QUANTUM CHROMODYNAMICS SUM RULES

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KOCAELI UNIVERSITY

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Faculties

- Faculty of Dentistry
- Faculty of Education
- **Faculty of Arts and Sciences**
- Faculty of Fine Arts
- Faculty of Law
- Faculty of Economics and Administrative Sciences
- Faculty of Communication
- Faculty of Architecture and Design
- Faculty of Engineering
- Faculty of Technical Education
- Faculty of Medicine
- Faculty of Aviation and Space Sciences
- Faculty of Theology

Institutes

- Natural and Applied Sciences
- Health Sciences
- Social Sciences

- 6 schools
- 22 vacational schools

81.877 STUDENTS

FACULTY OF ARTS AND SCİENCES

- It was founded in 1993
- The focus of faculty is handling teaching and research together
- 3732 undergraduate students
- 151 MSc/MA and PhD students

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Departments

- Biology
- Chemistry
- **Physics**
- Mathematics
- Philosophy
- Turkish Language and Literature
- Western Languages and Literatures
- Archaeology
- History

DEPARTMENT OF PHYSICS

MAJOR AREAS

- ❖ Atomic and molecular physics
- ❖ Solid state physics
- ❖ Mathematical physics
- ❖ Nuclear physics
- ❖ **High energy and plasma physics**

- ❑ 250 undergraduate students
- ❑ 21 MSc/MA and PhD students

RESEARCH LABORATORIES

- ✓ Solid state physics research laboratory
- ✓ Nuclear physics research laboratory
- ✓ **High energy physics numeric laboratory**

High Energy Group Research Subjects:

- Quantum Chromodynamics (QCD)
- QCD Sum Rules
- Thermal QCD
- Two Higgs Doublet Model
- Chiral and Deconfinement Phase Transitions
- Quark-Gluon Plasma Ossilations

OUTLINE

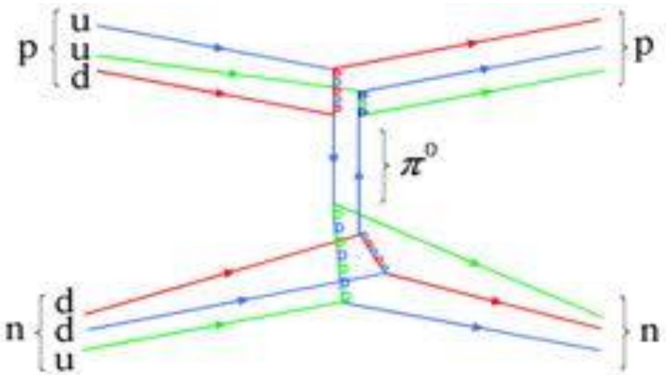
- Introduction to fundamental interactions and fundamental particles
- Hadrons
- Introduction to QCD: perturbative & non-perturbative
- QCD Sum Rules
- Leptonic Decay Constant of D^*_2 Tensor Meson
- Semileptonic Transition of $B \to D^*_2 l\nu$ in QCD Sum Rules
- Exotic Hadrons: Z_c and X(5568)
- Conclusion

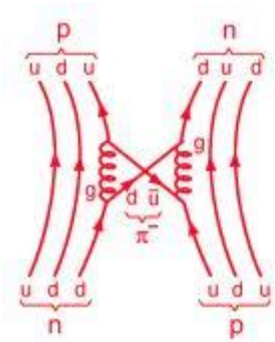
Fundamental Interactions

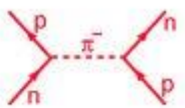
Interaction	Current Theory	Mediators	Relative Strength	Long-Distance Behavior	Range (m)
Strong	Quantum chromodynamics (QCD)	gluons	10^{38}	1	10^{-15}
Electromagnetic	Quantum electrodynamics (QED)	photons	10^{36}	$\frac{1}{r^2}$	∞
Weak	Electroweak Theory	W and Z bosons	10^{25}	$\frac{e^{-m_{W,Z}r}}{r}$	10^{-18}
Gravitation	General Relativity (GR)	Gravitons (hypothetical)	1	$\frac{1}{r^2}$	∞

SM

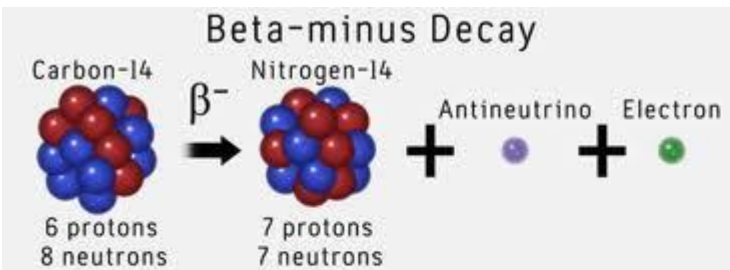
QCD is a theory of the strong interactions, i.e. a theory of fundamental interactions between colored quarks and gluons







Weak interaction: It is responsible for the radioactive decay of subatomic particles and initiates the process known as hydrogen fusion in stars.



$$^{14}_{6}C \longrightarrow {}^{14}_{7}N + e^{-} + \overline{\nu}_{e}$$

Electron antineutrino

$$n \rightarrow p + e^{-} + \overline{\nu}_{e}$$

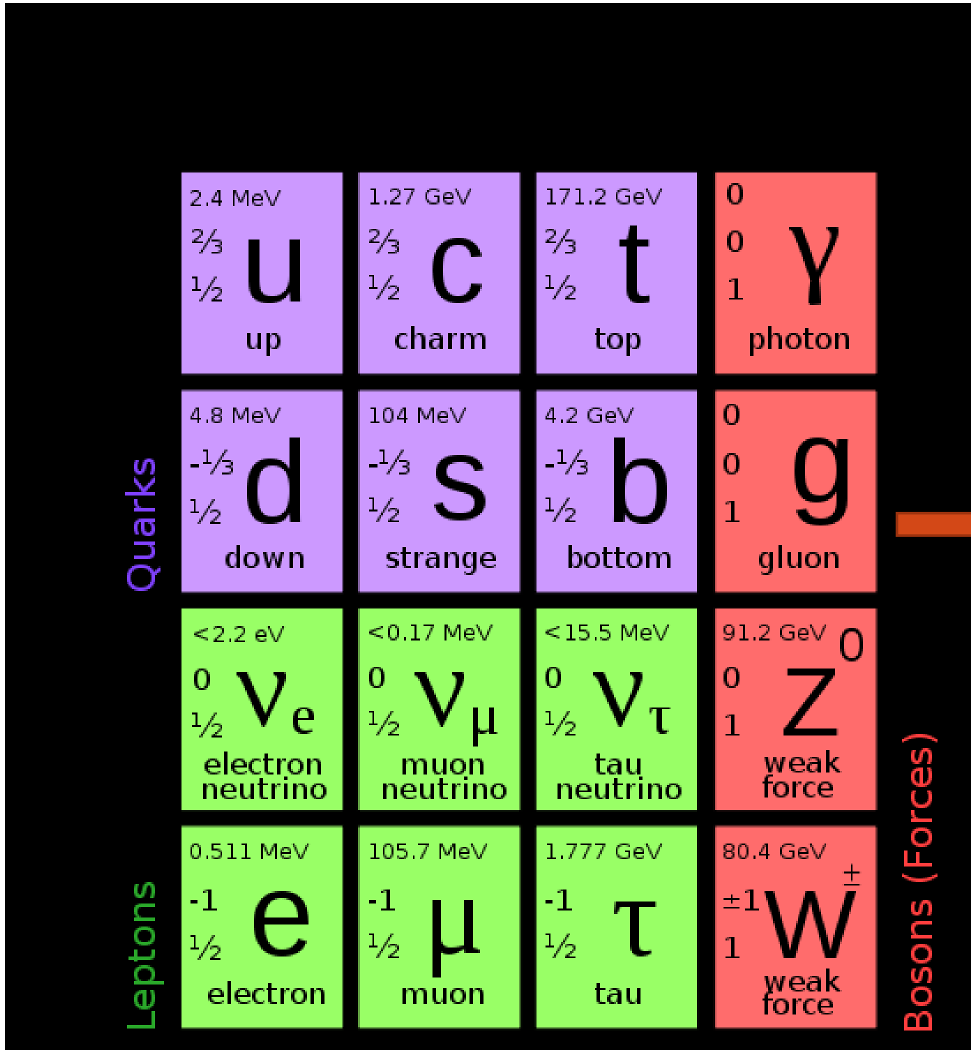
Nuclear level

$$d \rightarrow u + e^{-} + \overline{\nu}_{e}$$

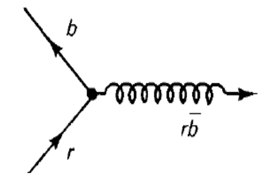
Quark level

Fundamental Particles

Periodic table of SM



Quarks come in three different colors: r, g, b



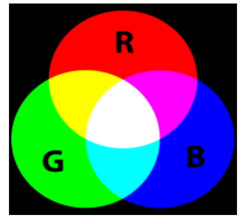
$$3 \otimes 3 = 8 \oplus 1$$

The gluons are massless vector bosons; like the photon, they have spin of 1. However against the photons gluons interact with each other due to color charge

The particles acquire their masses interacting with the Higgs boson which was recently discovered at Large Hadron Colliders

Hadrons: -subatomic colorless particles made of quarks

Hadrons
- Mesons: one quark and one anti-quark (color+anticolor=white)
- Baryons: 3 quarks



