CRACKLING NOISE IN NON-DESTRUCTIVE MATERIAL TESTING

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Abstract

Materials of high mechanical performance are often fabricated by embedding strong fibers in a relatively weak matrix. Under a constant or slowly increasing external load such reinforced composites undergo a damage process of gradual microcrack accumulation which then leads to localization and macroscopic fracture. The process of damaging can be followed experimentally by recording acoustic signals emitted by cracks. This so-called crackling noise is a very important diagnostic tool in non-destructive fracture testing and can even be used to predict the imminent failure event. We show by means of analytical calculations and computer simulations that the presence of two subsets of materials in the system with widely different mechanical strength has a crucial effect on the characteristics of crackling noise. We demonstrate that the fracture process of such two-component systems has two universality classes characterized by different distributions of the noise amplitudes.

I. Introduction

Damage and fracture of materials occurring under various types of external loads is an interesting scientific problem with an important technological impact. During the last two decades the application of statistical physics has revealed that heterogeneities of materials' microstructure play a crucial role in fracture processes [1]. To capture the effect of disorder, recently several stochastic fracture models have been proposed such as the fiber bundle model (FBM) and lattice models of fuses or springs [1, 2, 3, 4, 5, 6]. Based on these models, analytic calculations and computer simulations revealed that macroscopic fracture of disordered materials shows interesting analogies with phase transitions and critical phenomena having several universal features independent of specific material details [3, 4, 7, 5, 8]. It has been found that under a slowly increasing external load macroscopic failure is preceded by a bursting activity due to the cascading nature of local breakings [2, 3]. Since the bursts can be recorded experimentally by the acoustic emission technique, these precursors addressed the possibility of forecasting the imminent failure event [9, 10, 11]. The size distribution of bursts was proven to be a power law with an exponent 5/2 which is universal for a broad class of disorder distributions [2, 3].

The goal of this project is to analyze the properties of crackling noise under realistic loading and material conditions with special emphasis on novel high performance materials. The results of our research can be used to understand crackling noise spectra measured by on-field monitoring systems [13, 14].

II. Macroscopic response

We consider a set of N fibers which are loaded in parallel. Under an increasing external load σ_o the fibers have a linearly elastic response with a Young modulus $E = 1$ fixed for all the fibers. In order to capture the large variation of disordered material properties, we assume that the bundle is composed of two subsets of fibers with strongly different breaking characteristics: A fraction $0 \leq \alpha \leq 1$ of fibers is *strong* in the sense that they have an infinite load bearing capacity so that they never break. However, fibers of the remaining $1 - \alpha$ fraction are *weak* and break when the load on them σ exceeds a threshold value σ_{th}^i, $i = 1, \ldots, N_w$, where $N_w = (1 - \alpha)N$ is the number of weak fibers. The strength disorder of weak fibers is characterized by the probability density $p(\sigma_{th})$ and distribution function $P(\sigma_{th}) = \int_0^{\sigma_{th}} p(x)dx$ of the failure thresholds. After a weak fiber breaks in the bundle, its load has to be overtaken by the remaining intact ones.

For simplicity, we assume global load sharing (GLS) (also called equal load sharing) which means that all the intact fibers share the same load σ, hence, no stress concentration occurs around failed regions. Under these conditions the constitutive equation of the model can be written as

$$\sigma_o = (1-\alpha)\,[1-P(\sigma)]\,\sigma + \alpha\sigma, \tag{1}$$

where σ_o is the external load acting on the sample and σ denotes the load of single fibers which is related to the strain ε of the system as $\sigma = E\varepsilon$. The first term of Eq. (1) accounts for the load bearing capacity of the surviving fraction of *weak* elements, and the second one represents the stress carried by the *unbreakable* subset of the system. In the following calculations it is instructive to consider two different strength distributions for the weak fibers, namely, a uniform distribution between 0 and 1 and a Weibull distribution will be used with the distribution functions $P(\sigma) = \sigma$ and $P(\sigma) = 1 - \exp\left[-\left(\sigma/\lambda\right)^m\right]$, respectively.

