IMPACT FRAGMENTATION OF PLATE-LIKE OBJECTS

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Abstract

We study the impact fragmentation of plate-like solids in three dimensions focusing on the spatial distribution and mass-velocity correlation of fragments. Based on large scale computer simulations we show that the position of fragments inside the original body with respect to the impact site determines their mass and velocity in the final state. A novel relation of the mass and velocity of fragments is revealed: In the damaged phase of breakup strong mass-velocity correlation is obtained, however, in the fragmented phase the correlation disappears. The correlation function decays as a power law with different exponents in the regime of small and large fragment masses separated by highly energetic detached pieces. Our results have interesting implications for the understanding of the formation of meteoroid clouds and planetesimal accretion in the solar system.

I. Introduction

Fragmentation, *i.e.* the breaking of particulate materials into smaller pieces is a ubiquitous process that underlies many natural phenomena and industrial processes. The length scales involved in it range from the collisional evolution of asteroids through the scale of geological phenomena down to the breakup of heavy nuclei. The most striking observation about fragmentation is that the size distribution of fragments shows power law behavior independent on the microscopic interactions and on the relevant length scales of the fragmenting system [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Most of the theoretical and experimental investigations over the past decades focused on the origin of the power law fragment size distribution and on the analogy of fragmentation to critical phenomena and phase transitions [1, 2, 3, 4, 5, 7, 8, 10, 11, 12]. Much less is known about the velocity of fragments and its relation to the fragment mass which play a crucial role to understand the formation of meteoroid clouds and planetesimal accretion in the solar system, furthermore, to describe the time evolution of space debris generated by on-orbit fragmentation events.

The experimental determination of fragment velocities requires 2-3 high speed cameras with appropriate triggering, hence, only recent technological achievements made possible to measure simultaneously the velocity of a large number of fragments in laboratory experiments. Kadono et al. investigated the impact fragmentation of a thin glass plate by means of the high speed imaging techniques [6]. In the experiments a high velocity pro-

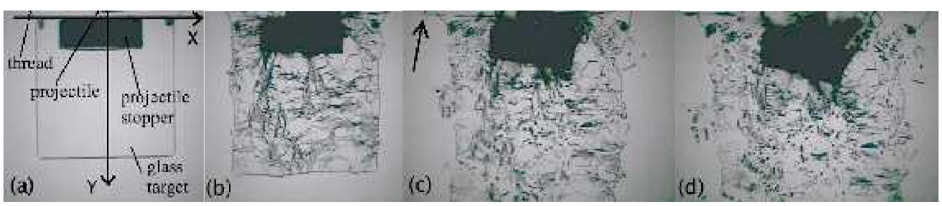

Figure 1: The experimental system with the thin glass plate. (a) is before, and (b..d) are 0.25, 0.5 and 1 ms after the impact.

jectile was shot into a Pyrex glass plate parallel to the plane of the glass. The penetration of the projectile into the plate was hindered by a stopping device so that the projectile only created a high pressure compressive wave in the material (see Fig. 1). The process of breakup was followed by two high speed cameras which made possible to identify individual fragment, to determine their velocity vectors and to track the fragments back to their original location inside the solid. As the main outcome of the experiments, no correlation was found between the mass and velocity of fragments, while a strong correlation was revealed between the velocity of fragments and their position inside the original body. Three dimensional fragmentation experiments have been performed by Nakamura et al. [13] where due to the limitations of the experimental techniques only large fragments could be captured with a reliable precision. Surprisingly, in the limit of large fragments a strong mass-velocity correlation was found which decays as a power law with a universal exponent 1/6 [13].

In the present paper we study the impact fragmentation of disordered solids in three dimensions focusing on the mass and velocity of fragments. We give numerical evidence that in the damage phase of the process fragment mass and velocity are correlated, however, in the fragmented phase no correlation can be pointed out. The correlation function has a power law decay, however, the exponent has different values for small and large fragments.