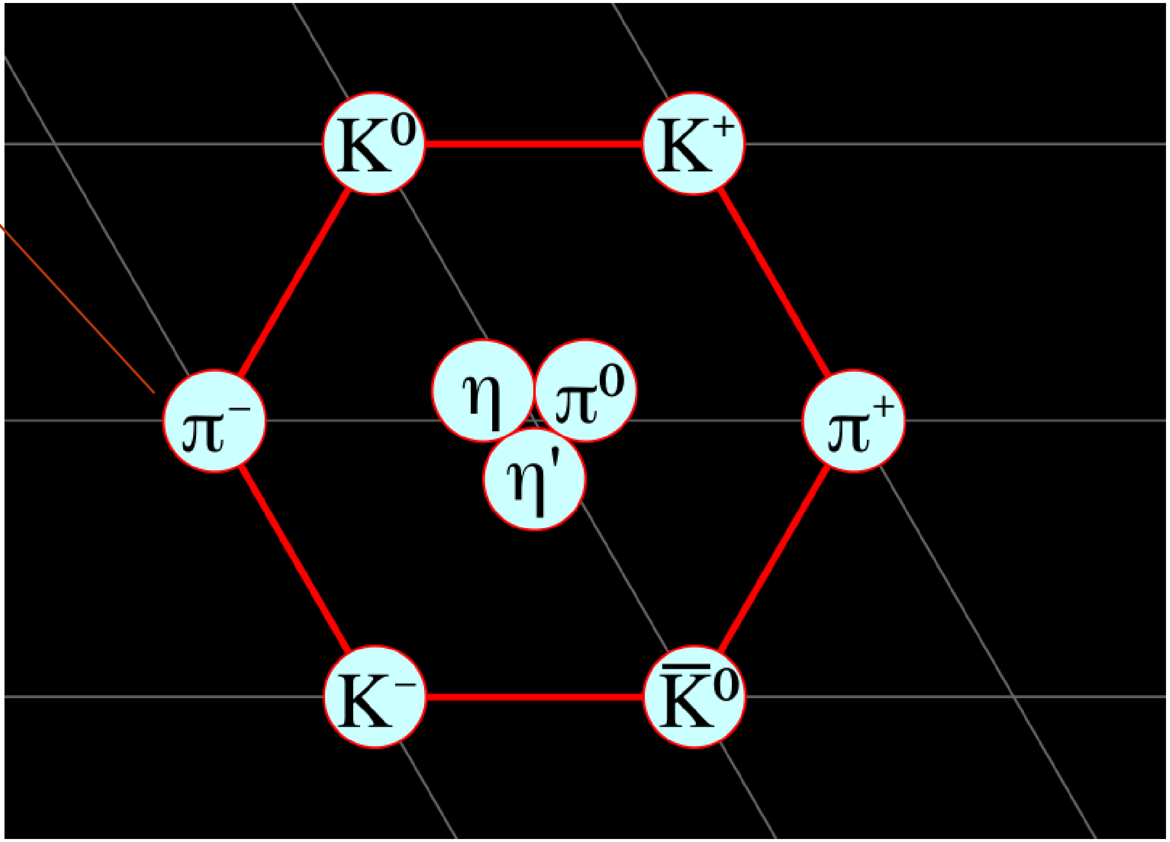
Exotic particles (newly experimentally seen!!!!!!): Glueballs, tetraquark (dimeson), pentaquarks, hexaquarks (dibaryon), ...

Types of mesons

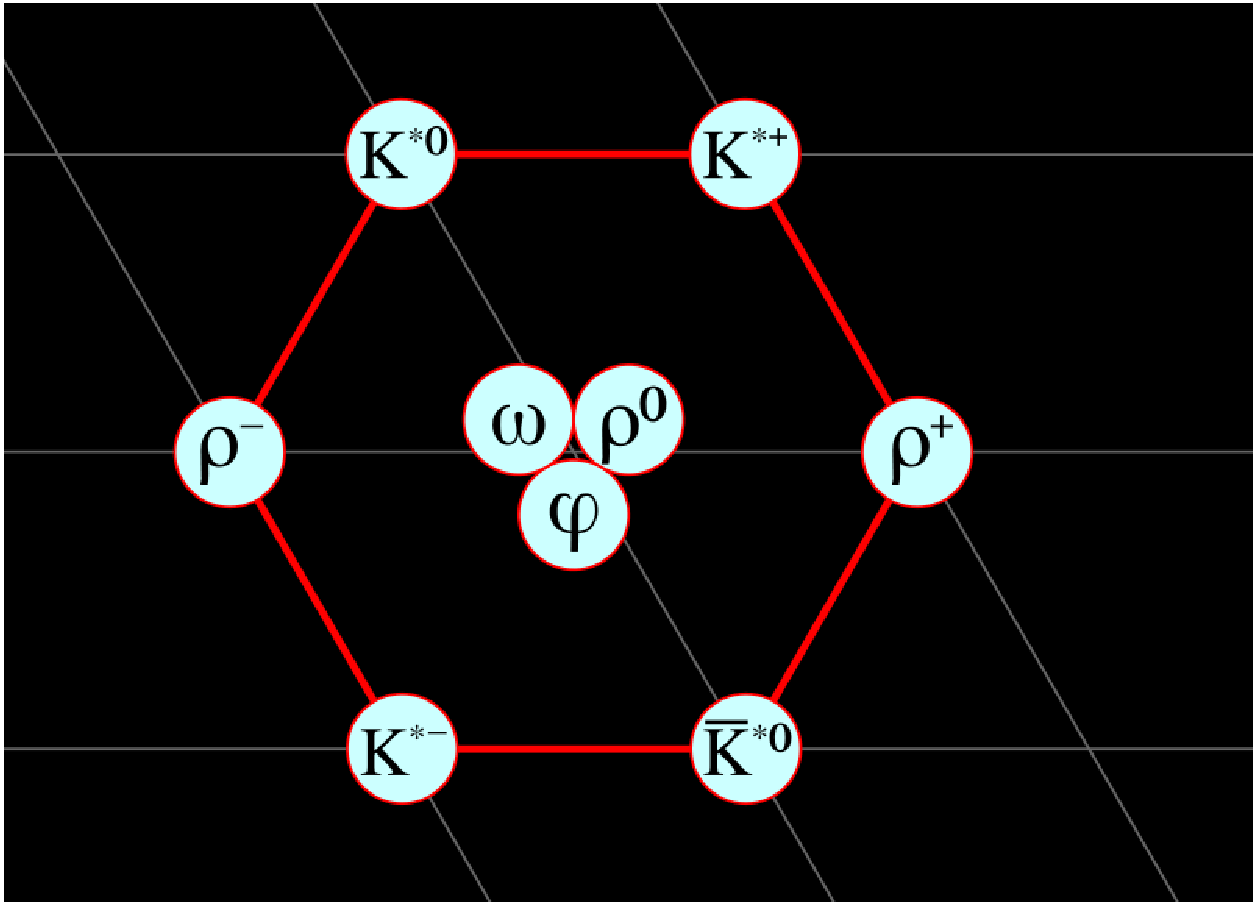
	S	L	P	J	J^P
Scalar	1	1	+	0	0^+
Pseudoscalar	0	0	–	0	0^-
Vector	1	0	–	1	1^-
Axial-vector	0	1	+	1	1^+
Tensor	1	1	+	2	2^+
Pseudotensor	1	1	-	2	2^-

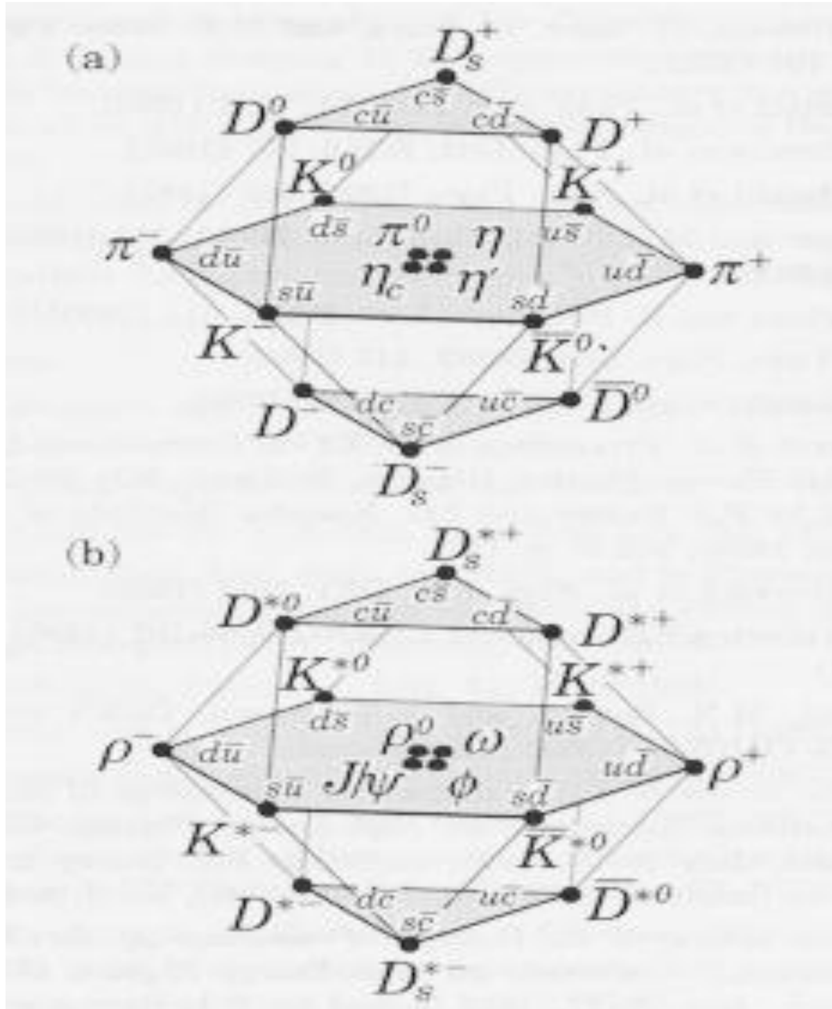
PS Mesons (light)

anti u-d



Vector Mesons (light)