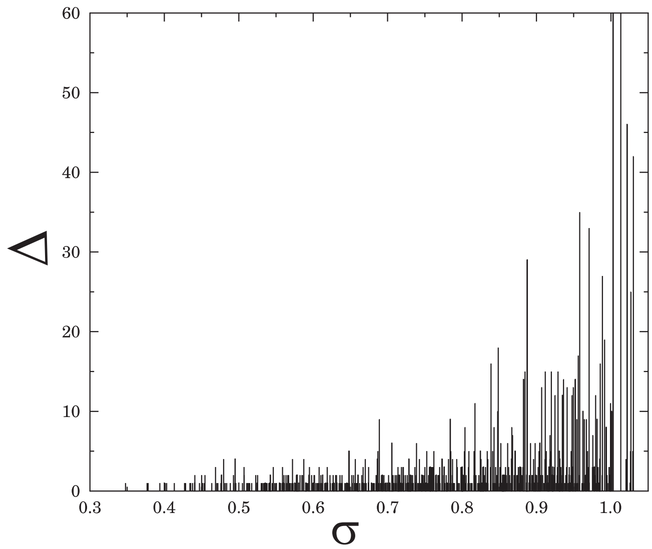

Figure 1: Example of bursts of fiber breakings recorded under an increasing external load σ. The size of bursts Δ has strong fluctuations with an increasing average when approaching macroscopic failure.

Varying the fraction of the two components α in the model, we showed analytically that the presence of unbreakable elements has a substantial effect on the fracture process of the system both on the micro- and macro-scales. We found a critical fraction α_c where a transition occurs between two qualitatively different regimes: below the critical point $\alpha < \alpha_c$ the macroscopic constitutive curve $\sigma_0(\sigma)$ has a single maximum, while above α_c the macroscopic response becomes monotonous [13]. Very interestingly the monotonicity drastically changes the microscopic process of fracture [13, 14].

III. Structure of crackling noise

Under stress controlled loading, *i.e.* when increasing σ_o the breaking of a single fiber can induce additional breakings which in turn may trigger an entire avalanche of breaking events. On the microlevel the failure proceeds in such breaking bursts which are analogous to microcracks giving rise to acoustic signals in experiments. Figure 1 presents an example of the failure process of a fiber bundle where the bursting activity can be observed. One of the most important characteristics of the microscopic process of failure is the size distribution of bursts, *i.e.* the hight distribution of peaks in Fig. 1. We showed analytically that below the critical fraction α_c of the two components the burst size distribution $D(\Delta)$ is a power law with the usual mean field exponent $\tau = 5/2$. However, above α_c the distribution $D(\Delta)$ can be cast in the form

$$\frac{D(\Delta)}{N} \simeq \frac{\Gamma\left(\frac{3}{4}\right)}{24\sqrt{3\pi a''_\sigma}3^{1/4}}\Delta^{-9/4}, \tag{2}$$

where a''_σ depends on the strength disorder P. Our derivation demonstrates that increasing α the behavior of the system changes both on the macro- and the micro-scales. We showed that while the quadratic maximum of $\sigma_o(\sigma)$ prevails, *i.e.* below the critical point α_c, the asymptotic behavior of the burst size distribution $D(\Delta)$ is controlled by the vicinity of the maximum resulting in a power law functional form $D(\Delta) \sim \Delta^{-\tau}$ with an universal exponent $\tau = 5/2$. However, at α_c the constitutive curve becomes monotonically increasing $d\sigma_o/d\sigma > 0$ and the avalanche statistics is dominated by the inflexion point of $\sigma_o(\sigma)$, giving rise to a different value of the exponent $\tau = 9/4$. Varying the control parameter α, the exponent

τ suddenly switches between the two values 5/2 and 9/4 when passing the critical point α_c. Note that in the derivation the only assumption we made is that the constitutive curve of the system has a single maximum and an inflexion point. It follows that the change of the exponent τ of the avalanche size distribution can be observed for a large variety of disorder distributions defining a novel universality class of breakdown phenomena. This universality class is narrower than the one in which the power law behavior of $D(\Delta)$ emerges with the exponent $\tau = 5/2$. For instance, the Weibull distributions do present the above switching of exponents, however, the uniform distribution does not [13].