II. Model system

Recently, we have worked out a three-dimensional dynamical model of deformable, breakable granular solids, which enables us to perform molecular dynamics simulation of fracture and fragmentation of solids in various experimental situations [11, 12]. Our model is an extension of those models which are used to study the behavior of granular materials applying polydisperse spherical particles to describe grains [11, 12]. In the model the sample is represented as a random packing of spheres with a size distribution. The particles interact via the Hertz contact law when they are pressed against each other. Cohesive interaction is provided by beams which connect the particles along the edges of a Delaunay triangulation of the initial particle positions. In 3D the total deformation of a beam is calculated by the superposition of elongation, torsion, as well as bending and shearing in two different planes [11, 12]. The beams, modeling cohesive forces between grains, can be broken according to a physical breaking rule, which takes into account the stretching and bending of the connections. The breaking rule contains two parameters t_ϵ, t_Θ controlling the relative importance of the stretching and bending breaking modes, respectively. The energy stored in a beam just before breaking is released in the breakage giving rise to energy dissipation. The average value of the energy dissipated by the breakup of one contact defines the crack surface energy E_s in the model solid as a function of the breaking parameters t_ϵ, and t_Θ. At the broken beams along the surface of the spheres cracks are generated inside the solid and as a result of the successive beam breaking the solid falls apart. The fragments are defined as sets of discrete particles connected by remaining intact beams. The time evolution of the fragmenting solid is obtained by solving the equations of motion of the individual particles until the entire system relaxes, *i.e.* there is no breaking of the beams during some hundreds consecutive time steps and there is no energy stored in deformation. For more details of the model's definition see Refs. [11, 12].

In order to make a realistic implementation of the impact process, in the simulations a plate-like sample was constructed with rectangular basis of side length L and height H. Simulations were carried out with linear extensions $L = 50$ and $H = 5$ measured in units of the average particle diameter. The projectile was shot into the sample parallel to the plane of the plate. To capture the effect of the stopping device used in the experiments, a single surface particle was chosen in the middle of one of the sides of the solid, which got a large initial velocity pointing towards the center of mass of the body. Representative snapshots of the time evolution of the fragmenting system are presented in Fig. 2.

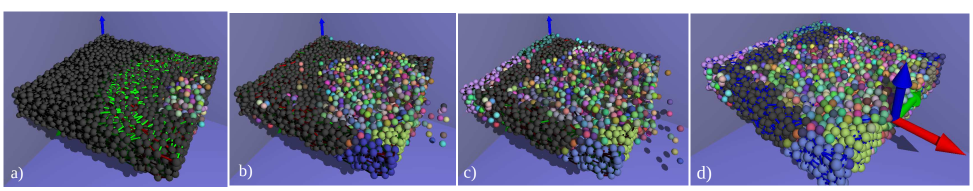

Figure 2: (a) Plate-like sample of size $L = 50$ and $H = 5$. Colored lines represent the stressed beams. Snapshots of the evolving system at (b) 7 time units and (c) 30 time units after impact. (d) In the final fragmented state we reassemble the system to identify the location of fragments in the original body.

III. Results

We performed a large number of simulations varying the impact velocity v_0 in a broad range at a fixed system size $L = 50$ and $H = 5$ (the number of spherical particles of the system was about 10000). The impacting sphere was chosen in the middle of the right hand side of the sample (see Fig. 2a). In order to improve the statistics, at each parameter set the simulations were repeated 1000 times with different realizations of the disordered microstructure. For a summary of the precise value of the material parameters of the model system see Refs. [11, 12]. In the final fragmented state of the system we determined the mass and velocity components of the fragments and the coordinates of their center of mass in the original body.

