TIME SERIES ANALYSIS OF CREEP RUPTURE

Zs. Danku and F. Kun

[1]Department of Theoretical Physics, University of Debrecen,
H-4010 Debrecen, P. O. Box: 5, Hungary

Abstract

Based on a fiber bundle model of creep rupture we demonstrate that fracture processes exhibit strong analogies to earthquakes: interpreting the macroscopic rupture as a main shock, the preceding event series of the nucleation and propagation of cracks can be described as a sequence of foreshocks. Our simulations revealed that approaching macroscopic rupture the increasing rate of crackling events obeys the Omori law and the entire time series can be described as a non-homogeneous Poisson process.

I. Introduction

The fracture of heterogeneous materials proceeds in bursts which can be recorded in the form of crackling noise by acoustic or electromagnetic measuring techniques. Crackling noise is the primary source of information about the microscopic dynamics of fracture: the measured signal is typically decomposed into a trail of pulses which can be assigned to elementary events of crack nucleation and propagation. Characteristic quantities of pulses such as amplitude, area, or energy provide a measure of the size of events, while, the pulse duration and the time elapsed between consecutive pulses reveals the internal dynamics of the process. Due to the disordered micro-structure of materials, characteristics of crackling noise exhibit strong fluctuations so that meaningful description can only be achieved in terms of their probability distributions. During the last two decades fracture experiments have revealed that crackling noise is characterized by power law distributions of burst sizes and of the waiting times between consecutive bursts. Creep and quasi-static loading experiments provided exponents in the range $1-2.5$ and $1-2$ for burst sizes and waiting times [1, 2, 3], respectively, which motivated a large amount of theoretical investigations. However, practically all these studies evaluated integrated distributions: events were accumulated from the beginning of measurements up to macroscopic failure washing out all information about the time evolution of the fracture process. The approach to macroscopic failure was characterized by the functional form of the cumulative dissipated energy which revealed a time-to-failure power law [3].

In the present project we demonstrate that due to the non-stationarity of the fracture process comprehensive description of crackling noise can only be obtained by analyzing the evolution of the time series of events as the system approaches macroscopic failure. Based on analytic calculations and computer simulations of a fiber bundle model we show that the creep rupture of heterogeneous materials exhibits a strong analogy to earthquakes: identifying the global rupture of the sample as the main shock, the preceding breaking avalanches can be described as a foreshock sequence. Our calculations revealed that in the mean field limit the time evolution of creep rupture can be described as a non-homogeneous Poisson-process, where the rate of events obeys the Omori law with a high precision.

II. Fiber bundle model

To investigate the creep rupture of heterogeneous materials we use a fiber bundle model, composed of N parallel fibers having a brittle response with identical Young modulus E (Fig. $1(a,b)$) [4, 5, 6]. It is a crucial element of the model that under a constant subcritical external load σ_0 the fibers break due to two physical mechanisms: immediate breaking occurs when the local load σ_i on fibers exceeds their fracture strength σ_{th}^i, $i = 1, \ldots, N$, which is considered to be a random variable. We assume that those fibers, which remained intact, undergo an aging process accumulating damage $c(t)$. The rate of damage accumulation Δc_i is assumed to have a power law dependence on the local load $\Delta c_i = a\sigma_i^\gamma \Delta t$, where a is a constant and the exponent γ controls the time scale of the accumulation process. The total amount of damage $c_i(t)$ accumulated up to time t is

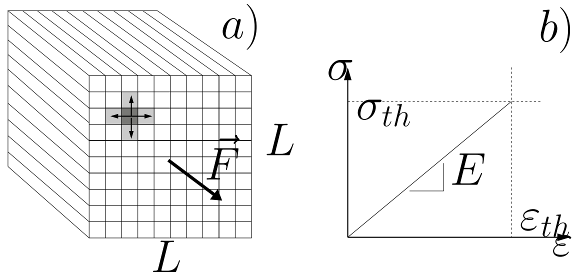

Figure 1: (a) The fiber bundle is composed of N parallel fibers. The constant external load parallel to the fibers. [8]. (b) The fibers have a brittle response [8].

obtained by integrating over the entire loading history of fibers $c_i(t) = a\int_0^t \sigma_i(t')^\gamma dt'$. Fibers can tolerate only a finite amount of damage so that when $c_i(t)$ exceeds a local damage threshold c_{th}^i the fiber breaks. The two breaking thresholds c_{th}^i and σ_{th}^i, $i = 1, \ldots, N$ of fibers are independent of each other being uniformly distributed between 0 and 1. Each breaking event is followed by a redistribution of load over the remaining intact fibers. For simplicity, we consider only equal load sharing, i.e. intact fibers share always the same load $\sigma(t) = N\sigma_0/[N - N_b(t)]$, where $N_b(t)$ denotes the number of fibers broken up to time t. It has been shown in Refs. [4, 5, 6] that the separation of time scales of the slow damage process and of immediate breaking leads to a highly complex time evolution: damaging fibers break slowly one-by-one, gradually increasing the load on the remaining intact fibers. After a certain number of damage breakings the load increment becomes sufficient to induce the immediate breaking of a fiber which in turn triggers an entire burst of immediate breakings. As a consequence, the time evolution of creep rupture occurs as a series of bursts corresponding to the nucleation and propagation of cracks, seperated by silent periods of slow damaging. An example of the time series of bursts is presented in Fig. 2 for a small system of $N = 100000$ fibers.

