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FISHER INFORMATION AND QUANTUM MECHANICS

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Information theoretical concepts gain growing importance in the study of properties of quantum mechanical systems. In 1925 Fisher [1] introduced a measure of 'intrinsic accuracy' in statistical estimation theory. Since that time Fisher information has proved to be very useful in studying several systems. The importance of Fisher information in quantum chemistry and density functional theory was first emphasized by Sears, Parr and Dinur [2]. Fisher information turned to be proportional to the Weizsäcker kinetic energy [3].

It turned out that there exists a principle concerning the Fisher information: the principle of extreme physical information [4]. For example the equations of nonrelativistic quantum mechanics were derived using this principle[5, 6]. Several years ago the present author derived the Euler equation of the density functional theory from the principle of minimum Fisher information [7]. Both time-independent and time-dependent cases were analyzed. It was shown that Fisher information is a measure of the quality of the approximate density. Nalewajski [8] derived Kohn-Sham equations using this principle. Fisher information of single-particle systems with a central potential was determined [9].

In a recent paper the present author [10] studied Fisher information for a two-electron 'entangled artificial' atom proposed by Moshinsky. A relationship between the Fisher and the Shannon information was explored. It was shown that the Fisher information for this model is proportional to the Kullback-Leibner (or relative) information that measures the deviation of the density from a constant, the value of the density at $r = 0$. The Pauli kinetic energy was also determined.

In another recent paper we investigated the atomic Fisher information [11]. It was shown that the Thomas-Fermi Fisher information is negative. A slightly more sophisticated model proposed by Gáspár provides a qualitatively correct expression for the Fisher information: Gáspár's Fisher information is proportional to the two-third power of the atomic number. Accurate numerical calculations show an almost linear dependence on the atomic number.

We have just intoduced the idea of phase-space Fisher information [12]. The theorem that the phase-space Fisher information is not less than the sum of the position space and the momentum space Fisher information was stated and proved. Fisher information sum was illustrated for the hydrogen-like atoms and the isotropic harmonic oscillator. Uncertainty-like relationship for the Fisher information sum was presented.

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