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SIMPLE ANALYTIC EXPRESSIONS FOR THE POLE PARTS OF DOUBLE VIRTUAL CORRECTION TO $e^+e^- \to 3$ PARTONS

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Abstract

We present simple formulae for the pole part contribution of the interference of two-loop and tree amplitudes to the two-loop QCD matrix element for $e^+e^- \to 3$ jets. The complexity of our formulae and computational cost are significantly smaller then in the case of the original formulae.

I. Introduction

The cross section of the m-jet at next-to-next-to-leading order (NNLO) accuracy is a sum of leading order (LO), next-to-leading order (NLO) and NNLO correction terms,

$$\sigma = \sigma^{\mathrm{LO}} + \sigma^{\mathrm{NLO}} + \sigma^{\mathrm{NNLO}} . \tag{1}$$

The NNLO correction is a sum of the doubly-real, the one-loop singly-unresolved real-virtual and the two-loop doubly-virtual terms,

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{VV}}_{m} J_{m} . \tag{2}$$

Here J_m is the jet function that defines the physical quantity. It must be IR safe, see Eqs. (9.7), (9.8) and (9.11–9.14) in Ref. [1]. The terms $\mathrm{d}\sigma^{\mathrm{RR}}_{m+2}$ and $\mathrm{d}\sigma^{\mathrm{RV}}_{m+1}$ are defined in Eq. (4.1) of Ref. [2] and in Eq. (2.19) of

Ref. [3], respectively. Here we focus on the squared matrix element, which was already presented in Ref. [12], of the double virtual contribution $\mathrm{d}\sigma^{VV}_{m=3}$.

In this paper we make a short contribution to the implementation of a new NNLO subtraction algorithm, which was sugested in [1]-[9], to the specific case of three jet production in electron-positron annihilation. In the framework of this scheme the NNLO correction to the m-jet cross section can be written as follows (see Eq. (3.4) and Eq. (3.7) in [2]).

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m} \tag{3}$$

where the last term is:

$$\begin{aligned}\mathrm{d}\sigma^{\mathrm{NNLO}}_{m} &= \Big\{\mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \Big[\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2}\Big] + \\ &\quad \int_1 \Big[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \Big(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2}\Big)^{A_1}\Big]\Big\}_{\epsilon=0} J_m \,. \end{aligned}\tag{4}$$

Definitions of the subtraction terms $\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2}$, $\mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2}$ are presented in Ref. [2], while $\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1}$ and $\Big(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2}\Big)^{\mathrm{A_1}}$ are defined in Ref. [3]. After integrating over the phase space of the unresolved parton(s), these can be written in the following way, see Eqs. (4.24), (5.21) of Ref. [4], Eq. (3.13) of Ref. [7] and Eq. (3.5) of Ref. [11],

$$\int_1 \mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} = \mathrm{d}\sigma^{\mathrm{V}}_{m} \otimes \mathbf{I}^{(0)}_1(\{p\}_m;\epsilon) + \mathrm{d}\sigma^{\mathrm{B}}_{m} \otimes \mathbf{I}^{(1)}_1(\{p\}_m;\epsilon) \tag{5}$$

$$\begin{aligned}\int_1 \Big(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2}\Big)^{\mathrm{A_1}} &= \mathrm{d}\sigma^{\mathrm{B}}_{m} \otimes \Big[\frac{1}{2}\Big\{\mathbf{I}^{(0)}_1(\{p\}_{m+1};\epsilon), \mathbf{I}^{(0)}_1(\{p\}_{m+1};\epsilon)\Big\} + \\ &\quad \mathbf{I}^{(0,0)}_{1,1}(\{p\}_{m+1};\epsilon)\Big]\end{aligned}\tag{6}$$

$$\int_2 \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} = \mathrm{d}\sigma^{\mathrm{B}}_{m} \otimes \mathbf{I}^{(0)}_{12}(\{p\}_m;\epsilon) \tag{7}$$

$$\int_2 \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} = \mathrm{d}\sigma^{\mathrm{B}}_{m} \otimes \mathbf{I}^{(0)}_{2}(\{p\}_m;\epsilon) \tag{8}$$

