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EFFECT OF SPATIAL DIMENSION ON IMPACT FRAGMENTATION

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Abstract

We study the impact fragmentation of heterogeneous materials in three dimensions varying the shape of the specimen from quasi two-dimensional plates to three-dimensional cubic bulk objects. Based on large scale molecular dynamics simulations we show that the damage-fragmentation transition prevails for each shapes, however, the mechanism of breakup and the resulting mass distribution of fragments have strong dimensional dependence. For plate-like objects in the fragmented regime power law mass distributions are obtained, however, the exponent is not universal, it increases with the imparted energy. For three-dimensional bulk specimens the energy dependence gradually disappears and a universal mass distribution emerges. Our detailed analysis revealed that the breakup process is the result of three competing mechanisms which all lead to universal mass distributions, however, their overall contributions change with the impact energy.

I. Introduction

Fragmentation phenomena are ubiquitous in nature and they are exploited in the industry, e.g. for ore processing. The length scales involved in it range from the collisional evolution of asteroids through the scale of geological phenomena down to the breakup of heavy nuclei. The most striking observation about fragmentation is that the size distribution of fragments shows power law behavior independent on the microscopic interactions and on the relevant length scales of the fragmenting system [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. The exponent of the fragment mass distribution is mainly determined by the dimensionality of the system based on which fragmenting systems have been organized into universality classes. Most of the theoretical and experimental investigations over the past decades focused on the origin of the power law fragment size distribution and on the analogy of fragmentation to critical phenomena and phase transitions [1, 2, 3, 4, 5, 6, 7, 9, 10, 11]. However, recent experiments and computer simulations questioned the universality of fragment mass distributions: it was found that the exponent of the distribution increases with the imparted energy.

In the present project we want to settle this debate by investigating the impact induced breakup of heterogeneous materials with brittle response. Based on a discrete element model we investigate how the breakup mechanism of solids changes when the shape of the object changes from quasi two-dimensional plates to nearly cubic shaped bulk objects. We give numerical evidence that the fragment mass distribution is determined by three mechanisms which get gradually activated as the impact energy increases. In all the cases universal power law fragment mass distributions are obtained, however, the interplay of the three mechanisms gives rise to an energy dependent overall behavior.

II. Discrete element model of brittle materials

In the model the sample is represented as a random packing of spherical particles with a log-normal size distribution [10, 11, 12]. The interaction of contacting particles is described by the Hertz contact law. Cohesive interaction is provided by beams which connect the particles along the edges of a Delaunay triangulation of the initial particle positions. In 3D the total deformation of a beam is calculated as the superposition of elongation, torsion, as well as bending and shearing [10, 11, 12]. Crack formation is captured such that the beams, modeling cohesive forces between grains, can be broken according to a physical breaking rule, which takes into account the stretching and bending of beams

$$\left(\frac{\varepsilon_{ij}}{\varepsilon_{th}}\right)^2 + \frac{\max\left(|\Theta_i|, |\Theta_j|\right)}{\Theta_{th}} > 1, \tag{1}$$

where the two parameters ε_{th} and Θ_{th} controll the relative importance of the two breaking modes. Here ε_{ij} denotes the axial strain, while Θ_1 and Θ_2 are the bending angles of the beam ends. In the model there is only structural disorder present: the breaking thresholds are constants, however, the physical properties of beams are determined by the random particle packing. The energy stored in a beam just before breaking is released in the breakage giving rise to energy dissipation. At the broken beams along the surface of the spheres cracks are generated inside the solid and as a result of the successive beam breaking the solid falls apart. The fragments are defined as sets of discrete particles connected by remaining intact beams. The time evolution of the fragmenting solid is obtained by solving the equations of motion of the individual particles until the entire system relaxes meaning that no beam breaking occurs during some hundreds consecutive time steps and there is no energy stored in deformation. For more details of the model construction see Refs. [10, 11, 12]. The shape of the sample was controlled in the simulations such that random particles packings were constructed on a rectangular basis of side length $L = 30$ and the height of the sample H was changed from $H = 3$ to $H = 15$, where all lengths are given in units of the average particle diameter $\langle D \rangle$. A single surface particle was selected in the middle of one of the side walls of the sample, which together with its contacting neighbors got a large initial velocity v_0 pointing towards the center of mass of the body [10, 11, 12].

III. Results

We performed a large number of simulations varying the impact velocity v_0 in a broad range for all values of the specimen height H. The impact velocity was varied in the range $v_0/c = 0.03-0.5$, where c denotes the sound speed of the material. The number of spherical particles of the system was about 15000. In order to improve the statistics, at each parameter set the simulations were repeated 1000 times with different realizations of the disordered microstructure [10, 11, 12]. The final state of the breakup of a

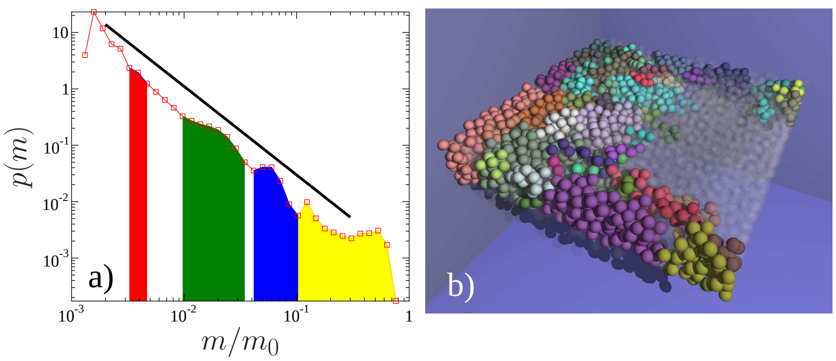

Figure 1: (a) Mass distribution $p(m)$ of fragments at the critical point of damage-fragmentation transition. The straight line represents a power law of exponent 1.8. (b) In the final fragmented state we reassemble the system to identify the location of fragments in the original body. Particles are colored according to the fragment they belong to. The colors in (a) identify different spatial regions of the specimen in (b) which give the dominant contribution to that range of $p(m)$.

