ACTA PHYSICA DEBRECINA XLVIII, 100 (2014)

ON THE DISPERSION CORRECTION TO THE DEUTERON CHARGE RADIUS. EXPERIMENTAL ($r_{el} - r_{mu}$) DIFFERENCES IN NUCLEI

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Abstract

The deuteron-electron scattering may contribute to the solution of the *”proton radius puzzle”*. A necessary ingredient to this is the dispersion correction to the deuteron charge radius. To estimate its value, results of theoretical calculations, extrapolation from an experiment performed on ^{12}C, as well as from ($r_{el} - r_{mu}$) differences are taken into account. Results are conflicting. Therefore, they are discussed not precluding even the idea of a new short-range muon-nucleon interaction.

I. Introduction

The *”proton radius puzzle”* (see [1] and references therein) initiated several new experimental programs. Among these, the measurement of the deuteron *rms* charge radius by fast electron scattering may also yield an independent information on the proton charge radius, because the proton-deuteron difference has already been determined by optical isotope shift [2]. However, the determination of charge radii by electron scattering suffers from a systematic error source: the uncertainty of the *dispersion correction* due to the two-photon exchange between the deuteron and the electron. In what follows, a short review is given of the methods and results of different calculations on the deuteron. The experimental result on ^{12}C, and an extrapolation from experimental ($r_{el} - r_{mu}$) differences obtained from complex nuclei to the deuteron is also presented. Finally, some questions are posed in connection with the contradictory results.

II. Overview of theoretical calculations for the deuteron

Electron scattering on nuclei is generally described in one-photon exchange approximation: the nucleus is regarded as a static charge distribution. In a better approximation, its internal degrees of freedom should also be taken into account. This is because during the scattering the nucleus may be excited and de-excited by the exchange of two virtual photons. This process changes the differential elastic scattering cross section, mainly at the diffraction minima, and decreases the „effective" radius measured. To compensate for this decrease, a *dispersion correction* Δr_{dc} should be added.

The *theoretical calculation* of this effect is a complicated mathematical procedure, and cannot be performed without assumptions (e.g. low-Z, low-q) and simplifying models to substitute the actual excitation spectrum of the nucleus investigated. One of the main results of the theoretical investigations is, that the effect *increases towards low Z values*. This last property is also supported by Bottino and Ciocchetti [3] together with the conclusion that rather accurate evaluation of this effect can be done for light nuclei using experimental photodisintegration cross sections. Assuming virtual giant dipole excitations, they have calculated the relative dispersion correction ε to the *ms* radii for light elements, and arrived at the result $\varepsilon = 1.1\ \%$ for electrons of $E_{el} = 50$ MeV; in the case of the deuteron this corresponds to $\Delta r_{dc} \cong 11.8$ am. Using the more rapid but less precise procedure (eq. (42) in [3]) we have calculated the dispersion corrections for the energy range where most *e-d* scattering experiments are performed, see second row in table 1. This procedure is acceptable for deuterium, but provides a rather rough estimate for higher mass numbers. Normalizing these data to the more precise value at 50 MeV, one has the approximate energy dependence of the dispersion correction $\Delta r_{dc,n}$: third row of table 1.

Table 1.

Dispersion corrections to deuteron *rms* charge radius calculated from (42) of [3] (second row), and the values normalized to the more precise calculation at 50 MeV (third row).

E_{el}	50 MeV	100 MeV	200 MeV	300 MeV	400 MeV
Δr_{dc}	13 am	12 am	9 am	7 am	6 am
$\Delta r_{dc,n}$	11.8 am	10.0 am	7.2 am	6.4 am	5.4 am

With similar physical assumptions as Bottino and Ciochetti but with different mathematical procedure, Friar [4] found the mass number independent result $\Delta r_{dc} \approx 7$ am. However, he used a phenomenological parametrization that works best for heavy nuclei; for light nuclei a higher value is expected.

In contrast to the above results, the more detailed numerical calculations of Herrmann and Rosenfelder [5] – avoiding approximations like the closure approximation or mixing of different models for ground and excited states - yield a small and *negative* (!) dispersion correction: $\boldsymbol{\Delta r_{dc} = -3}$ **am** for the deuteron.

III. Experimental investigation of the dispersion correction

Three *experimental methods* can be used for the determination of dispersive processes in electron scattering [6]:

1) measurement of the strength of a $0^+ \rightarrow 0^+$ transition,
2) comparison of electron and positron scattering for the same nucleus,
3) study of a sharp diffraction minimum at different electron energies; for the present case, this last method deserves a few sentences.

III.1. Investigation of electron scattering in diffraction minima

Dispersive effects in electron scattering from the deuteron has not been investigated experimentally. Because of the Z-dependence, we can still use the experimental result from any other light nucleus as a lower limit. An experimental value of the dispersion correction for E_{el} = 238-431 MeV electrons on the nucleus ^{12}C has been given by Offermann [6, 7]: $\Delta r_{dc,exp}(^{12}C)$ = 7 am, making use of the extremely improved angle resolution in e-^{12}C scattering in the first diffraction minimum. References to earlier experiments and theoretical papers can be found in [6, 7, 8].

