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STATISTICS OF RECORDS IN CRACKLING TIME SERIES

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Abstract

We present a theoretical study of the creep rupture of heterogeneous materials in the framework of a fiber bundle model. Focusing on the mean field limit of the system we studied the record breaking statistics of the time series of crackling bursts occurring during the failure process. Record breaking events have a larger size than all the previous ones in the time series. We showed that the number of records grows logarithmically with the total number of bursts for most of the failure process, however, close to macroscopic failure an exponential dependence is obtained. Both the size of records and their increments proved to have power law statistics with a common exponent significantly smaller than the one obtained for the complete burst size distribution. We also found power law distribution for the waiting times between consecutive records, however, the value of the exponent is different for low and high external loads. By studying the evolution of the record sequence we revealed the existence of a load-dependent characteristic record number of the system which separates the regimes of slow-down and acceleration in the occurrence of record events.

I. Introduction

Creep rupture processes occurring under constant sub-critical external loads are often responsible for the failure of engineering constructions and can lead to natural catastrophes such as snow and stone avalanches, landslides, earthquakes. During the fracture process micro-cracks form and grow in the material inducing soundwaves which may be recorded by sensitive microphones. These acoustic emissions can be used to monitor the damage state of materials. Usually, the analysis of recorded crackling events is focused on the integrated statistics determining the probability distribution of the magnitude of events and that of the waiting times in between [7].

Here we study the record breaking statistics of crackling burst time serieses to reveal interesting aspects of the dynamics of the creep rupturing system. In this method a special sub-series of bursting events is constructed considering solely those events which are larger than any previous ones. Statistical properties of such record breaking (RB) bursts can provide useful information about underlying trends and correlations in the failure process [7].

II. The fiber bundle model

We studied the creep rupture process by computer simulations in a fiber bundle model of heterogeneous materials. The sample is described as a bundle of N parallel fibers arranged on a square lattice. Two physical mechanisms could cause the failure of fibers: fibers have a brittle response so that they break immediately when the value of local load on them exceeds their fracture strength σ_{th}^i. Additionally, due to the local load intact fibers accumulate damage in the form of micro-cracks. We assume that the damage accumulation rate depends on the local load as a power law $\Delta^i c(t) = a\sigma_i^{\gamma}(t)\Delta t$. Fibers break when the amount of accumulated damage $c_i(t)$ exceeds their local damage threshold c_{th}^i. To consider the disorder in the material, random breaking thresholds c_{th} and σ_{th} are assigned to the fibers which are uniformly distributed between 0 and 1. After each breaking event intact fibers have to take over the load of the broken one. The load redistribution controls the range of interaction between fibers. We performed computer simulations in the equal load sharing limit (ELS), which is the mean field limit of the model. Under ELS the load of the broken fiber is equally redistributed over all the remaining intact fibers which

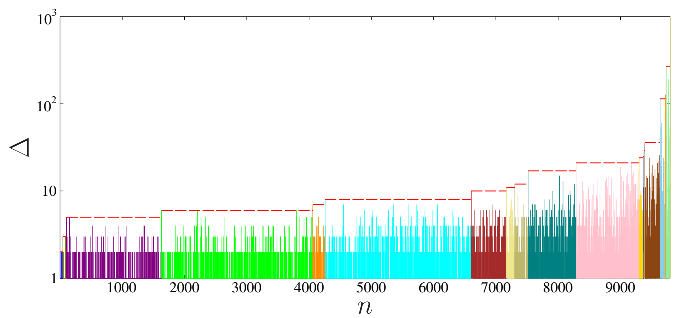

Figure 1: An example of a time series of bursts in the mean field limit (ELS) where the number of fibers in the bundle is $N = 10^5$. Sub-sequences starting with record breaking events are marked by different colors. The horizontal lines demonstrates the size increments between successive records [7].

leads to a homogeneous stress field. Further details of model construction can be found in Refs. [1, 2, 3].