It can be seen in the snapshot of the final state of the solid (Fig. 2c) that the system is expanding, all the fragments are moving outward the solid. Already by pure eye a highly non-trivial spatial distribution of mass and velocity of fragments can be suspected: around the impact site the solid is completely destroyed, all the fragments are single spheres in this regime. Surprisingly, quite a large number of particles are moving opposite to the impact velocity, *i.e.* particles around the impact site are back-scattered. Due to wave interference, a surface layer of the body opposite to the impact

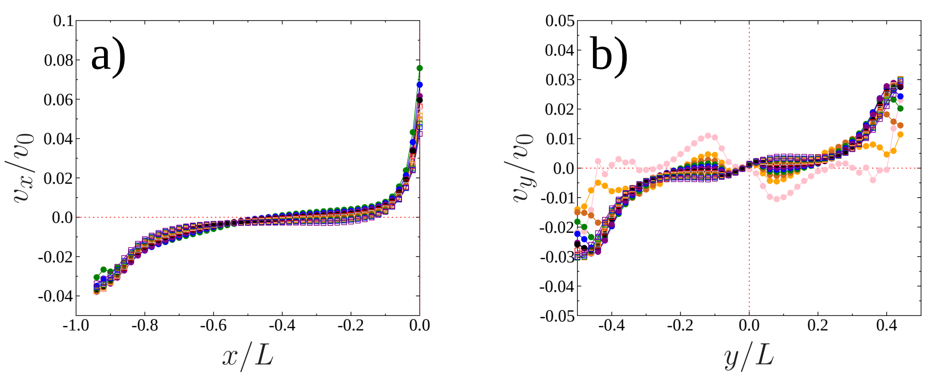

Figure 3: Velocity components of fragments as a function of their coordinates in the original body. For the definition of the coordinate system see Fig. 2(d).

point breaks off whose fragments fly at a relatively high speed. Inside the solid, however, the fragments are significantly larger than on the surface and move at a lower speed.

In order to give a quantitative characterization of the observed features, we determined the average value of the velocity components v_x and v_y of fragments as a function of their x and y coordinates in the original body.

It is interesting to note that inside the body there are fragments which practically do not move, *i.e.* their velocity is zero. Due to momentum conservation, $v_y(y)$ is a symmetric function with respect to $y = 0$. Hence, the fragment of zero velocity are along the line $y = 0$, while their x coordinate depends on the impact velocity v_0. It can be seen in Fig. 3 that both velocity components are monotonically increasing with the distance from the zero velocity fragments. Note that the back-scattered fragments and the ones detached along the surface of the body can even exceed the velocity of impact.

Fig. 2*c* shows that the mass of fragments strongly depends on their position inside the original solid. It is generally valid that fragments along

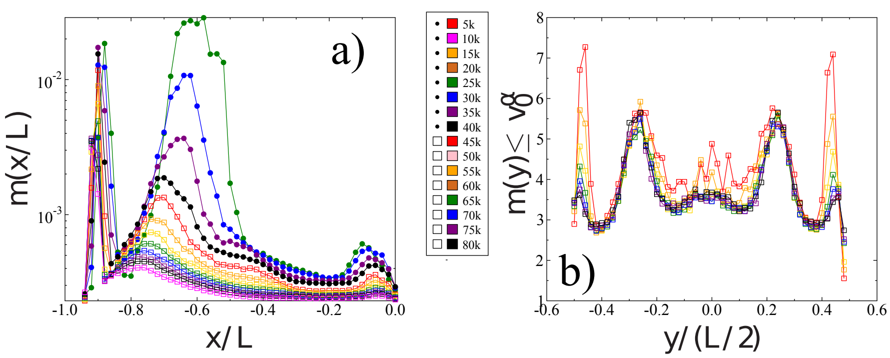

Figure 4: (a) Fragment mass as a function of the x (a) and y (b) coordinates in the original body at different impact velocities v_0. In (b) the two axis are rescaled.

