ACTA PHYSICA DEBRECENIENSIS XLI, 59 (2007)

MOMENTS OF THE TWO-PARAMETER FERMI CHARGE DISTRIBUTION

I. Angeli

Institute of Experimental Physics, University of Debrecen, H-4010 Debrecen 1, Pf. 105. Hungary

Abstract

Moments $\langle r^m \rangle$ and isotopic differences $\delta\langle r^m \rangle$ of the spherical two-parameter Fermi charge distribution are presented for $m =$ 1 to 10 as a function of the half-density radius c and diffusity a; the introduction of the parameter $\beta \equiv \pi a/c$ proved to be useful. First, the Fermi integrals $F_n(k)$ are derived. Then, even and odd orders are treated separately for $F_n(k)$ as well as for $\langle r^m \rangle$ and $\delta\langle r^m \rangle$; in calculating $\delta\langle r^m \rangle$ the diffusity a is assumed to be constant. The differences $\delta\langle r^m \rangle$ for even m are also given in terms of $\delta\langle r^2 \rangle$.

I. Introduction. The Fermi integral $F_n(k)$

Moments of model charge distributions are often used in atomic and nuclear physics. Isotope shifts $\delta\nu$ in optical and characteristic X-ray frequencies can be expressed with differences in even moments $\delta\langle r^m \rangle_{A2,A1}$ for two isotopes [1]. Another example: the energy of transitions in muonic atoms determine the *Barrett moment* $\langle r^k e^{-\alpha r} \rangle$ [2], which can be approximated by a series of moments $\langle r^m \rangle$ $(m = 2, 3, \ldots)$ [3]. A frequently used charge distribution is the spherically symmetric *two-parameter Fermi distribution (2pF)*:

$$\rho_F(r; c, a) = \frac{\rho_0}{1 + e^{\frac{r-c}{a}}} \equiv \frac{\rho_0}{1 + e^{x-k}} \qquad (r \geq 0) \tag{1}$$

where c is the half-density radius, a is the surface diffusity, $x \equiv r/a$, $k \equiv c/a$; the dimensionless quantity $\beta \equiv \pi/k = \pi a/c$ is also used where practical. The m-th absolute moment of the $2pF$ charge distribution is

$$\langle r^m \rangle = \frac{\int r^m \rho_F(r;c,a)dv}{\int \rho_F(r;c,a)dv} = \frac{\int r^{m+2} \rho_F(r;c,a)dr}{\int r^2 \rho_F(r;c,a)dr} = a^m \frac{F_{m+2}(k)}{F_m(k)} \tag{2}$$

where the *Fermi-integral*

$$F_n(k) = \int \frac{x^n}{1+e^{x-k}} dx \tag{3}$$

is introduced; the limits of integration in r and x are from zero to infinity. The Fermi integral can be expressed by the sums [4, Appendix C; note the misprint in the exponent of the last term!]:

$$\begin{aligned} F_n(k) &= \frac{k^{n+1}}{n+1} + \sum_{r=0}^{n} \left[1-(-1)^r\right] \frac{n!}{(n-r)!} k^{n-r} \left(1 - \frac{1}{2^r}\right) \zeta(r+1) \\ &\quad - \sum_{v=1}^{\infty} (-1)^{v+n} \frac{n!}{v^{n+1}} e^{-vk} \end{aligned} \tag{4}$$

where ζ is the *Riemann function* (see later).

In the present paper, formulae for the Fermi integrals $F_n(k)$, for moments $\langle r^m \rangle$ and for isotopic differences $\delta\langle r^m \rangle$ are given using the parameters c, a and β. In order to meet practical requirements (evaluation of optical isotope shift measurements) isotopic differences $\delta\langle r^m \rangle$ with even m are expressed by $\delta\langle r^2 \rangle$, too. Parts of this work have been published in [5, 3]. However, the present full version contains several completions.