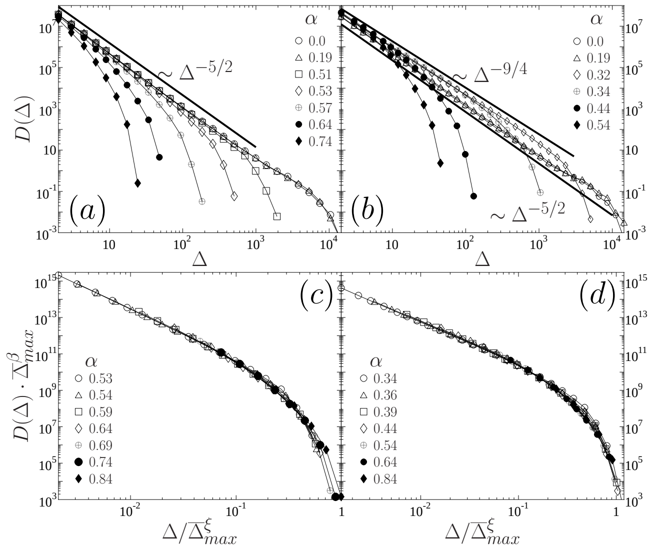

Figure 2: Non-normalized avalanche size distributions for uniform (a, c) and Weibull distributions (b, d) varying α below and above α_c. The straight lines in (a) and (b) represent the power laws obtained analytically. Rescaling the two axis according to the scaling formula Eq. (3), a very good quality data collapse is obtained in (c) and (d).

For the numerical verification of the above analytic results we carried out Monte Carlo simulations of the quasi-static fracture process of our FBM considering Weibull and uniform distributions for the breaking thresholds. Computer simulations were performed with $N = 10^6$ fibers averaging over

10^3 samples. The Weibull parameters were set to $m = 2$ and $\lambda = 1$ for which the critical point is $\alpha_c \simeq 0.3085$. For uniformly distributed threshold values it can be observed in Fig. 2(a) that below the critical point the numerically obtained distributions $D(\Delta)$ remain the same, even the cutoff of the distributions does not change with α. It is interesting to note that in this specific case the constitutive equation for $\sigma \leq \sigma_c(\alpha)$ is the same as if the system is composed of solely weak fibers with threshold values between zero and the upper bound $\sigma_{th}^{max} = 1/(1-\alpha)$. It follows that the entire failure process of the bundle obtained at different α values remains the same until there are enough weak fibers in the system $N_w > N/2$. Hence, for the parameter regime $\alpha < \alpha_c$ the avalanche statistics does not change, we obtain the typical power law distribution $D(\Delta) \sim \Delta^{-\tau}$ with the exponent $\tau = 5/2$ (Fig. 2(a)). For $\alpha > \alpha_c$ the parabolic shape of $\sigma_o(\sigma)$ prevails, however, due to the insufficient number of breakable fibers $N_w < N/2$ the system behaves as if the loading process was stopped before reaching the maximum of $\sigma_o(\sigma)$. Consequently, the cutoff of the distribution $D(\Delta)$ in Fig. 2(a) decreases with increasing α, however, the exponent τ has the same value as below α_c, in agreement with our predictions and also with Ref. [3].

We use the average size of the largest burst $\overline{\Delta}_{max}$ as the characteristic burst size of the system. It can be seen in Fig. 3(a) that for the uniform distribution below α_c, the value of $\overline{\Delta}_{max}$ is constant, while it rapidly decreases when α surpasses α_c. Figure 3(c) demonstrates that approaching α_c from above the characteristic burst size has a power law divergence $\bar{\Delta}_{max} \sim (\alpha - \alpha_c)^{-\nu}$, where for the value of the exponent $\nu = 1.56 \pm 0.07$ was obtained numerically. In the case of the Weibull distribution, below the critical point $\alpha < \alpha_c$ the burst size distribution has a power law behavior $D(\Delta) \sim \Delta^{-\tau}$, with the exponent $\tau = 5/2$ as it is expected (Fig. 2(b)). Increasing the value of α in this regime results in a slight increase of the cutoff burst size but the power law part of the distribution does not change. However, when α surpasses α_c the exponent of the power law regime of $D(\Delta)$ suddenly switches to the lower value $\tau = 9/4$, in a perfect agreement with our analytic predictions (see Fig. 2(b)). We again find that the characteristic burst size $\overline{\Delta}_{max}$ diverges as we approach α_c from above with the same value of the exponent ν as for the uniform case (see Fig. 3(d)). We emphasize that the value of the exponent of the power law regime of $D(\Delta)$ remains constant $\tau = 9/4$ when changing α above α_c [13, 14].