Each term on the right-hand side of Eq. (4) contains an insertion operator (in colour space) $\mathbf{I}(\epsilon)$ that can be written as a Laurent expansion in the dimensional regularisation parameter $\epsilon = (4-d)/2$ containing poles $1/\epsilon^i$, $i = 1, \ldots 4$. As the jet function J_m has to be IR safe, hence according to the Kinoshita-Lee-Nauenberg theorem, when we collect epsilon coefficients with a negative power, those coefficients have to vanish. In other words, poles are expected to cancel each other and only the finite part will remain. Terms proportional to ϵ^n, for $n > 0$, are out of interest, because after cancellation of poles we set $\epsilon = 0$. Our aim is to verify the cancellation explicitly for our specific case, hence it is useful to have an epsilon expansion of each term in Eq. (4). A numerical implementation of the epsilon expansion of insertion operators $\mathbf{I}_1^{(0)}$, $\mathbf{I}_1^{(1)}$, $\mathbf{I}_{1,1}^{(0,0)}$, $\mathbf{I}_{12}^{(0)}$, $\mathbf{I}_2^{(0)}$ is available, as well as an analytical implementation for the two highest poles. To reach our goals, we need to investigate the structure of $\mathrm{d}\sigma^{\mathrm{VV}}_{m=3}$, which led us to find very simple formulae of the pole part of the two-loop contribution to the squared matrix element for the $\gamma^* \to q\bar{q}g$ process.

II. Pole part of the interference of two-loop and tree amplitudes

The computation of a perturbative expansion of a squared amplitude of a virtual photon into $q\bar{q}g$ at $O(\alpha_S^3(q^2))$ can be found in [12], where the squared amplitude is denoted by $T(x,y,z)$ (see Eq. (2.6) in Ref. [12]).

$$\langle M \,|\, M \rangle = T(x,y,z) \tag{9}$$

The perturbative expansion is given in Eqs. (2.7) and (2.13) of Ref. [12],

$$\begin{aligned} T(x,y,z) \;=\;& 16\pi^2\alpha \sum_q e_q^2 \alpha_S(\mu^2) \Bigg\{ T^{(2)}(x,y,z) \\ &+ \left(\frac{\alpha_S(\mu^2)}{2\pi}\right) \left[T^{(4)}(x,y,z) + b_0 T^{(2)}(x,y,z) \ln\left(\frac{\mu^2}{q^2}\right) \right] \\ &+ \left(\frac{\alpha_S(\mu^2)}{2\pi}\right)^2 \Bigg[T^{(6)}(x,y,z) + \Bigg(2b_0 T^{(4)}(x,y,z) + \\ & b_1 T^{(2)}(x,y,z) \Bigg) \ln\left(\frac{\mu^2}{q^2}\right) + b_0^2 T^{(2)}(x,y,z) \ln^2\left(\frac{\mu^2}{q^2}\right) \Bigg] \\ &+ O(\alpha_S^3) \Bigg\} \end{aligned} \tag{10}$$

where $T^{(2)}$, $T^{(4)}$ and $T^{(6)}$ in the equation above are defined in Eqs. (2.8–2.10) of Ref. [12], e_q is the fractional electric charge of quark q, α and α_S are QED and strong couplings, the latter in the $\overline{\text{MS}}$ renormalisation scheme, respectively, while μ is the renormalisation scale. The constants b_0, b_1 are known coefficients of the two-loop beta functions and q^2 the total centre-of-mass energy squared. Eq. (1) is expansion in the strong coupling, where σ^{NNLO} is proportional to α_S^3, for $m = 3$. Then from Eq. (10) we see that

$$\begin{aligned} \mathrm{d}\sigma_{m=3}^{\text{VV}} \quad \propto \quad & \left(\frac{\alpha_S(\mu^2)}{2\pi}\right)^3 \Bigg[T^{(6)}(x,y,z) + \Big(2b_0 T^{(4)}(x,y,z) + b_1 T^{(2)}(x,y,z)\Big) \\ & \ln\left(\frac{\mu^2}{q^2}\right) + b_0^2 T^{(2)}(x,y,z) \ln^2\left(\frac{\mu^2}{q^2}\right)\Bigg] \,. \end{aligned} \tag{11}$$