II. Estimation of the exponential terms

It is worth estimating the value of the last sum of exponential terms

$$E_n(k) = -\sum_{v=1}^{\infty} (-1)^{v+n} \frac{n!}{v^{n+1}} e^{-vk} = (-1)^n n! e^{-k} \left(1 + \frac{e^{-k}}{2^{n+1}} + - \dots\right) \tag{5}$$

Even for the very light nucleus ^4He [6, p.489] $k = 1.0/0.32 = 3.12$, i.e. $e^{-k} \approx 0.045$, the second and any further terms in the parentheses can be neglected; i.e.:

$$|E_n(k)| \approx n! e^{-k} \tag{6}$$

Compare the $|E_n(k)|$ values with the sum of the leading terms:

$$\begin{aligned} F_2(k) &\approx \frac{k^3}{3}(1+1) = \frac{30}{3} \times 2 = 20 \\ & \quad |E_2(k)| \approx 2 \times 0.045 \approx 0.09 \\ F_6(k) &\approx \frac{k^7}{7}(1+7+16.3+10.3) = \frac{2873}{7} \times 34.6 = 14200 \\ & \quad |E_6(k)| \approx 32 \\ F_{10}(k) &\approx \frac{k^{11}}{11}(1+18+154+682+1397+852) = 7.7 \times 10^8 \\ & \quad |E_{10}(k)| \approx 1.6 \times 10^5 \end{aligned} \tag{7}$$

It can be seen that for ^4He the values $|E_n(k)|$ are three orders of magnitudes less than $F_n(k)$; consequently, $E_n(k)$ can be neglected. For heavier nuclei the difference between $E_n(k)$ and $F_n(k)$ is even more significant.

III. $F_n(k)$ for $m = 2, 4, \ldots, 10$ and $m = 1, 3, \ldots, 9$

In the first sum, for even r the value of the bracket vanishes: $[1-(-1)^r] = 0$; therefore, only terms with odd r remain. For odd r i.e. for even arguments $(r+1)$ the value of the *Riemann ζ function* is [7, p.807]:

$$\zeta(r+1) = \frac{(2\pi)^{r+1}}{2(r+1)!} |B_{r+1}| \tag{8}$$

with the *Bernoulli numbers* B_{r+1} [7, pp.804, 810]:

$$\begin{aligned} & B_2 = \frac{1}{6}, \quad B_4 = -\frac{1}{30}, \quad B_6 = \frac{1}{42}, \\ & B_8 = -\frac{1}{30}, \quad B_{10} = \frac{5}{66}, \quad B_{12} = -\frac{691}{2730} \end{aligned} \tag{9}$$

$$
\begin{aligned}
&\zeta(2) = \frac{(2\pi)^2}{2\times 2!}\frac{1}{6} = \frac{\pi^2}{6}, \quad \zeta(4) = \frac{(2\pi)^4}{2\times 4!}\frac{1}{30} = \frac{\pi^4}{90}, \\
&\zeta(6) = \frac{(2\pi)^6}{2\times 6!}\frac{1}{42} = \frac{\pi^6}{945}, \quad \zeta(8) = \frac{(2\pi)^8}{2\times 8!}\frac{1}{30} = \frac{\pi^8}{9450}, \\
&\zeta(10) = \frac{(2\pi)^{10}}{2\times 10!}\frac{5}{66} = \frac{\pi^{10}}{93555}, \quad \zeta(12) = \frac{(2\pi)^{12}}{2\times 12!}\frac{691}{2730}
\end{aligned} \tag{10}
$$

(The value of $\zeta(12)$ will be necessary for the closed expression of the 9th moment $\langle r^9 \rangle$.)

For even n

$$
\begin{aligned}
&F_n(k) \\
&= \frac{k^{n+1}}{n+1} + \sum_{r=0}^{n} \left[1-(-1)^r\right] \frac{n!}{(n-r)!} k^{n-r} \frac{2^r-1}{2^r} \frac{(2\pi)^{r+1}}{2(r+1)!} \left|B_{r+1}\right| \\
&= \frac{k^{n+1}}{n+1} \left[1 + \sum_{r=1,3,\ldots}^{n-1} 2(2^r-1) \left|B_{r+1}\right| \frac{(n+1)!}{(n-r)!(r+1)!} \left(\frac{\pi}{k}\right)^{r+1} \right]
\end{aligned} \tag{11}
$$

