CASIMIR ENERGY ON A LATTICE

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As an exploratory study we examine the possibilities of calculating the Casimir energy using field–theoretical simulations on lattice. The Casimir energy as the physical vacuum energy of a system of interacting quantum fields is formally defined as the difference between the vacuum energies in the presence and absence of external conditions. The latter may be understood as given geometrical boundary conditions imposed on the quantum fields. Since the vacuum structure of systems undergoing phase transitions strongly depends on the phase considered, the Casimir-energy could also differ in the various phases. In QCD and in nontrivial cases of other field theoretical models nonperturbative methods are needed to calculate this quantity. In the literature so far the Casimir energy has been analyzed extensively in continuum quantum field theory in the presence of external constraints [1, 2]. First attempts to explore the role of the Casimir effect on a lattice have been presented later for a simple scalar field theory [3]. However, in [3] the lattice was introduced only to obtain a finite set of Schroedinger equations and only the simplest, $D = 1 + 1$ dimensional problem was treated as a quantum-mechanical many-body system.

Within our ongoing project our goal is to develop a systematic field–theoretical method, based on Monte Carlo simulations on lattice, to calculate the Casimir energy in various field–theoretical models. Here we summarize the results of our method for noninteracting scalar fields. For these systems analytical approaches exist so a comparison of them to our results provides a test for the calculations on the lattice. The method, in principle, can be generalized to any lattice field–theoretical model, like QCD, which is physically more important.

As a generic scenario we imposed both Dirichlet boundary conditions and periodic boundary conditions to determine the Casimir-energy of a scalar field confined in the region between parallel plate configurations. A code was developed in order to calculate the stress-energy tensor numerically on a lattice using Monte Carlo simulations. We determined the Casimir-energy as a function of the plate separation for different lattice sizes between $24^3 \times 32$ and $40^3 \times 60$. We have shown that the behaviour of the Casimir energy calculated analytically can be qualitatively reproduced using lattice simulations in the simplest system of a massless scalar field, when periodic boundary conditions are applied. For Dirichlet boundary conditions, however, the effect cannot be seen due to statistical noise. Similar situation occurs for massive fields. Moreover, preliminary results show that with increasing mass the effect is becoming less prominent. Work is in progress aiming a more precise calculation for the above cases in lower dimensions. Later we plan to investigate physically more interesting cases, like strongly interacting nontrivial scalar fields and SU(2), SU(3) gauge fields.

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