(a)
D_s^+
D^0
$c\bar{u}$
$c\bar{s}$
$c\bar{d}$
D^+
K^0
K^+
$d\bar{s}$
$u\bar{s}$
π^-
π^0
η
π^+
$d\bar{u}$
$u\bar{d}$
η_c
η'
$s\bar{u}$
$s\bar{d}$
K^-
$\bar{K}^0$
$d\bar{c}$
$u\bar{c}$
D^-
$\bar{D}^0$
$s\bar{c}$
D_s^-
(b)
D_s^{*+}
D^{*0}
$c\bar{u}$
$c\bar{s}$
$c\bar{d}$
D^{*+}
K^{*0}
K^{*+}
$d\bar{s}$
$u\bar{s}$
ρ^-
ρ^0
ω
ρ^+
$d\bar{u}$
$u\bar{d}$
J/ψ
ϕ
$s\bar{u}$
$s\bar{d}$
K^{*-}
$\bar{K}^{*0}$
$d\bar{c}$
$u\bar{c}$
D^{*-}
$\bar{D}^{*0}$
$s\bar{c}$
D_s^{*-}

Light Baryons

To construct the light baryons, we consider the SU(3) flavor symmetry with quarks $q^1 = u$, $q^2 = d$ and $q^3 = s$.

$$\boxed{3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8' \oplus 1}$$

$$\begin{aligned}
q^i \times q^j \times q^k &= \frac{1}{6}(q^i q^j q^k + q^j q^i q^k + q^i q^k q^j + q^j q^k q^i + q^k q^j q^i + q^k q^i q^j) \\
&+ \frac{1}{6}(2q^i q^j q^k + 2q^j q^i q^k - q^i q^k q^j - q^j q^k q^i - q^k q^j q^i - q^k q^i q^j) \\
&+ \frac{1}{6}(2q^i q^j q^k - 2q^j q^i q^k + q^i q^k q^j - q^j q^k q^i + q^k q^j q^i - q^k q^i q^j) \\
&+ \frac{1}{6}(q^i q^j q^k - q^j q^i q^k - q^i q^k q^j + q^j q^k q^i - q^k q^j q^i + q^k q^i q^j) \\
&= T^{\{ijk\}} + T^{\{ij\}k} + T^{[ij]k} + T^{[ijk]},
\end{aligned}$$

i, j and k goes from 1 to 3.

Light decuplet baryons: (Spin 3/2)

$$
\begin{array}{lccccccc}
s = 0 & \Delta^- & & \Delta^0 & & \Delta^+ & & \Delta^{++} \\
s = -1 & & \Sigma^{*-} & & \Sigma^{*0} & & \Sigma^{*0} & \\
s = -2 & & & \Xi^{*-} & & \Xi^{*0} & & \\
s = -3 & & & & \Omega^{*-} & & & \\
 & I_3 = -\frac{3}{2} & -1 & -\frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{3}{2}
\end{array}
$$

quark contents for decuplet baryons are

$$
\begin{array}{c}
ddd \quad udd \quad uud \quad uuu \\
sdd \quad sud \quad suu \\
ssd \quad ssu \\
sss
\end{array}
$$

Light octet baryons: (spin 1/2)

$$
\begin{array}{lccccc}
s = 0 & & n & & p & \\
s = -1 & \Sigma^- & & (\Sigma^0, \Lambda) & & \Sigma^+ \\
s = -2 & & \Xi^- & & \Xi^0 & \\
 & I_3 = -1 & -\frac{1}{2} & 0 & \frac{1}{2} & 1
\end{array}
$$

or in terms of the quark contents:

$$
\begin{array}{ccc}
 & udd \quad uud & \\
sdd & sud & suu \\
 & ssd \quad ssu &
\end{array}
$$

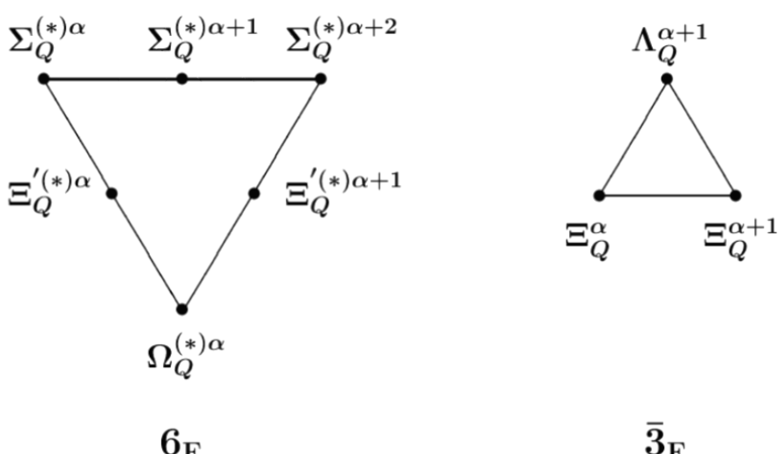
Light singlet (spin 1/2): $\Lambda(uds)$

Heavy Baryons: contain one-two or three heavy "b" or "c" quarks

The baryons containing single heavy quark can be classified according to the spin of the light degrees of freedom in the heavy quark limit, $m_Q \to \infty$.

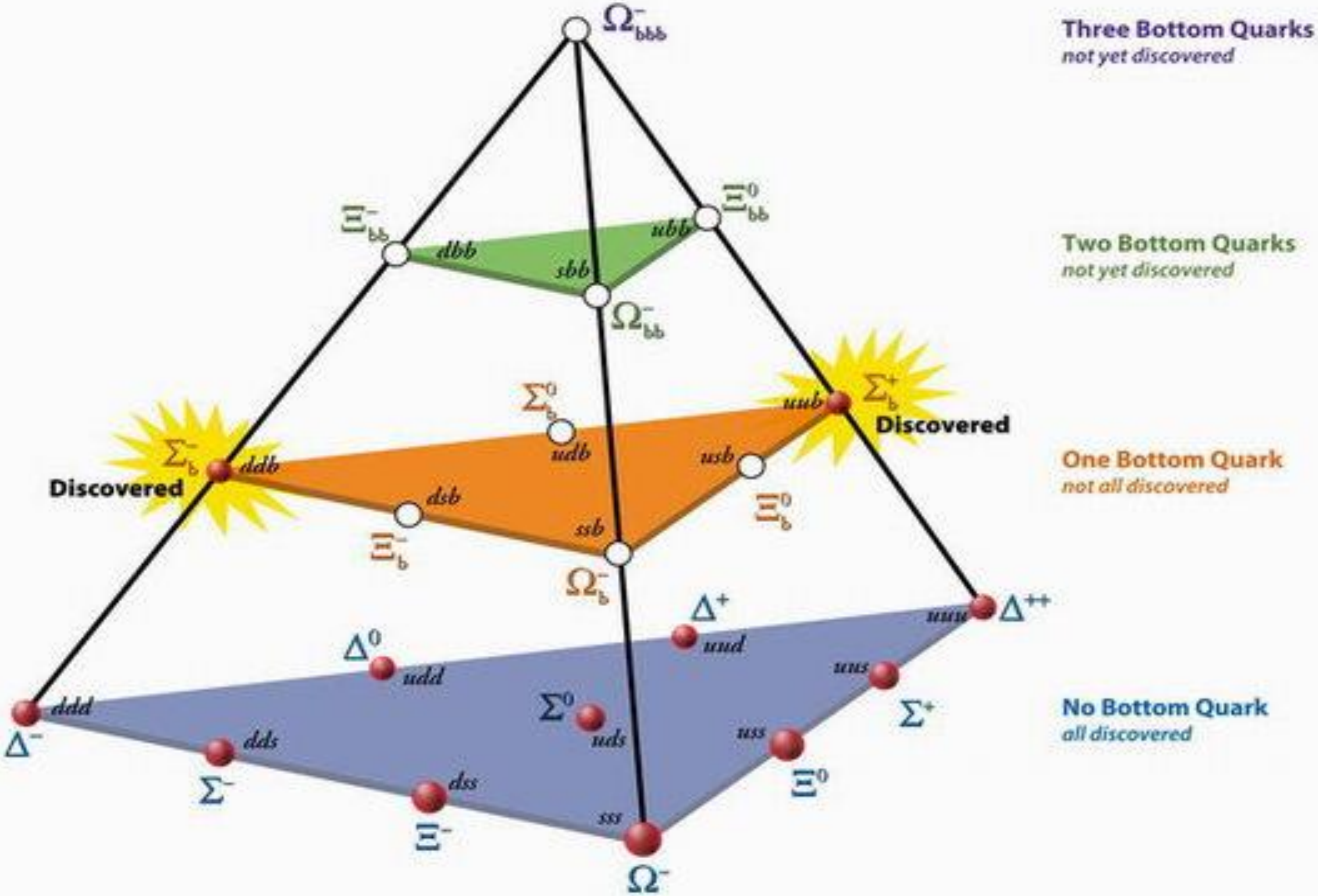
$$\boxed{3 \otimes 3 = \bar{3}_F \oplus 6_F}$$