plate-like object is presented in Fig. 1(b) for the height $H = 5$. Note that the specimen has been reassembled in the figure, i.e. the particles are placed back to their original position to have a better view on the fragments and cracks. Figure 2 presents the mass distribution $p(m)$ of fragments at several impact velocities v_0/c for $H = 5$. It can be observed that at low impact velocities the mass distribution are composed of two separated regimes, i.e. for small masses $p(m)$ rapidly decreases with an exponential form, while big fragments generate a peak of the distribution. As v_0 increases the distribution becomes continuous and at the critical point of fragmentation a power law distribution emerges

$$p(m) \sim m^{-\tau}, \tag{2}$$

where the exponent is $\tau \approx 1.8$. It is interesting to note that the straight line of the power law is decorated by several bumps (see Fig. 2). The rea-

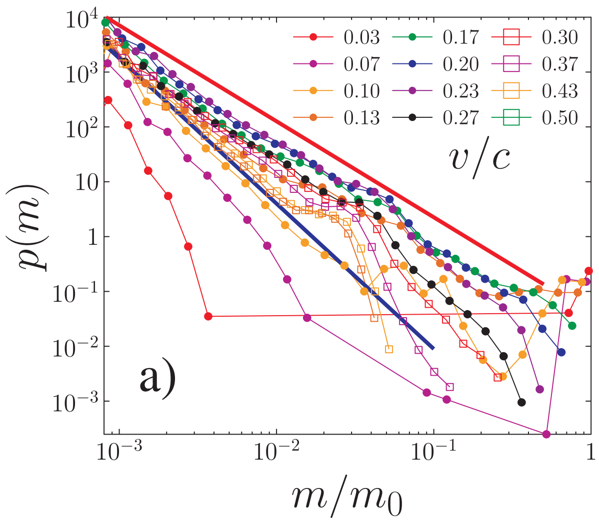

Figure 2: Mass distribution of fragments for a sample of height $H = 5$ at several impact velocities. The two straight lines represent power laws of exponents 1.8 and 2.3.

son is the special fragmentation mechanism which emerges due to the quasi two-dimensional shape of the specimen. To have a better overview of the functional form, Fig. 1(a) illustrates a single distribution $p(m)$ obtained nearly at the critical velocity v_c. The color code indicates that different regimes of $p(m)$ are dominated by well defined spatial regions of the specimen. Comparing to the reassembled sample in Fig. 1(b) it is clear that the breakup is caused by the regular crack pattern generated by the interference of tensile waves reflected from the free boundary of the body. The low value of the exponent τ is typical for two dimensional brittle materials [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. The most interesting outcome of the calculations is that above the fragmentation critical point the power law regime of the mass distribution prevails, however, the exponent gradually increases to a significantly higher value $\tau \approx 2.3$ which then remains stable (see Fig. 2). Further increasing v_0 only changes the cutoff of the distribution which finally turns into an exponential.

To understand why the value of τ increases with the imparted energy we carefully selected subsets of fragments which belong to the (i) bulk and (ii) surface of the body, and (iii) fragments which span the body along the direction perpendicular to the plan of the specimen. Figure 3 presents

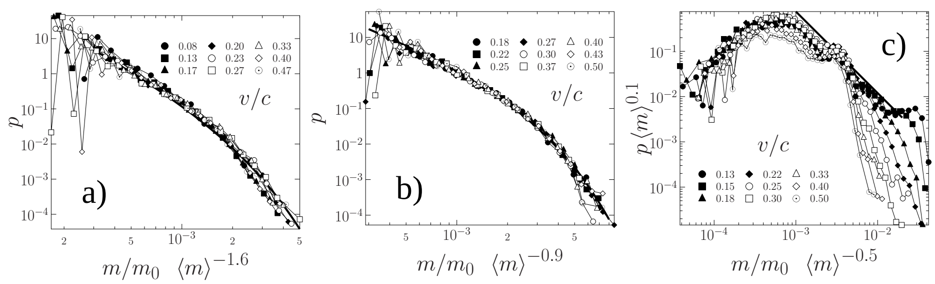

Figure 3: Mass distribution of (a) bulk, (b) surface, and (c) crossing fragments rescaled with the average fragment mass. The good quality collapse demonstrates the universality of the distributions.

the mass distribution of the three subsets of fragments rescaled with a power of the average fragment mass $\langle m \rangle$. The high quality collapse of the curves demonstrates the universality of the distributions, i.e. the exponent τ characterizing the subsets of fragments do not have any dependence on the impact velocity. The changing exponent observed for the complete distribution in Fig. 2 is caused by the fact that the contribution of the subsets to the complete distribution depends on the imparted energy. For bulk objects the behaviour of surface and bulk fragments become similar and a single exponent emerges which proved to be universal.

IV. Summary

We presented a detailed study of the fragmentation of three-dimensional brittle solids focusing on the mass distribution of fragments and on the underlying mechanism of breakup. Our study suggests that the energy dependent mass distribution exponent observed in experiments is caused by the presence of three competing mechanisms. The final distribution is a blend of the contributions of three subsets of fragments which all have universal distributions.

Acknowledgement

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