the surface created either in the distractive zone around the impact site or by the detachment effect along the outer surface of the body, are small compared to the original size of the body. Large fragments could survive, however, inside the body. In Fig. 4*a* the mass of fragments is shown as a function of their x coordinate. Due to the single spheres of the destroyed zone the mass has low values at $x \approx 0$, while the detached fragments give rise to the peak at $x \approx -L$ (see Fig. 4*a*). It is important to note that inside the body the fragment mass has a sharp maximum which gets smoothened and shifted with increasing impact velocity v_0. Fig. 4(b) shows that the fragment mass as a function of the y coordinates is symmetric with respect to $y = 0$. Two peaks of $m(y)$ occur symmetrically whose position does not depend on the impact velocity giving rise to a simple scaling structure of $m(y)$

$$m(y) \sim v_0^{-\beta} g(y/L). \qquad (1)$$

For the exponent $\beta = 1.0 \pm 0.1$ was obtained and g denotes the scaling function. The good quality data collapse obtained with the transformation Eq. (1) is illustrated in Fig. 4*b*.

Fragment velocities are crucial to understand the secondary evolution of fragmenting systems, *i.e.* the effect of secondary collisions of fragments or the time evolution of fragments created by asteroid impacts under a gravitational field. Based on the above results, in our model system important conclusions can be drawn for the fragment velocities and for their relation to other characteristic quantities of fragments like mass and spatial position. In order to get information on the possible correlation of fragment mas

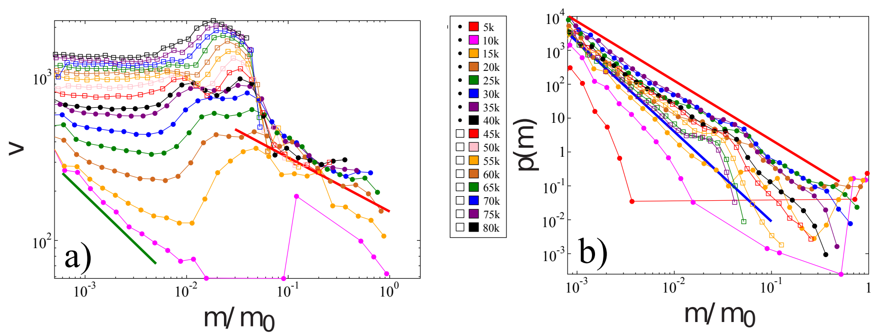

Figure 5: (*a*)Average fragment velocity as a function of fragment mass. (*b*) Fragment mass distributions $p(m)$ obtained at different impact velocities v_0. The slope of the upper and lower straight lines are 1.75 and 2.25, respectively.

and velocity, we calculated the average velocity of fragments with a given mass. It can be seen in Fig. 5*a* that in the fragmented phase (high impact velocities) for small fragment masses over almost two orders of magnitude the fragment velocity is constant, *i.e.* the mass and velocity of fragments is independent in this regime. In the damaged phase, however, the fragment velocity has a power law dependence on the mass

$$v \sim m^{-\gamma}, \tag{2}$$

indicating a strong mass-velocity correlation. The value of the exponent $\gamma \approx 2/3$ was obtained numerically for small fragments. It can be observed in the figure that for large fragments the exponent is different $\gamma \approx 1/3$. The two power law regimes are separated by a hump of detached fragments, indicated that these pieces have an anomalously high escape velocity. Our results imply that the discrepancy of experimental results on the mass-velocity correlation of fragments [6, 13] can arise from the difference of the imparted energy: correlations disappear at high energies where the solid is completely shattered.

It is important to note that irrespective of the presence or absence of mass-velocity correlations, the mass distribution of fragments is a continuous function which has a power law behavior (see Fig. 5*b*) as it is expected [11, 12].

IV. Summary

We presented a detailed study of the fragmentation of three-dimensional brittle solids focusing on the spatial distribution of fragment masses and velocities, and on their correlation. Computer simulations revealed that the position of fragments inside the original body with respect to the impact point determine the velocity and mass of fragments: the larger the distance is, the larger velocity the fragment attains in the final state. A scaling form was deduced characterizing the spatial distribution of fragment masses and velocities. The most important outcome of the present work is the relation of the mass and velocity fragments: we give numerical evidence that in the fragmented regime the velocity is independent of the mass, however, in the damaged phase a strong correlation emerges which has a power law decay. Our result provide a comprehensive explanation of recent contradictory experimental observations.

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