III. Results

In order to give a quantitative characterization of the time evolution of

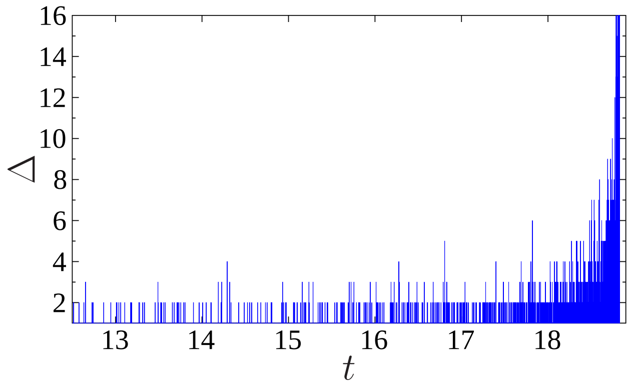

Figure 2: Time series of bursts in a relatively small bundle of $N = 100000$ at a load $\sigma_0/\sigma_c = 0.05$ for the damage accumulation exponent $\gamma = 1$. Approaching macroscopic failure the rupture process accelerates.

the system we determined the rate of bursts n as a function of time. In Fig. 3 the event rate $n(t)$ is presented as a function of the distance from the critical point $t_f - t$. It can be observed that at the beginning of the creep process the system accelerates, i.e. the rate of bursts monotonically increases having a power law functional form. As the system approaches catastrophic failure, the event rate $n(t)$ saturates and converges to a constant. The functional form of $n(t)$ can be described by the Omori law

$$n(t) = \frac{A}{\left(1 + \frac{t_f - t}{c}\right)^p}, \tag{1}$$

where A is the saturation rate at catastrophic failure, c denotes the characteristic time scale, and p is the Omori exponent. In the case of earthquakes, the Omori law describes the relaxation process following major earthquakes. For creep rupture we observe the inverse process: considering the macroscopic failure as the main shock, the breaking bursts are foreshocks whose increasing rate is described by the (inverse) Omori law.

A more detailed characterization of the statistics of the appearance of breaking bursts can be obtained by determining the probability distribution $P(T)$ of waiting times T. Figure 4 presents waiting time distributions

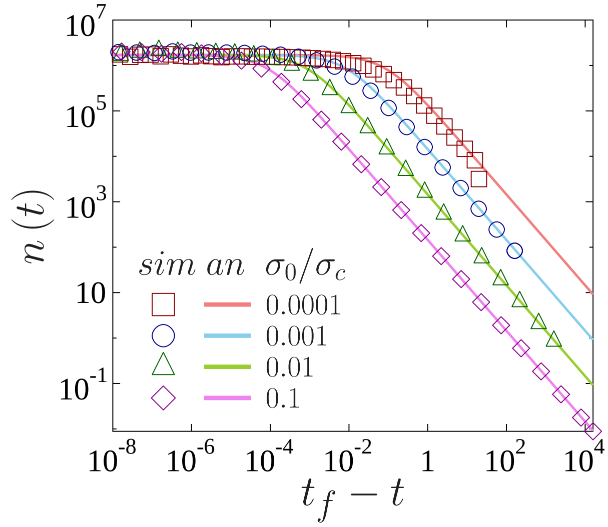

Figure 3: Event rate n as a function of the distance from the critical point $t_f - t$ for several different load values. The lines represent fits with the Omori law.

corresponding to the event rates of Fig. 3. It can be seen that along the distributions two characteristic time scales can be identified: for waiting times below a threshold $T < T_l$, the distributions have constant values, while in the limit of large waiting times $T > T_u$ a rapidly decreasing exponential form is obtained. For the intermediate regime $T_l < T < T_u$ the waiting time distributions exhibit a power law behavior

$$P(T) \sim T^{-z}. \tag{2}$$

where the exponent was determined numerically $z = 1$. Increasing the external load σ_0 the upper cutoff of the distribution shifts downwards, i.e. T_u decreases, however, both the lower characteristic time T_l and the exponent z proved to be independent of σ_0, furthermore, z does not depend on γ either.

