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RENORMALIZATION OF SINE-GORDON TYPE MODELS

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Abstract

We determine the infrared scaling laws of the sine-Gordon, the massive sine-Gordon and the layered sine-Gordon models for $d = 2$ of spacetime dimensions using the differential renormalization group in momentum space in the local potential approximation. We also obtain the phase structures of the models.

I. Introduction

The renormalization of sine-Gordon type models, which have relevance both in high-energy and low-temperature physics represents a challenge in quantum field theory. The usual strategies based on the Taylor expansion of the interaction Lagrangian fail, because the periodicity of the model is violated by any truncation of the Taylor expansion. The phase structure of the system depends on the symmetries of the interaction Lagrangian. In order to obtain the low-energy effective theory and to map out the phase structure of a SG type model one has to use a method which retains the periodicity of the system. Therefore we use the Wegner-Houghton renormalization group (WH-RG) method, in the local potential approximation to determine the phase strusture and the infrared scaling laws of the sine-Gordon (SG), the massive sine-Gordon (MSG) and the layered sine-Gordon (LSG) models.

The phase structure of the SG model which is periodic in the internal space spanned by the field variable has two phases separated by the critical value of the parameter, $\beta_c^2 = 8\pi$. The periodicity is spontaneously broken in the molecular (renormalizable) phase and a condensate appears when the restoring force acting on the field fluctuations to be eliminated is vanishing. In the ionized (non-renormalizable) phase the periodicity remains unbroken.

In the massive sine-Gordon (MSG) model due to the mass term of the Lagrangian the periodicity can be broken either spontaneously or explicitly. Therefore, the MSG model also has two phases.

It is an interesting subject to consider a SG type theory which combines the 'features' of the massless and massive SG models where the explicit breaking of periodicity is only partial. The corresponding generalization of the SG model which is called the layered sine-Gordon (LSG) model belongs to the latter category.

II. SG model

The well-known phase structure of the two-dimensional sine-Gordon model is reconstructed by studying the sensitivity of the dynamics on microscopic parameters without using topological disorder parameters to identify the phases of the model. Such an analysis resolves the apparent contradiction between the phase structure and the triviality of the effective potential in either phases [2]. The model provides a case where the usual classification of operators based on the linearization of the scaling relation around a fixed point is not available, a more complete study, based on the global features of the renormalization group trajectories is needed to determine the number of free parameters of the renormalized dynamics. Numerical results show that scaling laws characterized by critical exponents are actually recovered in a non-trivial, non-linear manner with no relevant parameter. This result is in agreement with the picture suggested by the analogy with the XY model where the vortex fugacity is found to be the only relevant parameter in this phase. Furthermore, it has been clarified that the sensitivity of the dynamics on the renormalizable coupling constants is washed away and a super-universality is generated in the IR limit of the molecular phase by the Maxwell-cut.

III. MSG model

The global features of the RG flow of the MSG model are discussed and it is shown that the model possesses two phases, (i) one containing no condensate, where the corresponding IR physics can be parametrized with the value of a single bare coupling, that of the fundamental mode of the periodic potential and (ii) the other with a condensate of elementary excitations with non-vanishing momentum, spinodal instability, for weak enough mass in the remnant of the molecular phase of the SG model. This condensate, the consequence of the presence of a periodic piece in the local potential, generates non-trivial blocked potential and phase structure despite of the explicit, stable mass of elementary excitations in the deep IR region. The sensitivity matrix allows us to study the way in which the UV parameters influence the IR physics. It was found that the suppression of the sensitivity on the non-renormalizable bare couplings, generated in the UV scaling regime, can be overturned by the increasing sensitivity piled up in the IR scaling regime if the UV and the IR cutoffs are removed in a coordinated manner. As a result, a non-trivial, global extension of the universality is found which goes beyond the local studies of the RG flow around the UV fixed point only. Based on our RG analysis one can recover the well-known phase structure of the bosonized version of QED_2 in the LPA. Therefore, one is lead to the conclusion that the phase structure, as well as the spinodal instability should survive wave-function renormalization.

IV. LSG model

The layered sine-Gordon (LSG) model with Josephson type interlayer coupling (i.e. with a tri-diagonal mass matrix) has been proposed in the literature to describe the vortex dynamics of Josephson coupled layered high-temperature superconducting materials. We restrict ourselves to small bare values of the periodic piece of the local potential in order to avoid the appearance of spinodal instability during the RG flow. The phase structure is found to be strongly related to the breaking of periodicity in the internal space. The LSG model has a special layered structure where the explicit breaking of periodicity is only partial. We conjectured by extrapolating the mass-corrected UV scaling laws to IR scales that there is a Kosterlitz-Thouless-Berezinsky-type phase transition accompanied by spontaneous breaking of the explicitly unbroken symmetry. The transition frequency $\beta_c^2(N) = N8\pi$ (inverse of the transition temperature, $\beta_c^2(N) \propto 1/T_c(N)$) shows a linear dependence on the number N of the layers. Recently, we have shown that these results are in agreement with those obtained for the 2-layer SG model by solving the WH-RG flow equations numerically [3]. The calculated and the experimentally observed dependences $T_c(N)$ of the transition temperature on N differ. Also the gas of topological excitations obtained by an appropriate transformation of the partition function of the LSG model [4] exhibits long-range interactions different of those in the vortex gas equivalent of the layered Ginzburg-Landau model. Therefore, we concluded that the LSG model with Josephson type interlayer coupling is not suitable to describe the Josephson coupled layered superconductors. We have shown that magnetically coupled layered superconductors can be described by an LSG model with an appropriately modified mass matrix (interlayer coupling) [5].

References

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