The matrix element can be decomposed into infrared poles and finite parts. The function $T^{(6)}(x,y,z)$ can be written as

$$T^{(6)}(x,y,z) = T^{(6,[2\times 0])} + T^{(6,[1\times 1])}\,, \tag{12}$$

where the contribution from the interference of two-loop and tree amplitudes can be decomposed into a sum of its finite and pole parts,

$$T^{(6,[2\times 0])} = Poles^{(2\times 0)} + Finite^{(2\times 0)}\,. \tag{13}$$

The same holds for one-loop self-interference

$$T^{(6,[1\times 1])} = Poles^{(1\times 1)} + Finite^{(1\times 1)}\,. \tag{14}$$

In the following we focus on $Poles^{(2\times 0)}$, which is defined in Eqs. (4.2–4.14) of Ref. [12] using master integrals defined in Eqs. (A.1–A.10) of Ref. [12]. The pole parts $Poles^{(2\times 0)}$, as presented originally in Ref. [12], is a non trivial function originally built from one and two-dimensional harmonic polylogarithms up to weight 4. The harmonic polylogarithms are defined in [15].

III. ϵ-expansion of $Poles^{(2\times 0)}$

Based on the explicit formulae in Ref. [12], we implemented $Poles^{(2\times 0)}$ in `Mathematica`. Using our `Mathematica` code we generated a Laurent-expansion in ϵ for $Poles^{(2\times 0)}$ about the point $\epsilon = 0$ to order 0,

$$Poles^{(2\times 0)} = \sum_{i=-4}^{0} c_i^{Poles(2\times 0)} \epsilon^i + o(\epsilon) \tag{15}$$

where the coefficients can be written in a compact form

$$c_j^{Poles(2\times 0)} = A_j C_j \tag{16}$$

with coefficients

$$A_{-4} = -\frac{2312}{9yz},\ A_{-3} = -\frac{68}{9yz},\ A_{-2} = \frac{2}{27yz}\,, \tag{17}$$

$$A_{-1} = \frac{1}{27yz},\ A_0 = -\frac{2}{405yz} \tag{18}$$

and fairly simple analytic functions

$$C_{-4} = 2x + y^2 + z^2\,, \tag{19}$$

$$\begin{aligned} C_{-3} &= 4\left(2x + y^2 + z^2\right)(\log(x) - 9\log(yz)) - \\ &\quad 110x - 89y^2 - 68yz - 89z^2\,, \end{aligned} \tag{20}$$

$$\begin{aligned} C_{-2} &= 4(2x + y^2 + z^2)\Big(6\log(x)\big(8\log(x) + 9\log(yz)\big) - 54\big(9\log(y) \\ &\quad \log(z) + 13\log^2(y) + 13\log^2(z)\big) + 1955\pi^2\Big) + 48\log(x) \\ &\quad \big(-9x + 4y^2 + 17yz + 4z^2\big) - 24\big(-128x + 89y^2 + 306yz + \\ &\quad 89z^2\big)\log(yz) - 1862x - 6541y^2 - 11220yz - 6541z^2 \end{aligned} \tag{21}$$

$$\begin{aligned} C_{-1} &= 8\left(2x + y^2 + z^2\right)\Big(-\log(x)\big(14\log^2(x) + 27\log^2(y) + \\ &\quad 27\log^2(z) - 230\pi^2\big) + 9\log(yz)\big(-3\log^2(x) - 17\log(y)\log(z) \\ &\quad +44\log^2(y) + 44\log^2(z)\big) + 578\zeta(3)\Big) - 4\big(15405x + 7237y^2 - \\ &\quad 931yz + 7237z^2\big) - 24\log(yz)\Big(4\log(x)\big(-4y^2 + y(9z+17) + \\ &\quad z(17-4z) - 17\big) + 2\big(690\pi^2 - 319\big)x + 690\pi^2\left(y^2 + z^2\right) - 191y^2 \\ &\quad +z(256y - 191z)\Big) + 12\log^2(x)\big(5y^2 - 2y(32z+37) + z(5z - \\ &\quad 74) + 74\big) - 8\log(x)\big(49y^2 - 2y(54z+103) + z(49z-206) + \\ &\quad 206\big) + 23\pi^2\big(231y^2 - 2y(680z+911) + z(231z-1822) + 1822\big) \\ &\quad -12\big(155y^2 - 2y(468z+623) + z(155z-1246) + 1246\big)\big(\log^2(y) \\ &\quad + \log^2(z)\big) - 432\big(7y^2 - 2y(9z+16) + z(7z-32) + 32\big) \\ &\quad \log(y)\log(z)\,, \end{aligned} \tag{22}$$