The spin of the light diquark is either $S = 1$ for 6_F, or $S = 0$ for $\bar{3}_F$. The ground state will have angular momentum $l = 0$. Therefore, the spin of the ground state is $1/2$ for $\bar{3}_F$, while it can be both $3/2$ or $1/2$ for 6_F

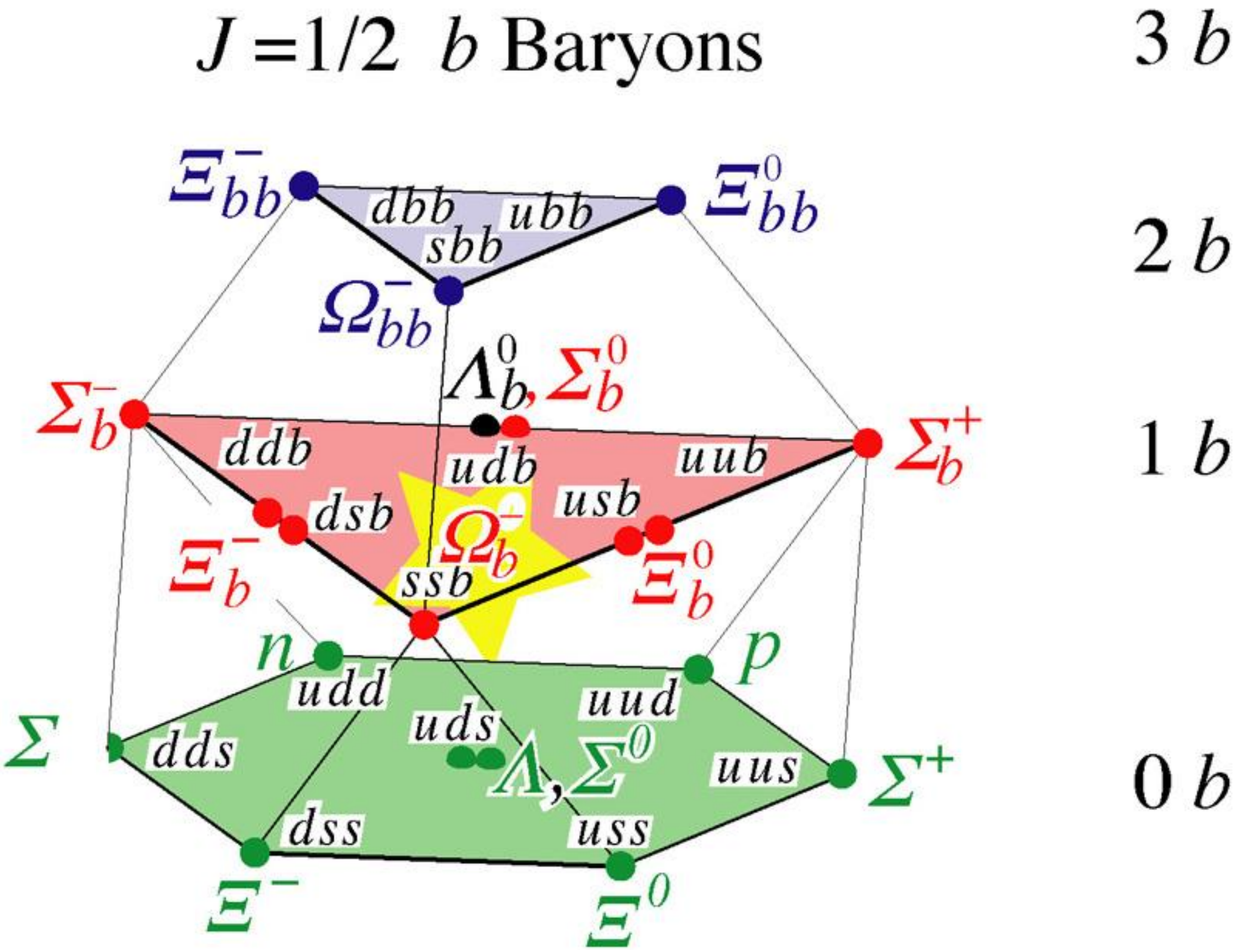


α, $\alpha+1$, $\alpha+2$ determine the charges of baryons ($\alpha = -1$ or 0), and the asterix $(*)$ denote $J^P = \frac{3}{2}^+$ states.

Baryons with Up, Down, Strange and Bottom Quarks and Highest Spin ($J = 3/2$)



Only symmetric part



All hadrons except protons are unstable and decay. Neutrons are stable only inside the atomic nuclei.

In most of non-perturbative approaches, hadrons are represented by their interpolating currents.

İnterpolating currents→ correspond to the wavefunctions in quark model.

interpolating currents of mesons

scalar: $J^S(x) = \bar{q}_1(x) q_2(x)$

Pseudoscalar: $J^{PS}(x) = \bar{q}_1(x) \gamma_5 \ q_2(x)$

Vector: $J^V_\mu(x) = \bar{q}_1(x) \gamma_\mu \ q_2(x)$

Axial-Vector: $J^{AV}_\mu(x) = \bar{q}_1(x) \gamma_\mu \gamma_5 q_2(x)$

interpolating current of each particle can create that particle from the vacuum with the same quantum numbers as the interpolating current.

Interpolating currents for the light and heavy baryons:

light spin ½ (octet) baryons:

$$\eta = A\varepsilon^{abc}\left\{(q_1^{aT}Cq_2^b)\gamma_5 q_3^c - (q_2^{aT}Cq_3^b)\gamma_5 q_1^c + \beta(q_1^{aT}C\gamma_5 q_2^b)q_3^c - \beta(q_2^{aT}C\gamma_5 q_3^b)q_1^c\right\}$$

	A	q_1	q_2	q_3
Σ^0	$-\sqrt{1/2}$	u	s	d
Σ^+	$1/2$	u	s	u
Σ^-	$1/2$	d	s	d
p	$-1/2$	u	d	u
n	$-1/2$	d	u	d
Ξ^0	$1/2$	s	u	s
Ξ^-	$1/2$	s	d	s

$$2\eta^{\Sigma^0}(d \to s) + \eta^{\Sigma^0} = -\sqrt{3}\eta^{\Lambda} \ ,$$

$$2\eta^{\Sigma^0}(u \to s) + \eta^{\Sigma^0} = \sqrt{3}\eta^{\Lambda} \ .$$

light spin 3/2 (Decuplet) baryons

$$\eta_\mu = A'\varepsilon^{abc}\left\{(q_1^{aT}C\gamma_\mu q_2^b)q_3^c + (q_2^{aT}C\gamma_\mu q_3^b)q_1^c + (q_3^{aT}C\gamma_\mu q_1^b)q_2^c\right\}$$

	A'	q_1	q_2	q_3
Σ^{*0}	$\sqrt{2/3}$	u	d	s
Σ^{*+}	$\sqrt{1/3}$	u	u	s
Σ^{*-}	$\sqrt{1/3}$	d	d	s
Δ^{++}	$1/3$	u	u	u
Δ^{+}	$\sqrt{1/3}$	u	u	d
Δ^{0}	$\sqrt{1/3}$	d	d	u
Δ^{-}	$1/3$	d	d	d
Ξ^{*0}	$\sqrt{1/3}$	s	s	u
Ξ^{*-}	$\sqrt{1/3}$	s	s	d
Ω^{-}	$1/3$	s	s	s

Heavy spin ½ baryons

$$\eta_Q^{(s)} = -\frac{1}{\sqrt{2}}\epsilon^{abc}\left\{\left(q_1^{aT}CQ^b\right)\gamma_5 q_2^c + \beta\left(q_1^{aT}C\gamma_5 Q^b\right)q_2^c - \left[\left(Q^{aT}Cq_2^b\right)\gamma_5 q_1^c + \beta\left(Q^{aT}C\gamma_5 q_2^b\right)q_1^c\right]\right\},$$

$$\eta_Q^{(anti-t)} = \frac{1}{\sqrt{6}}\epsilon^{abc}\left\{2\left(q_1^{aT}Cq_2^b\right)\gamma_5 Q^c + 2\beta\left(q_1^{aT}C\gamma_5 q_2^b\right)Q^c + \left(q_1^{aT}CQ^b\right)\gamma_5 q_2^c + \beta\left(q_1^{aT}C\gamma_5 Q^b\right)q_2^c\right.$$
$$\left. + \left(Q^{aT}Cq_2^b\right)\gamma_5 q_1^c + \beta\left(Q^{aT}C\gamma_5 q_2^b\right)q_1^c\right\}$$

Q=b or c

	$\Sigma_{b(c)}^{+(++)}$	$\Sigma_{b(c)}^{0(+)}$	$\Sigma_{b(c)}^{-(0)}$	$\Xi_{b(c)}^{-(0)'}$	$\Xi_{b(c)}^{0(+)'}$	$\Omega_{b(c)}^{-(0)}$	$\Lambda_{b(c)}^{0(+)}$	$\Xi_{b(c)}^{-(0)}$	$\Xi_{b(c)}^{0(+)}$
q_1	u	u	d	d	u	s	u	d	u
q_2	u	d	d	s	s	s	d	s	s

Heavy spin 3/2 baryons

$$\eta_\mu = A\epsilon^{abc}\left\{(q_1^a C\gamma_\mu q_2^b)Q^c + (q_2^a C\gamma_\mu Q^b)q_1^c + (Q^a C\gamma_\mu q_1^b)q_2^c\right\}$$