III. Time evolution of the bundle

When the bundle is subjected to the constant external load σ_0 below the critical value σ_c, weak fibers with lower mechanical strength break immediately. These initial breakings may cause further breakings as the load gets redistributed, but eventually a stable partially failed configuration is reached. Without the aging of fibers the system would have infinite lifetime. However, as fibers break slowly, one-by one due to the damage accumulation, the local load on intact fibers gradually grows until the level is high enough to trigger sequences of fiber breakings called bursts [1, 2, 3, 4, 5, 6]. In this model bursts correspond to the breaking events recorded in acoustic emission measurements. After a burst stops the local load is elevated by damage breakings again till the next burst. Ultimately the local load gets high enough to initiate the last, catastrophic burst in which all of the remaining intact fibers break and macroscopic failure occurs [1, 2, 3, 4, 5, 6].

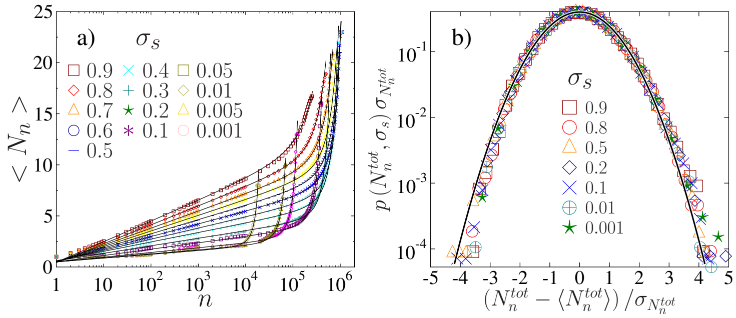

Figure 2: a): Average number of record breaking bursts N_n as a function of the order number n for different external loads. Two regimes can be seen characterized by a logarithmic and an exponential dependence, respectively. Black lines are fitting curves using Equation 1. b): Distribution of the total number of record busts N_n^{tot} for different load values rescaled with the corresponding average and standard deviation. The curves can be described by the Gauss distribution (black fitting curve) [7].

Figure 1 presents an example of the burst time series in the mean field limit where the burst size Δ (number of fibers breaking in the burst) is shown as a function of the order number n. This quantity strongly fluctuates during the fracture process, however, the average burst size increases towards macroscopic failure. Records are defined as bursts which have a size larger than any previous events. Such record breaking bursts divide the time series into sub-series's which are marked by using different colors in the figure. It can be observed that both the increments of record sizes and the number of bursts between successive records have strong fluctuations [7].

IV. Results

To study the statistics of record breaking bursts we performed computer simulations for systems containing $N = 10^7$ fibers and compared the results to the case of independent identically distributed (IDD) random variables[7].

Figure 2.a shows the average number of records $\langle N_n \rangle$ occurring till the

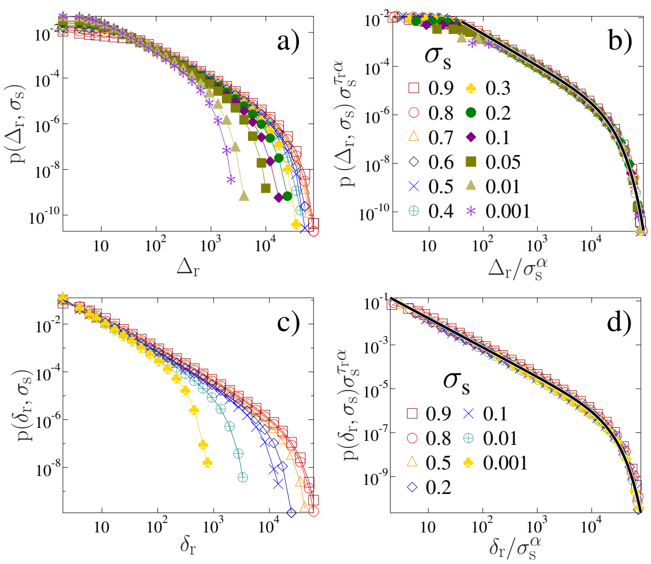

Figure 3: Record burst size (a) and size increment (c) distributions for different external loads. b) and d) show the rescaled distributions with the appropriate power of the load. Continuous lines are fits with Equation 3 [7].

