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EMERGENCE OF ENERGY DEPENDENCE IN THE FRAGMENTATION OF HETEROGENEOUS MATERIALS

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Abstract

In the fragmentation process of heterogeneous solids the mass (size) distribution of pieces is described by a power law functional form. The exponent of the distribution displays a high degree of universality depending solely on the dimensionality and on the brittle-ductile mechanical response of the system. Based on large scale computer simulations here we uncover a possible mechanism that can lead to the emergence of recently reported energy dependence in fragmentation processes resolving contraversal issues on the problem: studying the impact induced breakup of plate-like objects with varying thickness we show that energy dependence naturally occurs when a lower dimensional fragmenting object is embedded into a higher dimensional space. This apparent non-universality of fragmentation is the result of blending universal partial processes.

I. Introduction

Fragmentation occurs when a large amount of energy is imparted to a heterogeneous solid within a short time leading to rapid breakup [1]. Fragmentation processes have a high importance for industrial applications especially in mining and ore processing. During the past decades research on fragmentation focused on the statistics of fragment masses (sizes) [1]. The power law functional form describing the mass distribution of fragments was confirmed by a large number of experimental [1, 8, 7] and theoretical studies [5] . The exponent is determined by the dimensionality of the system [7, 5] and by the brittle or ductile mechanical response of the material [6], but is independent of the type of materials, amount of input energy and of the way the energy is imparted to the system [1].

Recently, in experiments on the fragmentation of long thin glass rods [2] and freely-hanging glass plates [3, 4] the exponent was found to increase logarithmically with the imparted energy [2, 3, 4]. These novel findings question the universality and hence the phase transition interpretation of fragmentation phenomena. Computer simulations of spheres impacting against a hard wall showed a counter example, i.e. steeper fragment mass distributions were obtained, however, rescaling with the average fragment mass the apparent energy dependence could be transformed out. This study highlighted the importance of finite size scaling in fragmentation studies [5].

In order to resolve these controversal issues on the universality of fragment mass distributions, here we study the impact induced breakup of heterogeneous materials by large scale computer simulations. Our results demonstrate that energy dependence naturally occurs when the fragmenting object is embedded in a higher dimensional space. We uncover a robust scenario which leads to the energy dependence of energy dependent mass distribution exponents but it still underlines the importance of universality.

II. Discrete element model of fragmentation

For the calculations we use a discrete element model (DEM) of heterogeneous materials which has recently been developed. A rectangular sample was generated by sedimenting spherical particles with randomly selected diameters. The interaction of contacting particles is described by the Hertz contact law. Cohesive interaction is introduced by beams which connect the particles along the edges of a Delaunay triangulation of the initial particle positions. In 3D the total deformation of a beam is calculated as the superposition of elongation, torsion, as well as bending and shearing. Crack formation is captured such that the beams can be broken according to a physical breaking rule, which takes into account the stretching and bending of contacts

$$\left(\frac{\varepsilon_{ij}}{\varepsilon_{th}}\right)^2 + \frac{\max(\Theta_i, \Theta_j)}{\Theta_{th}} \geq 1. \tag{1}$$

where ε_{ij} denotes the axial strain of the beam between particles i and j, while Θ_i, and Θ_j are the bending angles of the beam ends. The parameters ε_{th} and Θ_{th} control the relative importance of the two breaking modes [5]. Energy dissipation arises from the released energy stored in a beam just before breaking. The breaking thresholds are constant, therefore, only structural disorder is present, where the physical properties of beams are determined by the random particle packing. At the broken beams along the surface of the spheres cracks are generated inside the solid and as a result of the successive beam breaking the solid falls apart. The fragments are defined as sets of discrete particles connected by the remaining intact beams. The time evolution of the fragmenting solid is obtained by solving the equations of motion of the individual particles until the entire system relaxes meaning that no beam breaking occurs during one thousand consecutive time steps and there is no energy stored in deformation. For more details of the model construction see Ref. [5].

To initiate the breakup process, a surface particle with its contacting neighbors lying in the middle of one of the side walls got an initial velocity $\vec{v}_0$ pointing towards the center of mass of the sample. This is equivalent to an experimental setup where the impactor does not penetrate the target, as in Refs. [4]. Computer simulations were performed to determine the sound speed c of the model material. In the presentation of the results lengths and velocities are made dimensionless by dividing them with the average particle diameter $\langle d \rangle$ and with the sound speed c, respectively.

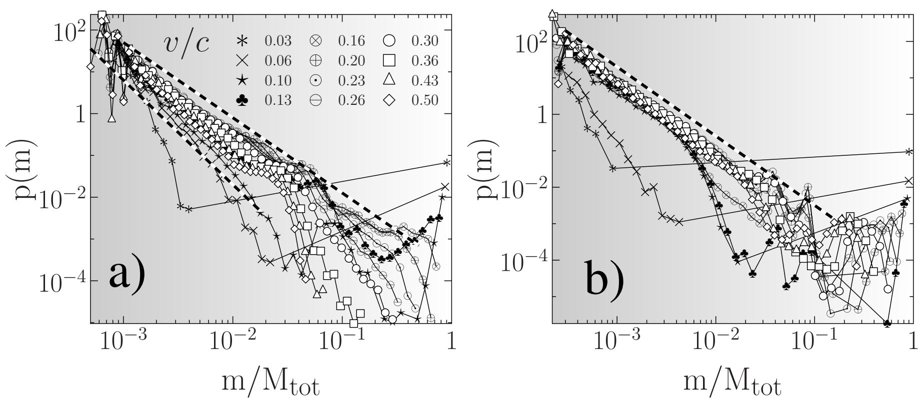

Figure 1: (a) For thin plates $H/\langle d\rangle = 5$ the value of the exponent changes from 1.7 to 2.4 depending strongly on the energy of the impact. (b) For thick plates $H/\langle d\rangle = 7, \ldots, 15$ a unique exponent 1.9 is obtained.