	$\Sigma_{b(c)}^{*+(++)}$	$\Sigma_{b(c)}^{*0(+)}$	$\Sigma_{b(c)}^{*-(0)}$	$\Xi_{b(c)}^{*0(+)}$	$\Xi_{b(c)}^{*-(0)}$	$\Omega_{b(c)}^{*-(0)}$
q_1	u	u	d	u	d	s
q_2	u	d	d	s	s	s
A	$\sqrt{1/3}$	$\sqrt{2/3}$	$\sqrt{1/3}$	$\sqrt{2/3}$	$\sqrt{2/3}$	$\sqrt{1/3}$

Interpolating currents for doubly heavy baryons

$$
\begin{aligned}
\eta^S &= \frac{1}{\sqrt{2}}\epsilon_{abc}\Big\{(Q^{aT}Cq^b)\gamma_5 Q'^c + (Q'^{aT}Cq^b)\gamma_5 Q^c + \beta(Q^{aT}C\gamma_5 q^b)Q'^c + \beta(Q'^{aT}C\gamma_5 q^b)Q^c\Big\}, \\
\eta^A &= \frac{1}{\sqrt{6}}\epsilon_{abc}\Big\{2(Q^{aT}CQ'^b)\gamma_5 q^c + (Q^{aT}Cq^b)\gamma_5 Q'^c - (Q'^{aT}Cq^b)\gamma_5 Q^c + 2\beta(Q^{aT}C\gamma_5 Q'^b)q^c \\
&+ \beta(Q^{aT}C\gamma_5 q^b)Q'^c - \beta(Q'^{aT}C\gamma_5 q^b)Q^c\Big\},
\end{aligned}
$$

T. M. Aliev, K. Azizi, M. Savci, Nucl.Phys. A895 (2012) 59-70 arXiv:1205.2873 [hep-ph]

Exotic Hadrons:

In recent years, beside the standard hadrons, some new objects composed of combinations of quarks and gluons but in different structures have been predicted and some of them have been detected and confirmed by different experiments.

These new hadrons are called the exotic particles. →

tetraquarks,
pentaquarks,
glueballs,
hybrids,
meson molecules,
dibaryons, etc.

$B^{\pm} \to J/\psi K^{\pm}\pi^{+}\pi^{-}$

Particle	J^{PC}	M (MeV)	Γ (MeV)	Decay Modes	EXP.
$X(3872)$	1^{++}	3871.4 ± 0.6	< 2.3	$\gamma J/\psi, D\bar{D}^*$	Belle, CDF, D0, BaBar (2003)
$Z_c(3900)$	1^{+}	3899 ± 6	46 ± 22	$\pi^{\pm} J/\psi$	Belle, BESIII* (2013)
$Z(3930)$	2^{++}	3929 ± 5	29 ± 10	$D\bar{D}$	Belle (2006)
$Z_c(4020)$	$1(?)^{+(?)-}$	4024 ± 2	10 ± 3	$h_c \pi^{+}$	BESIII (2013)
$Z_1(4050)$	?	4051^{+24}_{-23}	82^{+51}_{-29}	$\pi^{\pm}\chi_{cl}$	Belle (2008)
$Z(4200)$	1^{+-}	4196^{+35}_{-32}	370^{+99}_{-149}	$J/\psi \pi^{+}$	Belle (2014)
$Z_2(4250)$	?	4248^{+185}_{-45}	177^{+320}_{-72}	$\pi^{\pm}\chi_{cl}$	Belle (2008)
$Z(4430)$	?	4433 ± 5	45^{+35}_{-18}	$\pi^{\pm}\psi^{l}$	Belle (2007)
$X(3915)$	$0/2^{++}$	3914 ± 4	28^{+12}_{-14}	$\omega J/\psi$	Belle (2009)
$X(3940)$	$0^{?+}$	3942 ± 9	37 ± 17	$D\bar{D}^*$	Belle (2005)
$Y(3940)$	$?^{?+}$	3943 ± 17	87 ± 34	$\omega J/\psi$	Belle , BaBar (2005)
$Y(4008)$	1^{--}	4008^{+82}_{-49}	226^{+97}_{-80}	$\pi^{+}\pi^{-} J/\psi$	Belle (2005)
$Y(4140)$	$?^{?+}$	4144 ± 3	17 ± 9	$J/\psi \phi$	CDF, CMS (2011)
$X(4160)$	$0^{?+}$	4156 ± 29	139^{+113}_{-65}	$D^*\bar{D}^*$	Belle (2008)
$Y(4260)$	1^{--}	4264 ± 12	83 ± 22	$\pi^{+}\pi^{-} J/\psi$	Belle , BaBar* , CLEO (2005)
$Y(4350)$	1^{--}	4361 ± 13	74 ± 18	$\pi^{+}\pi^{-}\psi^{l}$	Belle, BaBar* (2007)
$X(4630)$	1^{--}	4634^{+9}_{-11}	92^{+41}_{-32}	$\Lambda_c^{+}\Lambda_c^{-}$	Belle (2008)
$Y(4660)$	1^{--}	4664 ± 12	48 ± 15	$\pi^{+}\pi^{-}\psi^{l}$	Belle (2007)
$Y_s(2175)$	1^{--}	2175 ± 8	58 ± 26	$\phi f_0(980)$ $\pi^{+}\pi^{-} J/\psi$	Belle, BaBar*, BESII (2006)
$Z_b(10610)$	1^{+}	$10{,}607 \pm 2$	18.4 ± 2.4	$\pi^{\pm} h_b(1{,}2P), \pi^{\pm}\Upsilon(1{,}2{,}3S)$	Belle (2011)
$Z_b(10650)$	1^{+}	$10{,}652 \pm 2$	11.5 ± 2.2	$\pi^{\pm} h_b(1{,}2P), \pi^{\pm}\Upsilon(1{,}2{,}3S)$	Belle (2011)
$Y_b(10890)$	1^{--}	$10{,}890 \pm 3$	55 ± 9	$\pi^{+}\pi^{-}\Upsilon(1{,}2{,}3S)$	Belle (2008)

Particles	J^{PC}	Structure	Mass	Residue	Strong Decay	Electromagnetic Decay	Weak Decay
$X(3872)$	1^{++}	$D\bar{D}^*$, dörtkuark, $(c\bar{c}q\bar{q})$,D_sD^*, $\bar{D}D^*$,hybrid	[54]	?	[52-54,61-74,76,119]	[145]	?
$Z_c(3900)$	1^+	dörtkuark	?	?	[76]	[145]	?
$Z(3930)$	2^{++}	$D\bar{D}$	?	?	[54]	?	?
$Z_c(4020)$	$1(?)^{+(?)-}$	$D^*\bar{D}^*$	?	?	[144]	[145]	?
$Z_c(4025)$	$1^-,2^+$	Dörtkuark,$D^*\bar{D}^*$, hadronik molekül	[79,82,83]	?	[76,79-83,91,144]	?	?
$Z_1(4050)$	?	D^*D^*	[60]	[60]	[60]	?	?
$Z(4200)$	1^{+-}	dörtkuark	?	?	[143]	?	?
$Z_2(4250)$	?	D_1D	[60]	[60]	[52,60]	?	?
$Z(4430)$	$0^-,1^-$	D^*D_1,$D^*\bar{D}_1$, dörtkuark	[55]	[55]	[52,55-59]	[145]	?
$X(3915)$	$0/2^{++}$	?	?	?	?	?	?
$X(3940)$	$0^{?+}$	$D^*\bar{D}$	?	?	[54]	?	?
$Y(3940)$	$?^{?+}$	?	?	?	?	[145]	?
$Y(4008)$	1^{--}	?	?	?	?	?	?
$X(4160)$	$0^{?+}$	?	?	?	?	?	?
$Y(4260)$	1^{--}	hybrid	?	?	[84-86]	[145]	?
$Y(4350)$	1^{--}	?	?	?	?	?	?
$X(4630)$	1^{--}	?	?	?	?	?	?
Y(4630)	?	$\Lambda_c\bar{\Lambda}_c$	?	?	[90]		
$Y(4660)$	1^{--}	dörtkuark	[76]	?	[76,78]	?	?
$Y_s(2175)$	1^{--}	?	?	?	?	?	?
$X(1835)$	0^{-+}	glueball	?	?	[141,142]	?	?
$X(3020)$	?	glueball	[130]	?	[130]	?	?
?	?	$ud\bar{s}\bar{s}$	[51]	?	[51]	?	?
?	?	$qqqqQ$	[89]	?	[89]	?	?
?	?	$\bar{D}^{(*)}NN$ ve $B^{(*)}NN$	[92]	?	[92]	?	?
?	?	hybrid $\bar{b}Gc$	[117]	?	[117]	?	?
$\pi_1(1400)$	1^{-+}	hybrid	[96-100]	[96,99,100]	[96-100]	?	?
$\pi_1(1600)$	1^{-+}	hybrid	[101,102]	?	[101,102]	?	?
$\pi_1(2000)$	1^{-+}	hybrid	[108,111]	?	[108,111]	?	?
$\pi_1(2015)$	1^{-+}	hybrid	[116]	?	[116]	?	?
$d_1^*(1956)$	?	dibaryon	?	?	[95]	?	?
$D_{03}(2370)$	?	$\Delta\Delta$	?	?	[94]	?	?
$\theta_c(3250)$	?	beşkuark	?	?	?	?	?
$Z_b(10610)$	1^+	?	?	?	?	?	?
$Z_b(10650)$	1^+	$B^*\bar{B}^*$	?	?	[144]	?	?
$Y_b(10890)$	1^{--}	Dörtkuark	[62]	?	[62]	?	?