With $\beta \equiv \pi/k = \pi a/c$ and using numerical values of the *Bernoulli numbers* B_{r+1}, we have

$$
\begin{aligned}
F_n(k) \quad \approx \quad & \frac{k^{n+1}}{n+1} \Bigg[1 + \frac{(n+1)!}{(n-1)!3!}\beta^2 + \frac{7}{3}\frac{(n+1)!}{(n-3)!5!}\beta^4 + \frac{31}{3}\frac{(n+1)!}{(n-5)!7!}\beta^6 \\
& + \frac{381}{5}\frac{(n+1)!}{(n-7)!9!}\beta^8 + \frac{2555}{3}\frac{(n+1)!}{(n-9)!11!}\beta^{10} \\
& + \frac{1414477}{105}\frac{(n+1)!}{(n-11)!13!}\beta^{12} + \ldots + (\ldots)\beta^n \Bigg]
\end{aligned} \tag{12}
$$

the sum in the brackets terminates with β^n. For $n = 2, 4, 6, 8$ and 10 we

have:

$$
\begin{aligned}
F_2(k) &= \frac{k^3}{3}\left(1+\beta^2\right) \\
F_4(k) &= \frac{k^5}{5}\left(1+\frac{10}{3}\beta^2+\frac{7}{3}\beta^4\right) \\
F_6(k) &= \frac{k^7}{7}\left(1+7\beta^2+\frac{49}{3}\beta^4+\frac{31}{3}\beta^6\right) \\
F_8(k) &= \frac{k^9}{9}\left(1+12\beta^2+\frac{294}{5}\beta^4+124\beta^6+\frac{381}{5}\beta^8\right) \\
F_{10}(k) &= \frac{k^{11}}{11}\left(1+\frac{55}{3}\beta^2+154\beta^4+682\beta^6+1397\beta^8+\frac{2555}{3}\beta^8\right) \quad (13)
\end{aligned}
$$

For odd n the sum over r ends on n, the terms in the brackets contain *even powers of* β the last term is $\sim \beta^{n+1}$:

$$
\begin{aligned}
F_1(k) &= \frac{k^2}{2}\left(1+\frac{1}{3}\beta^2\right) \\
F_3(k) &= \frac{k^4}{4}\left(1+2\beta^2+\frac{7}{15}\beta^4\right) \\
F_5(k) &= \frac{k^6}{6}\left(1+5\beta^2+7\beta^4+\frac{31}{21}\beta^6\right) \\
F_7(k) &= \frac{k^8}{8}\left(1+\frac{28}{3}\beta^2+\frac{98}{3}\beta^4+\frac{124}{3}\beta^6+\frac{127}{15}\beta^8\right) \\
F_9(k) &= \frac{k^{10}}{10}\left(1+15\beta^2+98\beta^4+310\beta^6+381\beta^8+\frac{2555}{33}\beta^8\right) \quad (14)
\end{aligned}
$$

IV. $\langle r^m \rangle$ for $m = 2, 4, \ldots, 10$ and $m = 1, 3, \ldots, 9$

Using $F_2(k)$ the ratio $F_{m+2}(k)/F_2(k)$ can be formed.

For *even* m:

$$\begin{aligned}\frac{F_{m+2}(k)}{F_2(k)} &= \frac{3}{m+3}k^m\frac{1}{1+\beta^2}\Bigg[1+\frac{(m+3)!}{(m+1)!3!}\beta^2+\frac{7}{3}\frac{(m+3)!}{(m-1)!5!}\beta^4 \\ &+\frac{31}{3}\frac{(m+3)!}{(m-3)!7!}\beta^6+\frac{381}{5}\frac{(m+3)!}{(m-5)!9!}\beta^8 \\ &+\frac{2555}{3}\frac{(m+3)!}{(m-7)!11!}\beta^{10}+\frac{1414477}{105}\frac{(m+3)!}{(m-9)!13!}\beta^{12} \\ &+\ldots+(\ldots)\beta^{m+2}\Bigg] \qquad (15)\end{aligned}$$

For odd m the last term contains $\sim \beta^{m+3}$.

