pieces. Simulations are carried out for the breakup of a plate-like solid embedded in the three-dimensional space. We give numerical evidence that in the damage phase of the breakup process fragment mass and velocity are correlated, however, in the fragmented phase no correlation emerges. The correlation function has power law decay, however, the exponent has different values for small and large fragments. Detailed analysis revealed that the position of fragments inside the original body determines both there mass and velocity being responsible for the observed mass-velocity correlation. Our results provide an explanation for some recent experiments on the fragmentation of heterogeneous brittle materials.

Discrete element model for fragmentation

In the model the sample is represented as a random packing of spherical particles with a log-normal size distribution [3,4]. The interaction of contacting particles is described by the Hertz contact law. Cohesive interaction is provided by beams which connect the particles along the edges of a Delaunay triangulation of the initial particle positions. In 3D the total deformation of a beam is calculated as the superposition of elongation, torsion, as well as bending and shearing [3,4,5]. Crack formation is captured such that the beams, modeling cohesive forces between grains, can be broken according to a physical breaking rule, which takes into account the stretching and bending of beams

$$\left(\frac{\varepsilon_{ij}}{\varepsilon_{th}}\right)^2 + \frac{\max(|\Theta_i|,|\Theta_j|)}{\Theta_{th}} > 1, \quad (1)$$

where the two parameters ε_{th} and θ_{th} controll the relative importance of the two breaking modes. Here ε_{ij} denotes the axial strain, while θ_i, and θ_j are the bending angles of the beam ends. In the model there is only structural disorder present: the breaking thresholds are constants, however, the physical properties of beams are determined by the random particle packing. The energy stored in a beam just before breaking is released in the breakage giving rise to energy dissipation. At the broken beams along the surface of the spheres cracks are generated inside the solid and as a result of the successive beam breaking the solid falls apart. The fragments are defined as sets of discrete particles connected by remaining intact beams. The time evolution of the fragmenting solid is obtained by solving the equations of motion of the individual particles until the entire system relaxes meaning that no beam breaking occurs during some hundreds consecutive time steps and there is no energy stored in deformation. For more details of the model construction see Refs. [3,4].
In order to make a realistic implementation of the impact process, in the simulations a plate-like sample was constructed with rectangular basis of side length L and height H. Simulations were carried out with linear extensions L=30 and H=5 measured in units of the average particle radius <R>. A single surface particle was selected in the middle of one of side walls of the sample, which together with its contacting neighbors got a large initial velocity v_0 pointing towards the center of mass of the body. The entire external surface of the body is free, no constraints are imposed on the motion of particles. Snapshots of the time evolution of the system is presented in Fig. 1.

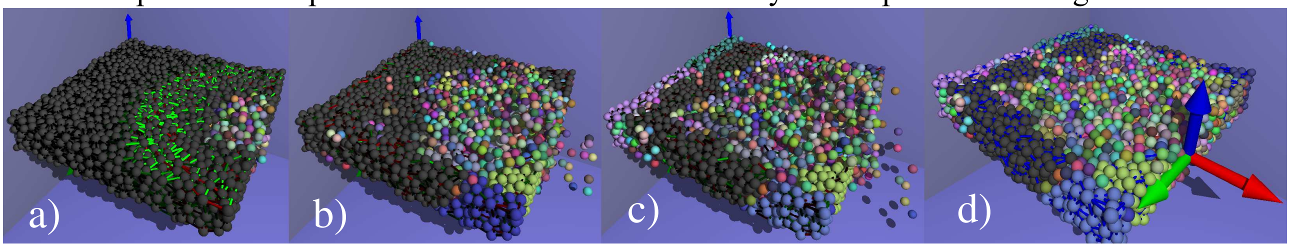

Fig. 1: Time evolution of the fragmentation process of a plate. Beams are colored according to their deformation so that the elastic wave is visible in (a) but they are virtually invisible in (b-d). The solid gradually breaks up in (b) and (c). Particles of different fragments have randomly selected colors. In (d) the body is reassembled to have a better view on the spatial arrangement of fragments and on the structure of cracks.

Transition from damage to fragmentation

At low impact velocities the body gets damaged around the impact site but a large residue remains which comprises the majority of the original mass. The impact velocity v_0 has to surpass a critical value v_c to obtain complete breakup, where even the largest fragment is significantly smaller than the original body. A large number of simulations were carried out varying the impact velocity v_0 in a broad range to cover both the damage and fragmentation phase of the system. Figure 2(a) presents that in the damage phase the mass distribution p(m) of fragments has a gap, i.e. there are a few big fragments and a certain amount of small ones with a rapidly decreasing mass distribution. When reaching the critical velocity v_c the mass distribution becomes a power law

$$p(m) \propto m^{-\tau}, \quad (2)$$

where the exponent τ has the value τ=1.6±0.06 at v_c. Further increasing the impact velocity the distribution gets somewhat steeper. (The impact velocity v_0 is given in terms of the sound speed c of the material throughout the manuscript.)

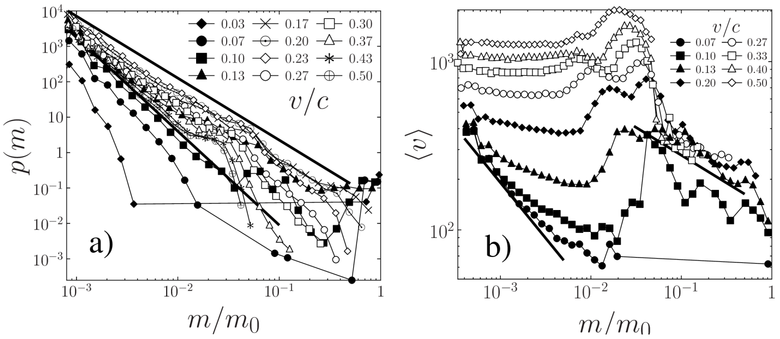

Fig. 2: (a) Mass distribution of fragments varying the impact velocity v_0 in a broad range. (b) The average value of the magnitude of the velocity of fragments as a function of their mass for different impact velocities. The straight lines with slope 2/3 and 1/3 are to guide the eye.