Introduction to QCD: perturbative & non-perturbative

- ✓ The aim of physics → to describe 4 fundamental interactions in a simple and unified way.
- ✓ Up to now → We have electroweak theory. GUT: electroweak+strong interaction
- ✓ Theory of strong interaction → QCD

Hence, first we need to understand the QCD.

- ✓ Properties of QCD → asymptotic freedom ----- confinement

asymptotic freedom → Has been proven:2004 Nobel

confinement → There is no analytic proof yet

- ✓ As a result of confinement → hadronization

To understand the QCD, the best laboratory is hadron physics.

Hadron or Particle Physics

- Experiment
- Lattice
- Theory
- Phenomenology

Experiment and Lattice are more reliable

But facilities in EXP. & Lattice are very limited

Phenomenological models play an important role

QCD Lagrangian:

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_q \bar{\psi}_q (i \not{D} - m_q) \psi_q,$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f^{abc} G^b_\mu G^c_\nu ,$$

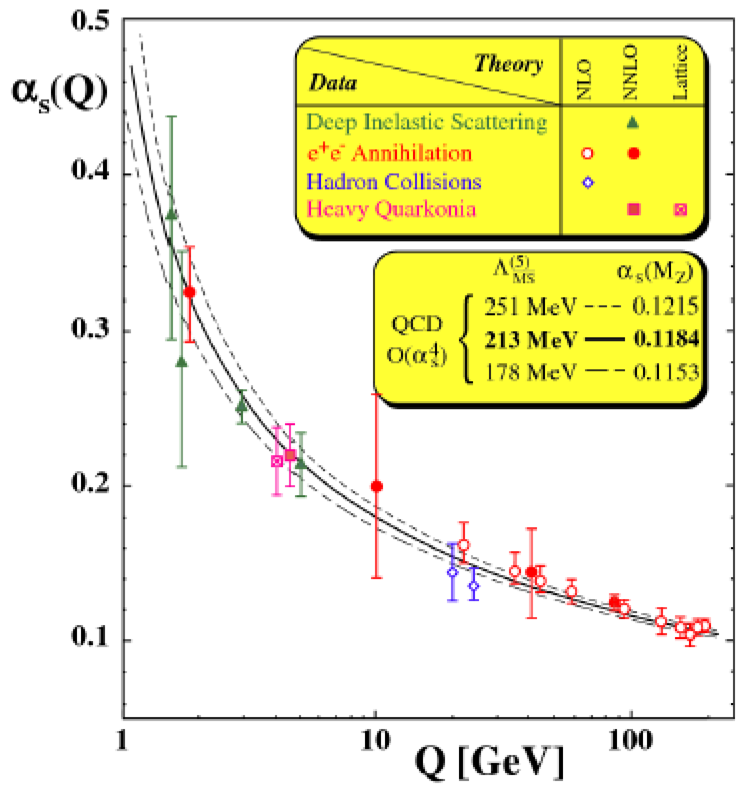
$$\not{D} = \gamma^\mu \mathcal{D}_\mu$$

$$\mathcal{D}_\mu = \partial_\mu + i \frac{g}{2} \lambda^a G^a_\mu$$

In principle, besides the dinamics of quarks and gluons, this lagrangian should be responsible for hadrons and determination of their properties. Unfortunately, it is valid only in a limited region.

➢In very large momentum transfers, due to “asymptotic freedom” we can use this Lagrangian and perturbation theory. However, when energy is decreased the coupling constant between quark-gluon becomes large and perturbation theory fails.

S. Bethke, J Phys G 26, R27 (2000)

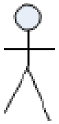


- Hadrons are formed in low energies far from the “asymptotic freedom” and perturbative region.

→ To investigate their properties, we need some nonperturbative approaches.

Some nonperturbative methods:

- ✓ Different “relativistic” and “nonrelativistic” quark models
- ✓ Chiral perturbation theory
- ✓ HQET
- ✓ Nambu–Jona-Lasinio model
- ✓ Lattice QCD
- ✓ ....
- ✓ QCD sum rules and its extension: light cone QCDSR.



One of the most applicable tools to hadron physics →

- does not include any free parameter
- is based on QCD Lagrangianin
- gives results in a good consistency with existing EXP. data
- its results agree with Lattice predi.
- can be expanded to thermal QCD

....

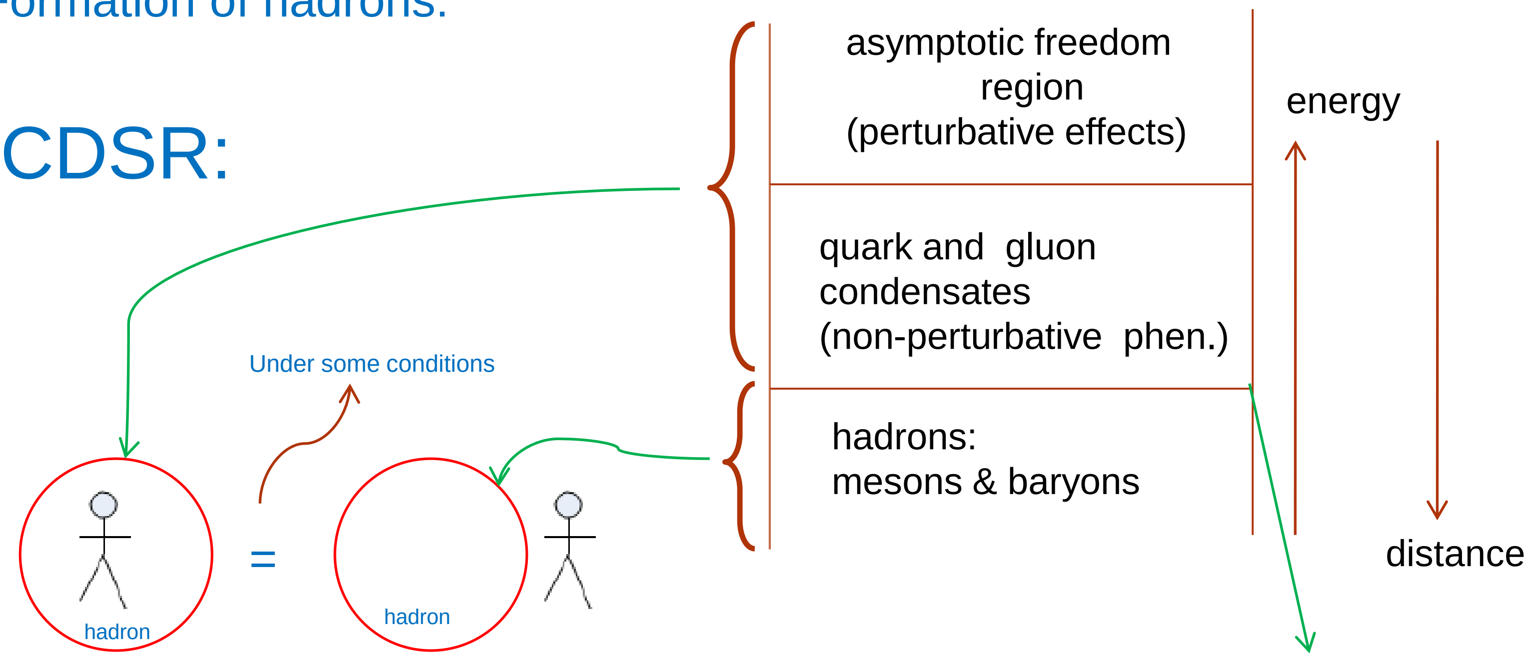
QCD Sum Rules:

- Among the non-perturbative methods, QCD Sum Rule provide the most powerful information about the hadron properties.
- In 1979, the QCD sum rule proposed by Shifman, Vainshtein, and Zakharov to calculate the non-perturbative contributions in meson physics
- This method is generalized to baryons by Ioffe in 1981
- In 1986, it is generalized to finite temperature and density by Shaposhnikov and Bochkarev

❖ Formation, internal structure and spectroscopy of hadrons

➢ Formation of hadrons:

QCDSR:



Λ_{QCD} ~ 200 MeV ~ 1 fm^{-1}

Hadronic scale

➢ internal structures of hadrons:

✓ Some hadrons like D_{sJ} mesons have not exactly known quark structure. Using QCD sum rules approach, we can get information about their internal structures as well as the strong interactions inside them.

QCD sum rules: technical details

As previously mentioned, in this method, hadrons are represented by their interpolating quark currents. The main object in this approach is the so called **correlation function** expressed in terms of these interpolating currents.

- Types of the corr. func.:
  1) two point

Time ordering product

$$T = i \int d^4x e^{ipx} \langle 0 \mid T\{\eta_1(x)\bar{\eta}_2(0)\} \mid 0 \rangle,$$

we obtain: mass, residue (lep. decay. cons.)
2) three point

$$T = i \int d^4x d^4y e^{ipx} e^{-ipy} \langle 0 \mid T\{\eta_1(x)\eta^{tr}(0)\bar{\eta}_2(y)\} \mid 0 \rangle,$$

we calculate: form factors used in decay rates, branching ratio ...