In the case of ^4He, $\beta \approx 1$, while for ^{12}C: $k = 2.36/0.52 = 4.54$ [6, p.489] $\beta \approx 0.69$ i.e. $\beta^2 \approx 0.48$. Therefore, the series expansion

$$\frac{1}{1+\beta^2} = 1-\beta^2+\beta^4-\beta^6+\beta^8-\beta^{10}+\beta^{12}-+\ldots \qquad (16)$$

is valid –except for the lightest elements (as perhaps Li,Be and B)–. Using Eq. (16) we have the general formula for the m-th moment:

$$\begin{aligned}&\langle r^m\rangle = a^m\frac{F_{m+2}(k)}{F_2(k)} = \frac{3}{m+3}c^m \times \\ &\Bigg\{1+\left[\frac{(m+3)(m+2)}{3!}-1\right]\beta^2+\Bigg[\frac{7}{3}\frac{(m+3)(m+2)(m+1)m}{5!}- \\ &\frac{(m+3)(m+2)}{3!}+1\Bigg]\beta^4+\Bigg[\frac{31}{3}\frac{(m+3)(m+2)\ldots(m-1)(m-2)}{7!} \\ &-\frac{7}{3}\frac{(m+3)(m+2)(m+1)m}{5!}+\frac{(m+3)(m+2)}{3!}-1\Bigg]\beta^6 \\ &+\Bigg[\frac{381}{5}\frac{(m+3)\ldots(m-3)(m-4)}{9!}-\frac{31}{3}\frac{(m+3)\ldots(m-2)}{7!}\end{aligned}$$

$$
\begin{aligned}
&+\frac{7}{3}\frac{(m+3)\ldots m}{5!}-\frac{(m+3)(m+2)}{3!}+1\Bigg]\beta^8\\
&+\Bigg[\frac{2555}{3}\frac{(m+3)\ldots(m-5)(m-6)}{11!}\\
&-\left(\frac{381}{5}\frac{(m+3)\ldots(m-3)(m-4)}{9!}-\ldots-1\right)\Bigg]\beta^{10}\\
&+\Bigg[\frac{1414477}{105}\frac{(m+3)\ldots(m-7)(m-8)}{13!}\\
&-\left(\frac{2555}{3}\frac{(m+3)\ldots(m-5)(m-6)}{11!}+\ldots+1\right)\Bigg]\beta^{12}+\ldots\Bigg\} \quad (17)
\end{aligned}
$$

It can be seen that for $\langle r^m\rangle$ the coefficients of $\beta^{m+2}, \beta^{m+4}, \ldots$ vanish exactly. Applying the above general formula to *even* m values and substituting $\beta = \pi a/c$ we have:

$$
\begin{aligned}
\langle r^2\rangle &= \frac{3}{5}c^2\left(1+\frac{7}{3}\beta^2\right)=\frac{3}{5}c^2+\frac{7}{5}(\pi a)^2\\
\langle r^4\rangle &= \frac{3}{7}c^4\left(1+6\beta^2+\frac{31}{3}\beta^4\right)\\
\langle r^6\rangle &= \frac{1}{3}c^6\left(1+11\beta^2+\frac{239}{5}\beta^4+\frac{381}{5}\beta^6\right) \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
\langle r^8\rangle &= \frac{3}{11}c^8\left(1+\frac{52}{3}\beta^2+\frac{410}{3}\beta^4+\frac{1636}{3}\beta^6+\frac{2555}{3}\beta^8\right)\\
\langle r^{10}\rangle &= \frac{3}{13}c^{10}\Bigg(1+25\beta^2+\frac{926}{3}\beta^4+\frac{46714}{21}\beta^6+\frac{910573}{210}\beta^8\\
&\qquad+\frac{19447}{210}\beta^{10}\Bigg) \qquad (19)
\end{aligned}
$$

For *odd* m, from β^{m+2} the signs alternate, with the same non-zero absolute

values. This alternating series can be written in the closed form $1/(1+\beta^2)$

$$
\begin{aligned}
\langle r\rangle &= \frac{3}{4}c\left[1+\beta^2-\frac{8}{15}\beta^4(1-\beta^2+\beta^4-\beta^6+\beta^8-\ldots)\right] \\
&= \frac{3}{4}c\left[1+\beta^2-\frac{8}{15}\frac{\beta^4}{1+\beta^2}\right] \\
\langle r^3\rangle &= \frac{1}{2}c^3\left[1+4\beta^2+3\beta^4-\frac{32}{21}\frac{\beta^6}{1+\beta^2}\right] \\
\langle r^5\rangle &= \frac{3}{8}c^5\left[1+\frac{25}{3}\beta^2+\frac{73}{3}\beta^4+17\beta^6-\frac{128}{15}\frac{\beta^8}{1+\beta^2}\right] \\
\langle r^7\rangle &= \frac{3}{10}c^7\left[1+14\beta^2+84\beta^4+226\beta^6+155\beta^8-\frac{2560}{33}\frac{\beta^{10}}{1+\beta^2}\right] \\
\langle r^9\rangle &= \frac{1}{4}c^9\left[1+21\beta^2+210\beta^4+1154\beta^6+3037\beta^8+2073\beta^8\right. \\
&\qquad \left.-\frac{1415168}{1365}\frac{\beta^{12}}{1+\beta^2}\right]
\end{aligned}
\tag{20}
$$