For the analysis of the spatial distribution of the mass and velocity of fragments, we determined the average value of the velocity components v_x and v_y of fragments as a function of their x and y coordinates in the original body. It can be observed in Fig. 3(a) and (b) that inside the body there are fragments which practically do not move, i.e. their velocity is zero. Due to momentum conservation, v_y is a symmetric function with respect to y=0. Hence, the fragment of zero velocity are along the line y=0, while their x coordinate depends on the impact velocity v_0. In Fig. 3(a) and (b) both velocity components are monotonically increasing with the distance from the zero velocity fragments. Note that the fragments which are back-scattered at the impact site and the ones detached along the surface of the body can even exceed the velocity of impact.

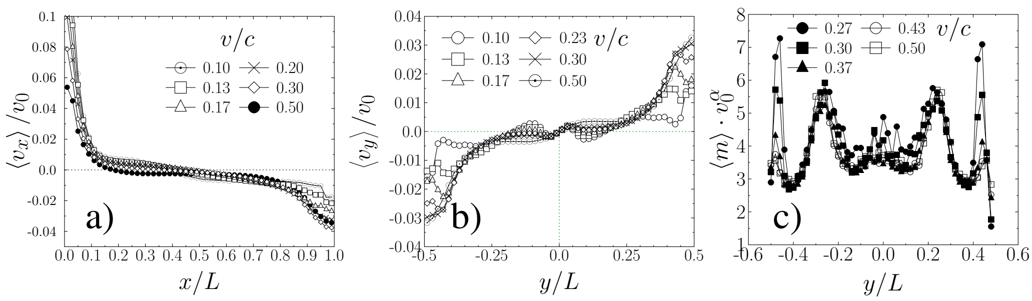

Fig. 3: Velocity components v_x (a) and v_y (b) rescaled with the impact velocity v_0 are plotted as a function of the spatial coordinates x and y of the fragments. (c) Mass of fragments as a function their y coordinate. Data collapse is obtained by rescaling the vertical axis with a power α of v_0 .

A general feature of our system is that fragments along the surface created either in the distractive zone around the impact site or by the detachment effect along the surface of the body, are small compared to the original size of the body. Large fragments could survive, however, inside the plate. Fig. 3(c) shows that the fragment mass as a function of the y coordinate is a symmetric function with respect to y=0 point. Two peaks of the m(y) function occur symmetrically whose position does not depend on the impact velocity, however, their height has a strong velocity dependence. A very interesting feature of the spatial distribution of fragments is that m(y) has the scaling structure in terms of the impact velocity

$$m(y) \propto v_0^{-\beta} g(y/L), \qquad (3)$$

where the exponent β=1.5±0.1 provides the best collapse and g denotes the scaling function. The good quality data collapse obtained with the transformation Eq. (3) is illustrated in Fig. 3(c). A detailed knowledge on the velocity of fragments is needed to understand the secondary evolution of fragmenting systems, i.e. the effect of secondary collisions of fragments or the time evolution of pieces created e.g. by asteroid impacts under a gravitational field. In the framework of our discrete element approach, important conclusions can be drawn for the fragment velocities and for their relation to other characteristic quantities of fragments like mass and spatial position. In order to get information on the possible correlation of fragment mass and velocity, we calculated the average velocity of fragments with a given mass. It can be observed in Fig. 2 (b) that in the fragmented phase $v_0 > v_c$ for small fragment masses the fragment velocity is constant over almost two orders of magnitude, i.e. the mass and velocity of fragments are independent in this regime. In the damaged phase, however, a strong mass-velocity correlation is found with power law functional form

$$m \propto v_0^{-\gamma} . \quad (4)$$

The value of the exponent $\gamma \approx 2/3$ was obtained numerically for small fragments. It can be observed in Fig. 2(b) that for large fragments the exponent is different $\gamma \approx 1/3$. The two power law regimes are separated by a hump of detached fragments, indicating that these pieces have an anomalously high escape velocity.

Summary

We presented a discrete element study of the fragmentation of plate-like brittle solids focusing on the spatial distribution of fragment masses and velocities, and on their correlation. Computer simulations revealed that the position of fragments inside the original body with respect to the impact point determine the velocity and mass of fragments: as the distance increases the velocity of the fragment becomes larger. We obtained a simple scaling form of the spatial distribution of fragment masses in the direction perpendicular to the impact velocity. The mass distribution of fragments proved to be a power law in agreement with experiments and with previous computational studies. Simulations give strong evidence that in the fragmented regime the velocity is independent of the mass, however, in the damaged phase a strong correlation emerges which has a power law decay.

Acknowledgements

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References

[1] T. Kadono, M. Arakawa, and M. Mitani: Phys. Rev. E Vol. 72 (2005) 0452106(R).
[2] F. Kun and H. J. Herrmann: Phys. Rev. E Vol. 59 (1999) 2623.
[3] G. Timár, J. Blömer, F. Kun, and H. J. Herrmann: Phys. Rev. Lett. Vol. 104 (2010) 095502.
[4] G. Timár, F. Kun, H. A. Carmona, and H. J. Herrmann: Phys. Rev. E Vol. 86 (2012) 016113.
[5] H. A. Carmona, F. K. Wittel, F. Kun, and H. J. Herrmann: Phys. Rev. E Vol. 77 (2008) 051302.