3) light cone: The main idea, here, is to expand the time ordered products of currents in the correlation function near the light cone, $x^2 \simeq 0$. Instead of the expansion of the long-distance effects in terms of operators with different mass dimensions in traditional three-point sum rules, in LCQSR , those effects are parameterized in terms of light-cone distribution amplitudes with different twists. Twist is defined as the difference between the mass dimension and the spin of local operators. In light cone sum rules, we consider T-product of two quark currents between vacuum and an on-shell state such as photon,

$$T = i \int d^4x e^{ipx} \langle \gamma \mid T\{\eta_1(x)\bar{\eta}_2(0)\} \mid 0 \rangle,$$

we obtain form factors used in electromagnetic moments, decay rates, branching ratio

This correlation function is calculated in two different approaches:

- 1) In the phenomenological side, it is saturated by a tower of hadrons with the same flavor quantum numbers.
- 2) On the quark level, it describes a hadron as quarks and gluons interacting in QCD vacuum (QCD side) via the operator product expansion (OPE), where the short- and long-distance quark-gluon interactions are separated. The former is calculated using QCD perturbation theory, whereas the latter are parameterized in terms of the vacuum condensates or light-cone distribution amplitudes.
- The physical quantities are determined matching two different representations of the correlation function.

Leptonic Decay Constant of $D_2^*(2460)$ Tensor Meson

The starting point is to consider the following two-point correlation function:

$$\Pi_{\mu\nu,\alpha\beta}(q^2) = i \int d^4x \; e^{iq\cdot(x-y)} \langle 0 \mid \mathcal{T}[J_{\mu\nu}(x)\bar{J}_{\alpha\beta}(y)] \mid 0 \rangle,$$

the interpolating current of the $D_2^*(2460)$ tensor meson:

$$J_{\mu\nu}(x) = \frac{i}{2}\left[\bar{u}(x)\gamma_\mu \overleftrightarrow{\mathcal{D}}_\nu (x)c(x) + \bar{u}(x)\gamma_\nu \overleftrightarrow{\mathcal{D}}_\mu (x)c(x)\right],$$

Physical Side:

In the physical side, the correlation function is obtained inserting complete set of hadronic state having the same quantum numbers as the interpolating current $j_{\mu\nu}$
After performing integral over four-x, we obtain the physical side of correlation function as following form:

$$\Pi_{\mu\nu,\alpha\beta} = \frac{\langle 0 \mid J_{\mu\nu}(0) \mid D_2^*(2460)\rangle\langle D_2^*(2460) \mid \bar{J}_{\alpha\beta}(0) \mid 0\rangle}{m^2_{D_2^*(2460)} - q^2} + \cdot(4$$

$$\langle 0 \mid J_{\mu\nu}(0) \mid D_2^*(2460)\rangle = f_{D_2^*(2460)} m^3_{D_2^*(2460)} \epsilon_{\mu\nu}. \qquad (5)$$

Combining Eq. (4) and Eq. (5) and performing summation over polarization tensor via:

$$\Pi_{\mu\nu,\alpha\beta} = \frac{f^2_{D_2^*(2460)} m^6_{D_2^*(2460)}}{m^2_{D_2^*(2460)} - q^2} \left\{ \frac{1}{2}(g_{\mu\alpha}\, g_{\nu\beta} + g_{\mu\beta}\, g_{\nu\alpha}) \right\} + \text{oth. str.}$$

QCD Side:

The correlation function in QCD side, is calculated in deep Euclidean region, $q^2 \ll 0$, by the help of operator product expansion (OPE) where the short and long distance contributions are separated. The short distance effects are calculated using the perturbation theory, while the long distance effects are parameterized in terms of quark and gluon condensates.

$$\Pi(q^2) = \int ds \frac{\rho^{pert}(s) + \rho^{nonpert}(s)}{s - q^2} \qquad (7)$$

the spectral density: $\rho(s) = \frac{1}{\pi} Im[\Pi(s)]$

Now, we proceed to calculate the spectral density ρ(s). Making use of the tensor current into the correlation function and contracting out all quark fields applying the Wick's theorem, we get:

$$
\begin{aligned}
\Pi &= \frac{i}{4}\int d^4x e^{iq(x-y)}\Big\{ Tr\Big[S_u(y-x)\gamma_\mu \overleftrightarrow{\mathcal{D}}_\nu(x) \overleftrightarrow{\mathcal{D}}_\beta(y) S_c(x-y)\gamma_\alpha\Big] \\
&\quad + [\beta\leftrightarrow\alpha] + [\nu\leftrightarrow\mu] + [\beta\leftrightarrow\alpha, \nu\leftrightarrow\mu]\Big\}.
\end{aligned}
$$

$$
S_c^{ij}(x-y) = \frac{i}{(2\pi)^4}\int d^4k e^{-ik\cdot(x-y)}\left\{\frac{\not{k}+m_c}{k^2-m_c^2}\delta_{ij}+\cdots\right\}
$$

$$
\begin{aligned}
S_u^{ij}(x-y) &= i\frac{\not{x}-\not{y}}{2\pi^2(x-y)^4}\delta_{ij} - \frac{m_u}{4\pi^2(x-y)^2}\delta_{ij} \\
&- \frac{\langle\bar{u}u\rangle}{12}\Big[1-i\frac{m_u}{4}(\not{x}-\not{y})\Big]\delta_{ij} \\
&- \frac{(x-y)^2}{192}m_0^2\langle\bar{u}u\rangle\Big[1-i\frac{m_u}{6}(\not{x}-\not{y})\Big]\delta_{ij}+\cdots
\end{aligned}
$$

After dimensional regularization and taking the imaginary part and selecting the coefficient of the aforesaid structure, the spectral densities are obtained as:

$$\rho^{pert}(s) = \frac{N_c}{960\,\pi^2 s^3}(m_c^2 - s)^4\,(2\,m_c^2 + 3\,s),$$

and

$$\rho^{nonpert}(s) = -\frac{N_c}{48s} m_c m_0^2 \langle \bar{u}u \rangle.$$

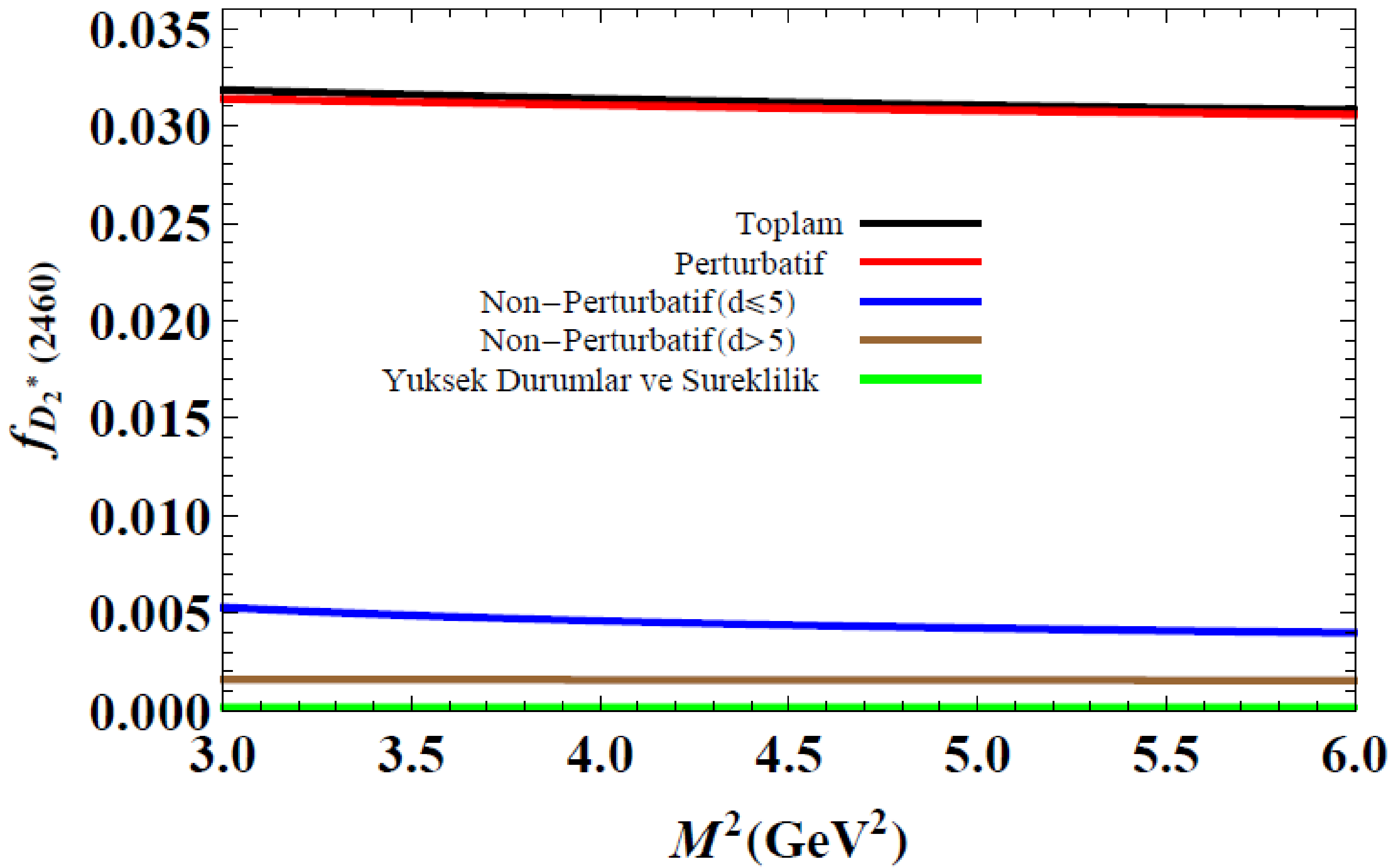
After achieving the correlation function in two different ways, we match these two different representations to obtain two-point QCD sum rules. In order to suppress contributions of the higher states and continuum, we apply Borel transformation with respect to the initial momentum squared, q^2, to both sides of the sum rules and use the quark-hadron duality assumption. As a result, the following sum rule for the meson-current coupling constant of the $D_2^*(2460)$ tensor meson is obtained:

$$f_{D_2^*}^2 e^{-m_{D_2^*}^2/M^2} = \frac{1}{m_{D_2^*}^6} \int_{m_c^2}^{s_0} ds \Big(\rho^{pert}(s) + \rho^{nonpert}(s) \Big) e^{-s/M^2},$$