nth burst as a function of the order number for different external loads. Two regimes can be observed. In the first regime the average record number depends on the event number logarithmically as the disorder dominates the occurrence of bursts, in agreement with the analytical prediction for the IID case. However, close to macroscopic failure the triggering of bursts becomes more efficient and the dependence turns into exponential. The $\langle N_n \rangle (n)$ curves can be described by the following functional form [7]:

$$\langle N_n \rangle = A + B \ln n + C \exp\left[(n/D)^{\xi}\right], \tag{1}$$

In Figure 2.a symbols denote the simulation data and lines are the analytical fits with the above formula [7]. The distribution of the total number of RB events occurring before macroscopic failure can be seen in Figure 2.b, where the curves of different external loads are rescaled by the respective averages and standard deviations. The scaling function can be characterized well

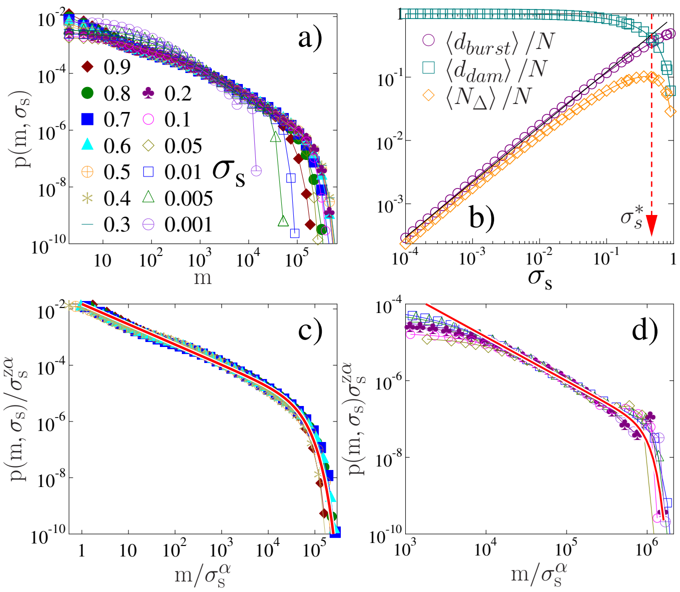

Figure 4: a) Waiting time distributions for different external load values. c) and d) shows the rescaled distributions with the external load in the high and low load regime, respectively. Continuous lines are the fittings with Equation 4. b) fraction of fibers breaking due to the damage accumulation $\langle d_{dam} \rangle$, and in bursts $\langle d_{burst} \rangle$, and the total number of burst $\langle N_\Delta \rangle$ as a function of the external load σ_s. The dashed line shows the position of the characteristic load σ_s^* [7].

with the standard normal distribution [7]:

$$p(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-x^2/2\right), \tag{2}$$

The same distribution has been predicted for IIDs [7].

Figure 3.a shows the distribution of record sizes $p(\Delta_r, \sigma_s)$ at different external loads. The distributions follow a power law form with an exponential cutoff. The characteristic exponents are independent of the value of the load so with the appropriate power of the external load the curves can be collapsed on each other (Figure 3.b) which implies the following scaling structure [7]:

$$p(\Delta_r, \sigma_s) = \Delta_r^{-\tau_r} \phi(\Delta_r/\sigma_s^\alpha), \tag{3}$$

where the value of α is 0.65, and the power law exponent $\tau_r = 1.33 \pm 0.03$ is significantly smaller than the exponent for the burst size distribution in which all of the bursts are included ($\tau_{ELS} = 2.5$) [7].

The size increments between successive record bursts are defined as $\delta_k = \Delta_r^{k+1} - \Delta_r^k$ where $k = 1, 2, 3...$ is the rank of the record in the sequence of records. For the distribution of size increments also a power law form is obtained with exponential cutoff (Figure 3.c) where the values of the characteristic exponents are the same as for the record size distribution (Figure 3.d) [7].