V. $\delta\langle r^m\rangle$ for $m = 2, 4, \ldots, 10$

Assuming a constant *surface diffusity* $a = const.$, and a mass number dependence $c = r_0 A^{1/3}$ for the *half-density radius* c, we have

$$\delta c = \frac{1}{3}c\frac{\delta A}{A}. \tag{21}$$

The change in the second moment corresponding to a change $\delta A \equiv A_2 - A_1$ in the mass number is

$$\delta\langle r^2\rangle = \frac{3}{5}2c\frac{1}{3}\frac{r_0}{A^{2/3}}\delta A = \frac{2}{5}c^2\frac{\delta A}{A} \tag{22}$$

and the shifts in the higher even moments:

$$
\begin{aligned}
\delta\langle r^4\rangle &= \frac{4}{7}c^4\left[1+3\beta^2\right]\frac{\delta A}{A} \\
\delta\langle r^6\rangle &= \frac{2}{3}c^6\left[1+\frac{22}{3}\beta^2+\frac{239}{15}\beta^4\right]\frac{\delta A}{A} \\
\delta\langle r^8\rangle &= \frac{8}{11}c^8\left[1+13\beta^2+\frac{205}{3}\beta^4+\frac{409}{3}\beta^6\right]\frac{\delta A}{A} \\
\delta\langle r^{10}\rangle &= \frac{10}{13}c^{10}\Bigg[1+20\beta^2+\frac{1852}{10}\beta^4+\frac{186856}{210}\beta^6 \\
&\qquad +\frac{1821146}{2100}\beta^8\Bigg]\frac{\delta A}{A}
\end{aligned}
\tag{23}
$$

VI. $\delta\langle r^m\rangle$ in terms of for $\langle r^m\rangle$ for $m = 4, 6, \ldots, 10$

For some applications e.g. for the evaluation of isotope shifts in optical and characteristic X-ray spectra the isotopic differences in higher moments $\delta\langle r^4\rangle, \delta\langle r^6\rangle, \ldots,$ should be expressed in terms of $\delta\langle r^2\rangle$ [5]. This can be done easily because the formula of the previous section allows to express c^2 by $\delta\langle r^2\rangle$:

$$
c^2 = \frac{5}{2}\frac{A}{\delta A}\delta\langle r^2\rangle = \frac{5}{4}\frac{A_1+A_2}{A_2-A_1}\delta\langle r^2\rangle \tag{24}
$$

where A is now replaced by $(A_1 + A_2)/2$ and δA by $A_2 - A_1$. Using Eq. (24), the isotope shifts of higher order are:

$$
\begin{aligned}
\delta\langle r^4\rangle &= \frac{25}{14}\frac{A_1+A_2}{A_2-A_1}\left(\delta\langle r^2\rangle\right)^2+\frac{30}{7}(\pi a)^2\delta\langle r^2\rangle \\
\delta\langle r^6\rangle &= \frac{125}{48}\left(\frac{A_1+A_2}{A_2-A_1}\right)^2\left(\delta\langle r^2\rangle\right)^3+\frac{275}{18}(\pi a)^2\frac{A_1+A_2}{A_2-A_1}\left(\delta\langle r^2\rangle\right)^2 \\
&\quad +\frac{239}{9}(\pi a)^4\delta\langle r^2\rangle \\
\delta\langle r^8\rangle &= \frac{625}{176}\left(\frac{A_1+A_2}{A_2-A_1}\right)^3\left(\delta\langle r^2\rangle\right)^4+\frac{1625}{44}(\pi a)^2\left(\frac{A_1+A_2}{A_2-A_1}\right)^2\left(\delta\langle r^2\rangle\right)^3 \\
&\quad +\frac{5125}{33}(\pi a)^4\frac{A_1+A_2}{A_2-A_1}\left(\delta\langle r^2\rangle\right)^2+\frac{8180}{33}(\pi a)^6\delta\langle r^2\rangle
\end{aligned}
$$