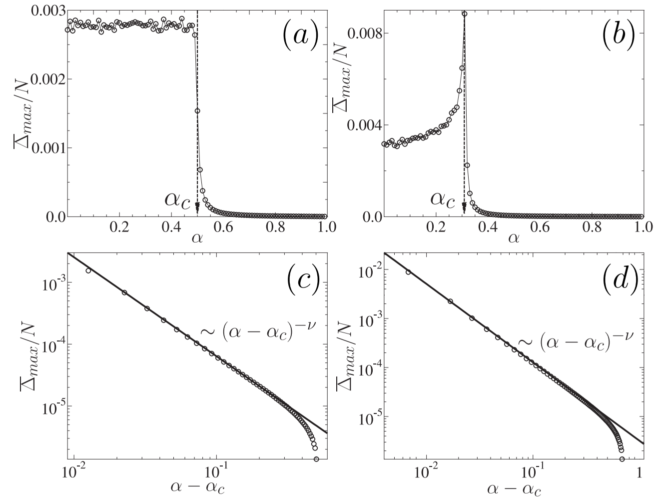

Figure 3: The average size of the largest burst $\overline{\Delta}_{max}$ as a function of α for uniform (a, c) and Weibull (b, d) distributions. The vertical straight lines in the figures indicate the corresponding critical point α_c. (c) and (d) show that approaching α_c from above, $\overline{\Delta}_{max}$ has a power law divergence as a function of $\alpha - \alpha_c$. The value of the exponent is $\nu = 1.56 \pm 0.07$ for both cases.

Using $\overline{\Delta}_{max}$ as a scaling variable, we introduce the scaling ansatz

$$D(\Delta) = \bar{\Delta}_{max}^{-\beta} g(\Delta/\overline{\Delta}_{max}^{\xi}) \tag{3}$$

for the burst size distributions above the critical point $\alpha > \alpha_c$. Here β and ξ are scaling exponents, which have the relation $\beta = \tau\xi$ with $\tau = 5/2$ and $\tau = 9/4$ for the uniform and Weibull distributions, respectively. Figures 2(c) and (d) present the rescaled burst size distributions plotting $D(\Delta)\overline{\Delta}_{max}^{\beta}$ as a function of $\Delta/\overline{\Delta}_{max}^{\xi}$. The high quality data collapse is obtained with the parameters $\beta = 3.25$, $\xi = 1.25$ and $\beta = 1.12$, $\xi = 2.52$, for the uniform and Weibull distributions, which are consistent with the two different values of the τ exponent [13, 14].

IV. Discussion

Our numerical and analytical calculations revealed that the presence of unbreakable elements gives rise to a substantial change of the fracture process of disordered materials both on the micro- and macro-scales. Astonishingly we found a critical fraction of the breakable and unbreakable components where the exponent of the burst size distribution switches from the well known mean field exponent of FBM $\tau = 5/2$ to a significantly lower value $\tau = 9/4$. The transition is conditioned to disorder distributions where the macroscopic constitutive response of the system has a single maximum and an inflexion point, implying a novel universality class of FBM [13]. Besides its theoretical importance, the problem has several implications for experimental studies. New materials of high mechanical performance are often fabricated by mixing components with widely different properties. For instance, fiber reinforced composites are composed of strong fibers which are embedded in a carrier matrix. In this case at the breaking of weak elements, the strong ones act as the unbreakable component of our model [14].

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