$$
\begin{aligned}
C_0 \;=\;& 6\left(2x+y^2+z^2\right)\Bigg(-680\zeta(3)(\log(x)-9\log(yz))-90\log^3(x)\\
&\log(yz)-90\log(x)\big(-13\pi^2\log(yz)+\log^3(y)+\log^3(z)\big)+5\log^2(x)\\
&\big(-27\log^2(y)-27\log^2(z)+226\pi^2\big)-25\log^4(x)+45\Big(18\log^3(y)\\
&\log(z)+\log^2(y)\big(27\log^2(z)-356\pi^2\big)+18\log(y)\log(z)\big(\log^2(z)-\\
&13\pi^2\big)+40\log^4(y)+40\log^4(z)-356\pi^2\log^2(z)\Big)+10336\pi^4\Bigg)-\\
&5\Big(86492x+89461y^2-\pi^2\big(62859yz+y(7316y+48227)+7316z^2+\\
&48227(z-1)\big)+92430yz+89461z^2\Big)-15\log^3(x)\big(-94y^2+\\
&4y(28z+75)-94z^2+300(z-1)\big)-15\log^2(x)\big(1586x+571y^2-\\
&444yz+571z^2\big)+45\log^2(z)\Big(4010x-36\big(7y^2-2y(9z+16)+\\
&z(7z-32)+32\big)\log(y)+759y^2-2492yz+759z^2\Big)-15\log^2(y)\\
&\Big(2\big(-1584yz+y(821y-3226)+821z^2-3226(z-1)\big)\log(y)-\\
&3\big(4010x+759y^2-2492yz+759z^2\big)\Big)-60\log(x)\Big(-388x-91y^2+\\
&\pi^2\big(-460yz+y(227y-914)+227z^2-914(z-1)\big)+206yz-\\
&91z^2\Big)-90\log(y)\log(z)\Big(550x+18\big(7y^2-2y(9z+16)+z(7z-32)+\\
&32\big)\log(y)+851y^2+1152yz+851z^2\Big)+30\log(yz)\Bigg(2\Big(-3032x-\\
&559y^2+4\pi^2\big(-1035yz+y(451y-1937)+451z^2-1937(z-1)\big)+\\
&1914yz-559z^2\Big)+3\log(x)\big(58x+97y^2+136yz+97z^2\big)\Bigg)+\\
&360\log(x)\big(4y^2-y(9z+17)+z(4z-17)+17\big)\big(\log(x)\log(yz)+\\
&\log^2(y)+\log^2(z)\big)-30\big(821y^2-2y(792z+1613)+z(821z-3226)+\\
&3226\big)\log^3(z)-30\zeta(3)\big(-2312yz+y(14121y-30554)+14121z^2-\\
&30554(z-1)\big)\,.
\end{aligned}
$$

The variables x, y, z are difined in Eq. (2.4) in [12].

IV. Check

We implemented $Poles^{(2\times0)}$, based on Ref. [12], twice. Our first implementation was done in `Fortran` using the subroutine `hplog` the numerical evaluation of harmonic polylogarithms, described in Ref. [14]. The second implementation is in `Mathematica`, where harmonic polylogarithms are computed analytically from their definitions. Our codes provide same output for any input with chosen precision. In our `Mathematica` code one can simply perform an expansion of our implementation of $Poles^{(2\times0)}$ at some specific point and by this we obtain numerical coefficients of the epsilon expansion in such a particular case. Finally, we implemented our simple formula, which we provide in this paper, of epsilon coefficients of $Poles^{(2\times0)}$. We verified that it provides the same numerical values as the implementations in `Fortran`.

V. Conclusion

We studied the pole part contribution of the interference of two-loop and tree amplitudes, $Poles^{(2\times0)}$, to the two-loop QCD matrix element for $e^+e^- \to 3$ jets. In particular we found that epsilon expansion of function $Poles^{(2\times0)}$ is a simple formula that is composed of polynomials and logarithms. Thus instead of an implementation of a complicated and costly formulae, given in Ref. [12] and containing one and two-dimensional harmonic polylogarithms up to weight 4, it is sufficient to use our simple formulae that are free of harmonic polylogarithms. We also investigated $Poles^{(1\times1)}$, but we did not find such a remarkable simplification, nevertheless some was achieved. We discussed an implementation of a new NNLO subtraction algorithm to the specific case of three jet production in electron-positron annihilation, where we will use our simple formulae of $Poles^{(2\times0)}$.

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