The functional form of the rate of bursts, i.e. the Omori law describing the acceleration of the system towards macroscopic failure suggests that the time evolution of crackling noise of heterogeneous materials undergoing

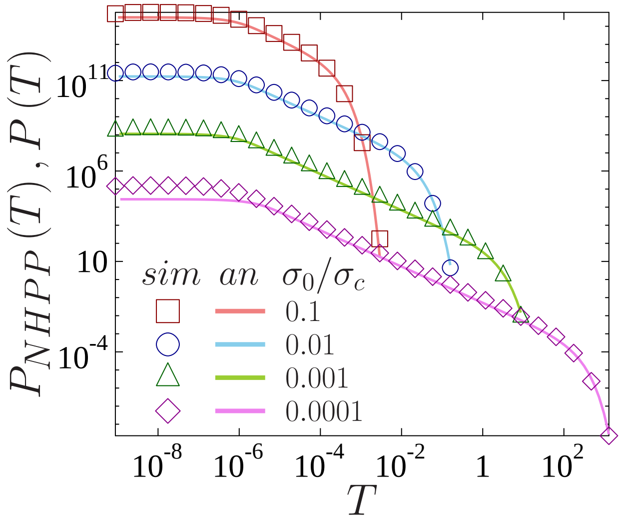

Figure 4: Waiting time distributions obtained from the simulations (symbols) and from the analytic formula of non-homogeneous Poissonian processes. Very good agreement is obtained at all load values. The curves of different load values are vertically shifted to separate them for clarity.

creep rupture can be described as a non-homogeneous Poissonian process (NHPP). For NHPP the waiting time distribution of the event series of duration t_f can be obtained analytically starting from the event rate [7] as

$$
\begin{aligned}
P(T) &= \frac{1}{N_\Delta}\int_0^{t_f-T} n(s)n(s+T)e^{-\int_s^{s+T} n(u)du} ds \\
&+ n(T)e^{-\int_0^T n(s)ds}. \qquad (3)
\end{aligned}
$$

To verify the consistency of the NHPP picture for the creep rupture of fiber bundles under equal load sharing conditions, first we fitted the event rate functions by the Omori law determining the value of the parameters A, c, and p. Then the analytic form Eq. (1) of the event rates $n(t)$ with the numerical parameters was plugged into the integral expression Eq. (3) and the integral was calculated numerically taking into account the load dependent lifetime $t_f(\sigma_0)$ of the sample. In Fig. 4 an excellent agreement

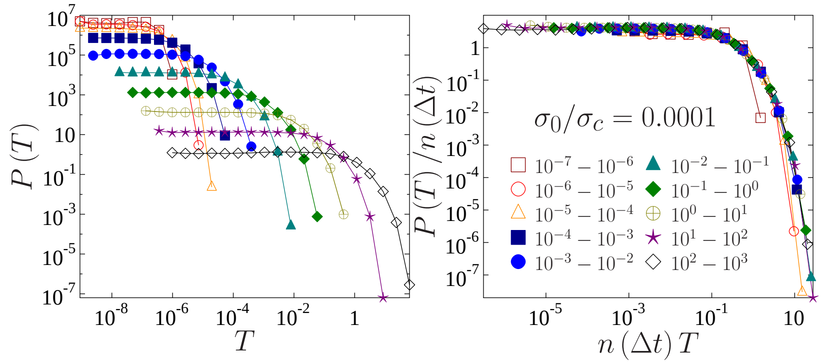

Figure 5: (a) Waiting time distributions obtained for bursts in narrow time windows Δt during the rupture process. (b) The curves can be collapsed on the top of each other by rescaling with the average waiting time of the window $1/n(\Delta t)$.

can be observed between the waiting time distributions obtained from the simulations of creep rupture and the analytic prediction of Eq. (3). Based on the NHPP nature of the time series we can understand the cutoff values T_l and T_u of waiting times: the lower cutoff T_l is determined by the saturation event rate

$$T_l = 1/A, \tag{4}$$

which does not depend on the external load. The upper cutoff T_u is determined by the characteristic time scale c, by the total duration of the time series, i.e. the lifetime of the sample, and by the Omori exponent in the form

$$T_u = \frac{1}{A}\left(\frac{t_f}{c}\right)^p. \tag{5}$$

An interesting consequence of the above arguments is that if we select bursts in narrow time windows Δt during creep rupture, the series of bursts should behave as a homogeneous Poissonian process, i.e. the waiting time distributions should have an exponential form. It is illustrated in Fig. 5(a) where all waiting time distributions $P(T)$ are exponentials. It can also be observed in Figure 5(b) that the distributions obtained at different times along the time evolution, can be collapsed on a master curve just by rescaling with the average waiting time $1/n(\Delta t)$.

IV. Conclusions

Using a fiber bundle model of creep rupture of heterogenous materials we gave numerical evidence that the time series of micro-fracturing events can be described as a non-homogeneous Poisson process, where the event rate obeys (inverse) Omori scaling. The results suggest that the analogy of rupture phenomena with earthquakes goes beyond the Gutenberg-Richter law of event magnitudes, i.e. considering macroscopic failure as a main shock, all preceeding fracturing events can be interpreted as a sequence of foreshocks. For the consistency of the NHPP picture we demonstrated that the waiting time distributions obtained from computer simulations have a perfect agreement with the analytic results of NHPPs using the functional form of the inverse Omori law for the event rates. In sufficiently small time windows we pointed out that the probability distribution of waiting times has an exponential form. It has the consequence that the power law form of the distribution measured for a wide variety of materials may not reveal any dynamical correlation, it is just the consequence of the summation of exponentials.

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