III.2. Estimate by extrapolation from experimental $(r_{el} - r_{mu})$ differences

In comparison of *rms* radii deduced from electron scattering and muonic X-rays [9] the result from the latter type of experiment is nearly always larger (and more precise) than that from electron scattering. The disagreement might be due to the neglect of dispersive contributions to the electron scattering data. A *systematic investigation* [8] of *rms* charge radius differences measured by electron scattering (r_{el}) and by muonic atom X-rays (r_{mu}), respectively, resulted in a weighted mean value if averaged over the whole mass number interval (at 85 mass numbers): $(r_{el} - r_{mu})_{av}$ = -9.3(1.5) am. If the $(r_{el} - r_{mu})$ differences are attributed to the effect of dispersion in electron scattering, this means an average value for the dispersion correction: Δr_{dc} = +9.3(1.5) am. This experimental value overlaps with those calculated by [3], but disagrees with that from [5]. In order to look for a mass number dependence of the experimental differences, a linear weighted least square fit was performed, which resulted in:

$$(r_{el} - r_{mu}) = -12.0(2.6) + 0.027(21)\times A \text{ (am)} \tag{1}$$

Using this extrapolation to the deuteron, we arrive to Δr_{dc} = +11.9(2.7) am.

In figure 1 only the most accurate data are plotted – together with their *±1 sigma* error interval. Note that a value at a given mass number, contains the weighted average of several experimental results. As can be seen on the figure, not only the average, but also most of the *individual data* are under zero, i.e. the dispersion corrections attributable to them, is positive.

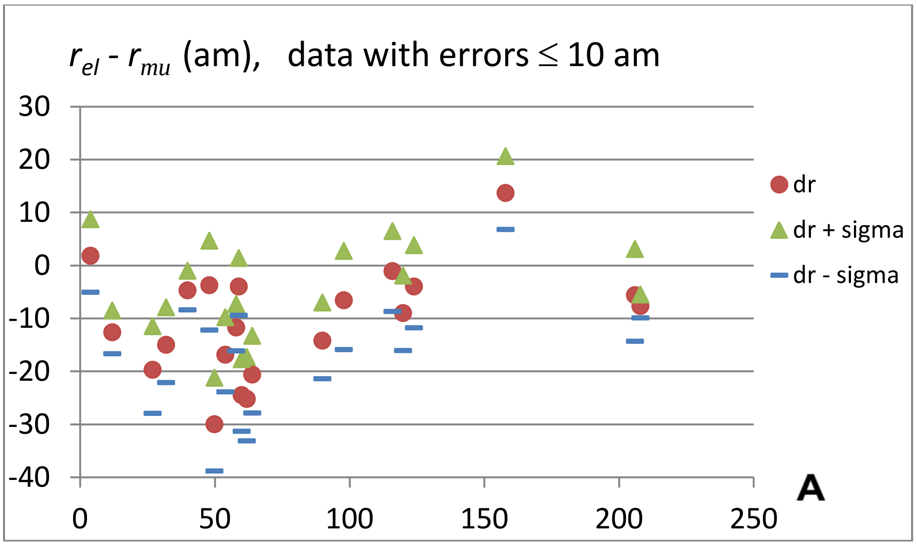

Figure 1. The most precise $(r_{el} - r_{mu})$ differences with *±1 sigma* error intervals

IV. Summary. Conclusions

An independent determination of the proton charge radius would be possible through the measurement of the *rms* charge radius of the deuteron by electron scattering, because the proton-deuteron radius difference has already been determined by optical isotope shift [2]. In order to have as accurate result as possible, the dispersion correction – i.e. the contribution of two-photon exchange processes – should also be added. *Theoretical calculations* of [3] gave $\Delta r_{dc} \approx 10$ am (see also table 1.), and [4]: $\Delta r_{dc} \approx 7$ am, while a more elaborate work [5] resulted in a *negative* value: $\boldsymbol{\Delta r_{dc} \approx -3}$ **am.**

As for *experiment*, there is no measurement for the deuteron. For ^{12}C $\Delta r_{dc} \approx 7$ am is obtained [6]. Taking into account the theoretically predicted Z-dependence, this suggests for the deuteron a value $\Delta r_{dc} > 7$ am. The 85 differences $(r_{el} - r_{mu})$ [8] – if attributed to dispersion effects – yield a dispersion correction $\Delta r_{dc} \approx 9.3$ am. The A-dependence as eq. (1), tends to about $\Delta r_{dc} \approx 12(3)$ am.

The early theoretical calculations are in harmony with the extrapolation of experimental data. However, the result of the more elaborate theory brings a dissonant tune into this harmony. The negative dispersion correction means that the deuteron is an exception: it does not follow the predicted Z-dependence; on the contrary, it differs even in sign from it.

The case of $(r_{el} - r_{mu})$ differences is even more interesting. What if they are not – or not completely – caused by dispersion effects? Note that Ruckstuhl [9] attributed this difference to a new, short-range interaction between muon and nucleon. In these days, this possibility can not be absolutely precluded until the *proton radius puzzle* – giving rise to similar suspicion -is not solved.

References

[1] Pohl, R., et al.: *Ann. Rev. Nucl. Part. Sci.,* **63** (2013) 175.
[2] Huber, A. et al.: *Phys. Rev. Lett.,* **80** (1998) 468.
[3] Bottino, A., Ciocchetti, G.: *Nucl. Phys.*, **A178** (1972) 593.
[4] Friar, J.L.: Dispersion and recoil effects in elastic electron scattering, in: *Proc. Int. School on Electron and Pion Interactions with Nuclei at Intermediate Energies* (Arricia, Italy, June 1979). Eds. W. Bertozzi et al., (Harwood, New York, 1980) 143.
[5] Herrmann, T., and Rosenfelder, R.: *Eur. Phys. J.,* **A2** (1998) 29.

[6] Offermann, E.A.J.M., *Ph.D. Thesis,* Univ. Amsterdam (1988).
[7] Offermann, E.A.J.M., et al.: *Phys. Rev. Lett.* **57** (1986) 1546.
[8] Angeli, I.: *Z. Physik,* **A334** (1989) 377.
[9] Ruckstuhl, W., et al: *Nucl. Phys*. **A430** (1984) 685.