$$
\begin{aligned}
\delta\langle r^{10}\rangle &= \frac{15625}{3328}\left(\frac{A_1+A_2}{A_2-A_1}\right)^4\left(\delta\langle r^2\rangle\right)^5 \\
&\quad + \frac{15625}{208}(\pi a)^2\left(\frac{A_1+A_2}{A_2-A_1}\right)^3\left(\delta\langle r^2\rangle\right)^4 \\
&\quad + \frac{57875}{104}(\pi a)^4\left(\frac{A_1+A_2}{A_2-A_1}\right)^2\left(\delta\langle r^2\rangle\right)^3 \\
&\quad + \frac{583925}{273}(\pi a)^6\frac{A_1+A_2}{A_2-A_1}\left(\delta\langle r^2\rangle\right)^2 + \frac{910573}{546}(\pi a)^8\delta\langle r^2\rangle
\end{aligned}
\tag{25}
$$

VII. $\delta\langle r^m\rangle$ for $m = 1, 3, \ldots, 9$

Here the derivative and the series sum

$$
\begin{aligned}
&\delta\left(\frac{\beta^3}{1+\beta^2}\right) = -\beta^3\frac{1+\frac{1}{3}\beta^2}{(1+\beta^2)^2}\frac{\delta A}{A}, \\
&1 - \frac{5}{3}\beta^2 + \frac{7}{3}\beta^4 - 3\beta^6 + \frac{11}{3}\beta^8 - + \ldots = \frac{1+\frac{1}{3}\beta^2}{(1+\beta^2)^2}
\end{aligned}
\tag{26}
$$

are used in the expressions for $\delta\langle r^m\rangle$:

$$
\begin{aligned}
\delta\langle r\rangle &= \frac{1}{4}c\left[1-\beta^2+\frac{8}{5}\beta^4\frac{1+\frac{1}{3}\beta^2}{(1+\beta^2)^2}\right]\frac{\delta A}{A} \\
\delta\langle r^3\rangle &= \frac{1}{2}c^3\left[1+\frac{4}{3}\beta^2-\beta^4+\frac{32}{21}\beta^6\frac{1+\frac{1}{3}\beta^2}{(1+\beta^2)^2}\right]\frac{\delta A}{A} \\
\delta\langle r^5\rangle &= \frac{5}{8}c^5\left[1+5\beta^2+\frac{73}{15}\beta^4-\frac{17}{5}\beta^6+\frac{128}{25}\beta^8\frac{1+\frac{1}{3}\beta^2}{(1+\beta^2)^2}\right]\frac{\delta A}{A} \\
\delta\langle r^7\rangle &= \frac{7}{10}c^7\left[1+10\beta^2+\frac{18}{5}\beta^4+\frac{226}{7}\beta^6-\frac{155}{7}\beta^8\right. \\
&\qquad \left. + \frac{2560}{77}\beta^{10}\frac{1+\frac{1}{3}\beta^2}{(1+\beta^2)^2}\right]\frac{\delta A}{A} \\
\delta\langle r^9\rangle &= \frac{3}{4}c^9\left[1+\frac{49}{3}\beta^2+\frac{350}{3}\beta^4+\frac{1154}{3}\beta^6+\frac{3037}{9}\beta^8-\frac{691}{3}\beta^{10}\right. \\
&\qquad \left. + \frac{1415168}{4095}\beta^{12}\frac{1+\frac{1}{3}\beta^2}{(1+\beta^2)^2}\right]\frac{\delta A}{A}
\end{aligned}
\tag{27}
$$

References

[1] E. C. Seltzer, Phys. Rev. **188**, 1916 (1969).

[2] R. C. Barrett, Physics Letters B **33**, 388 (1970).

[3] I. Angeli, Acta Phys. Hung., New Series, Heavy Ion Physics **15**, 87 (2002).

[4] L. R. B. Elton, *Nuclear Sizes, App C.* (Oxford University Press, 1961).

[5] I. Angeli, Hyperfine Interactions **136**, 17 (2001).

[6] C. W. deJager, et al., Atomic Data and Nuclear Data Tables **14**, 479 (1974).

[7] M. Abramowitz and I.A.Stegun, *Handbook of Mathematical Functions* (Publications, Inc., New York, 1970).