The waiting time between consecutive records is defined as $m_k = n_{k+1} - n_k$ where n_k is the order number of the kth record in the time series of bursts. The distribution of waiting times can also be characterized by a power law form with an exponential cutoff (Figure 4.a), however, in this case, the exponent has different values for low and high external loads [7].

Figure 4.d presents the fraction of fibers in the bundle breaking due to damage accumulation (teal curve) and in bursts (purple curve) as a function of the external load σ_s. A characteristic external load can be found where the two curves intersect each other. Below the characteristic load $\sigma_s^* \approx 0.48$ the fracture process is dominated by damaging, however, close to the macroscopic failure it is controlled by the occurrence of large bursts [7].

The analytical prediction for the value of the power law exponent of the waiting time distribution is $z_{IDD} = 1$. Figure 4.b,c shows that the waiting time distributions $p(m, \sigma_s)$ can be collapsed on each other with the appropriate power of the external load and the scaling structure is similar to the case of size and increments distributions [7]:

$$p(m) = m^{-z}\psi(m/\sigma_s^{\alpha}), \tag{4}$$

where below the characteristic load $z = 1.15$, $\alpha = 0.625$, however, above the characteristic load $z = 0.72$, $\alpha = -1.45$, so long waiting times occur more frequently than in the IID case [7].

We also determined average quantities of single records to get an understanding of the evolution of the series of record breaking bursts. Figure 5.a shows the average relative size increments of successive record bursts as the

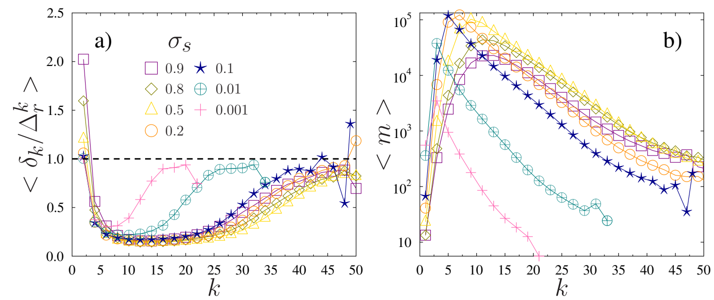

Figure 5: a) Average relative size increment $\langle \delta_k/\Delta_r^k \rangle$ and b) average waiting time $\langle m_k \rangle$ as a function of the rank k for several external load values [7].

function of the record rank k for different external load values. The relative size increment is defined as the ratio δ_k/Δ_r^k and gives information about the rate of increase of record sizes. In Figure 5.b the average waiting time is presented as a function of the record rank. It can be observed that at each load value during the fracture process at first the occurrence of records slows down, the average waiting times increase which is accompanied by a decrease in the average relative increments. However, at each load there is a characteristic rank $k*$ from which an acceleration can be seen with decreasing waiting times and increasing relative increments. This transition between the two regimes approximately coincides with the transition that can be observed in the curves of the average record number $\langle N_n \rangle$. Close to the macroscopic failure the average relative increments tends to 1 so at that time the size of successive records nearly doubles [7].

V. Conclusions

In this paper we used a fiber bundle model of creep rupture of heterogeneous materials to investigate the record breaking statistics of crackling bursts accompanying the failure process. In order to reveal trends and correlations in the time series of crackling bursts we compared the outcomes of large scale computer simulations to the analytical results obtained on the RB statistics of independent identically distributed random variables.

We showed that a load-dependent characteristic time scale emerges which separates two regimes during the failure process: the first regime is characterized by a logarithmic dependence of the number of record breaking bursts on the event number and by a slow-down of record breaking. In the second regime, close to the macroscopic failure, an acceleration can be observed with an exponentially increasing record number. We showed that the distribution of the size of records and their increments have a power law functional form characterized by the same values of exponents. For the waiting time distribution the same behavior is obtained with different exponents for low and high external loads [7]. The results demonstrate that in spite of the absence of spatial correlations between local failure events the competition of load increments and of the disorder in the micro-structure of the system leads to the emergence of interesting temporal correlation of bursts.

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