III. Results

Large scale computer simulations were performed in the three-dimensional space on plate-like objects varying the plate thickness $H/\langle d\rangle$ and the impact velocity $v_0/c = 0.03 - 0.5$ in broad ranges. Computer simulations revealed that for thin plates the power law exponent of the fragment mass distribution strongly depends on the impact velocity: power law with an exponent $\tau = 1.7$ appears at the critical velocity of impact, and further increasing the impact velocity gradually changes the exponent up to $\tau = 2.4$. However, for thick plates a unique exponent is obtained $\tau = 1.9$, dependence on the impact velocity can only be pointed out for the cutoff of the distributions. Figure 1 provides an overview of fragment mass distributions obtained at different impact velocities for two thicknesses. The velocity dependent exponent can be explained by the interplay of the geometry of the sample and of the dimensionality of the system which gives rise to a crossover between two different fragmentation mechanisms. In the vicinity of the critical velocity the interference of elastic waves generates a crack pattern which is essentially two-dimensional. The high degree of regularity

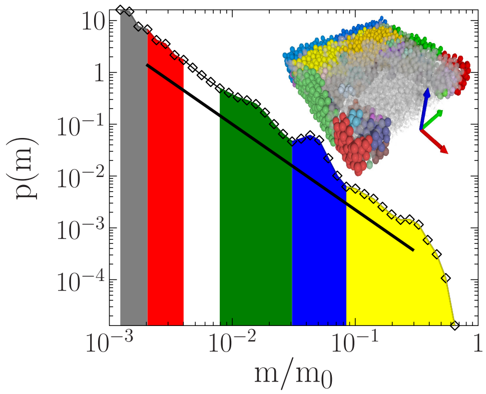

Figure 2: The probability distribution of the mass of fragments at the critical impact velocity v_c where complete breakup first occurs. The local maxima correspond to different spatial regions of origin. Inset: final reassembled sample where fragments are colored according to the regime of the distribution they give dominating contribution to.

of the crack structure gives rise to local maxima on the power law functional form of the fragment mass distribution (see Fig. 2). At increasing impact velocities bulk cracking gets activated so that the crack structure becomes three-dimensional with a high degree of randomness.

The fragments can be grouped into bulk, surface and spanning pieces depending on the location of their bounding boxes with respect to that of the complete sample as it is illustrated in Fig. 3(c). The formation of these subsets of fragments is governed by different cracking mechanisms. Scaling analysis showed a striking universality of the mass distributions of bulk and surface fragments, with exponents $\tau = 1.7$ and $\tau = 2.4$, respectively. The results imply that for thin plates the velocity dependence of the exponent of the complete mass distribution is caused by the mixing of the contributions of the subsets of fragments, where the mixing ratio depends on v_0 (see Fig. 3). Our results have the general consequence that energy dependence of the mass distribution exponent of fragmentation phenomena can be expected when a low dimensional object is embedded into a higher dimensional space

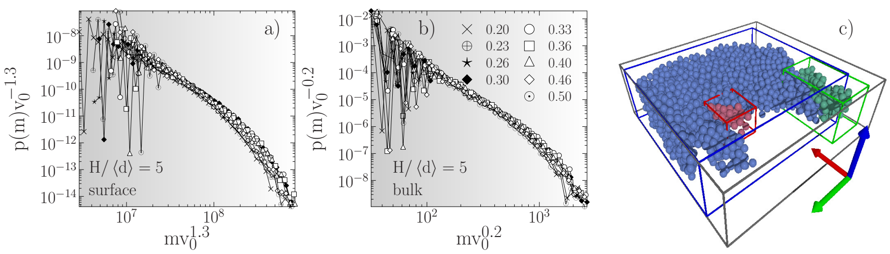

Figure 3: Scaling plot of the mass distribution of surface (a) and bulk (b) fragments. The high quality collapse implies that the mass distribution exponents of the subsets of fragments do not have any energy dependence. The non-universality of the complete distribution originates from the mixing of these universal contributions where the mixing ratio depends on v_0.

allowing for the emergence of a transition in the spatial structure of cracks generated by the initial shock wave [9].

IV. Summary

We presented a detailed study of the fragmentation of three-dimensional brittle solids focusing on the mass distribution of fragments and on the underlying mechanism of breakup. Large scale computer simulations revealed that energy dependence occurs when a lower dimensional fragmenting object is embedded into a higher dimensional space. The reason is a transition between two distinct cracking mechanisms driven by the impact velocity. In the vicinity of the critical impact velocity the interference pattern of elastic waves gives rise to a two-dimensional crack structure. Increasing impact velocity activates bulk cracking leading to a three-dimensional crack structure with a high degree of universality. Selecting fragments in different spatial regions of the sample, we showed that the cracking mechanisms result in universal mass distributions. The observed non-universality of the complete distributions is the consequence of blending the contributions of universal partial processes. Studying the impact induced breakup of thin glass plates, in the experiments of Ref. [3] an increase of the mass distribution exponent was reported with increasing impact velocity. The authors argued that the effect can be attributed to the increase of the fractal dimension of the crack pattern, i.e. as the crack structure gets more-and-more space filling the mass distribution exponent increases and approaches a limit value [3]. Our results clarify the background of these experimental findings unveiling the underlying mechanism [9].

IX. ACKNOWLEDGEMENTS

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