The sum rules contain two auxiliary parameters:

$7.6 \leq s_0 \leq 8.7$

$3\ GeV^2 \leq M^2 \leq 6\ GeV^2$



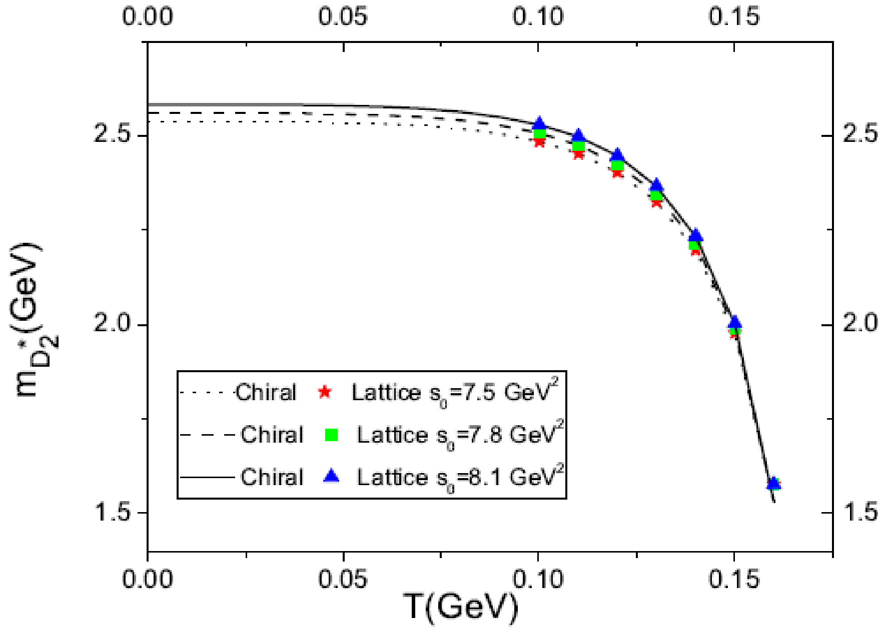
	Present Work	Experiment [7]
$m_{D_2^*(2460)}$	$(2.50 \pm 0.48)\ GeV$	$(2.4626 \pm 0.0007)\ GeV$
$f_{D_2^*(2460)}$	0.0317 ± 0.0092	-

PROPERTIES OF D_2* IN THERMAL QCD SUM RULES

$$
\begin{aligned}
S_q^{ij}(x-y) &= i\frac{\not{x}-\not{y}}{2\pi^2(x-y)^4}\delta_{ij} - \frac{m_q}{4\pi^2(x-y)^2}\delta_{ij} - \frac{\langle\bar{q}q\rangle}{12}\delta_{ij} + \frac{i}{3}\Big[(\not{x}-\not{y})\Big(\frac{m_q}{16}\langle\bar{q}q\rangle - \frac{1}{12}\langle u\Theta^f u\rangle\Big) \\
&+ \frac{1}{3}\Big(u.(x-y)\,\not{u}\langle u\Theta^f u\rangle\Big)\Big]\delta_{ij} + ...,
\end{aligned}
$$

where $\Theta^f_{\mu\nu}$ is the fermionic part of the energy momentum tensor and u_μ is the four-velocity of the heat bath. In the rest frame of the heat bath, $u_\mu = (1,0,0,0)$ and $u^2 = 1$.

	Present Work	Experiment [34]	Vacuum Sum Rules
$m_{D_2^*(2460)}(\mathrm{GeV})$	2.55 ± 0.46	2.4626 ± 0.0007	2.53 ± 0.45 [9]
$f_{D_2^*(2460)}$	0.027 ± 0.013	$-$	0.0228 ± 0.0068 [9]



Thermal average of the energy density obtained using the Lattice QCD:

$$\langle\Theta\rangle = 2\langle\Theta^f\rangle = 6\times 10^{-6} exp\Big[80(T-0.1)\Big]$$

$$\langle\bar{q}q\rangle = \langle 0|\bar{q}q|0\rangle\left[1-0.4\Big(\frac{T}{T_c}\Big)^4 - 0.6\Big(\frac{T}{T_c}\Big)^8\right]$$

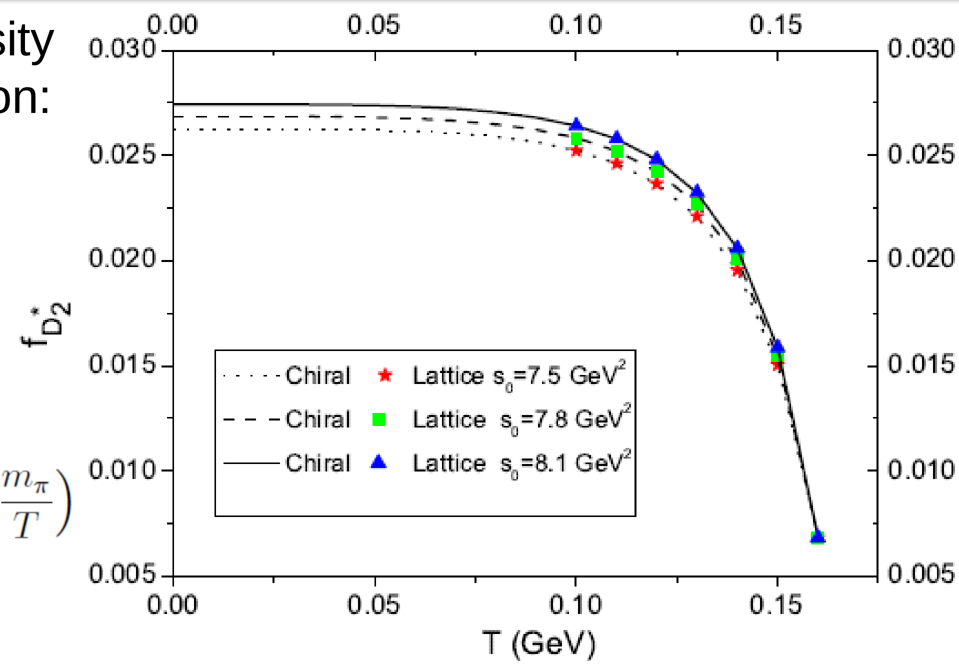
$$s_0(T) = s_0\frac{\langle\bar{q}q\rangle}{\langle 0|\bar{q}q|0\rangle}\Big(1-\frac{(m_c+m_q)^2}{s_0}\Big)+(m_c+m_q)^2$$

Thermal average of the energy density obtained using the Chiral Perturbation:

$$\langle\Theta\rangle = \langle\Theta^\mu_\mu\rangle + 3\ p$$

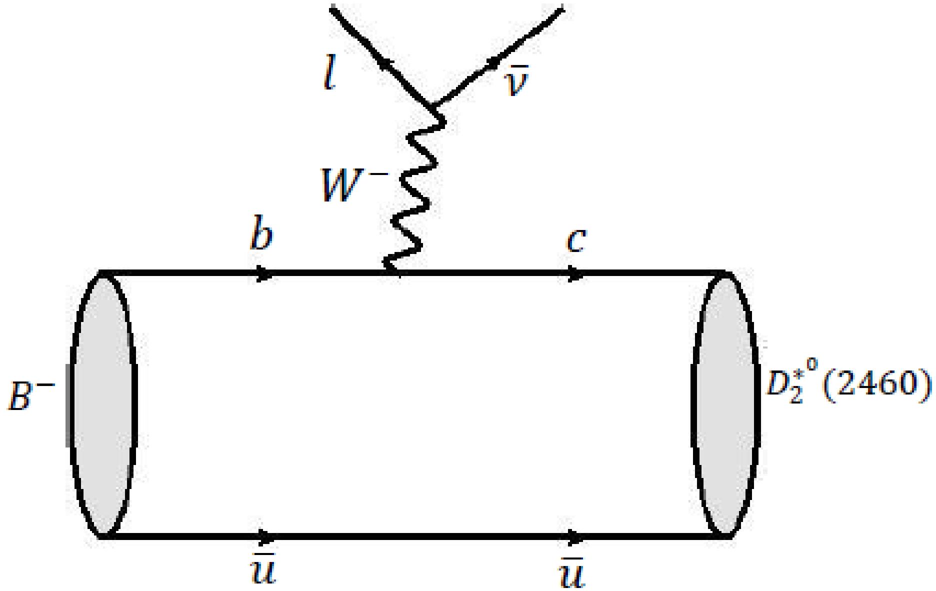
$$\langle\Theta^\mu_\mu\rangle = \frac{\pi^2}{270}\frac{T^8}{F_\pi^4}\ln\Big(\frac{\Lambda_p}{T}\Big)$$

$$p = 3T\Big(\frac{m_\pi\ T}{2\ \pi}\Big)^{\frac{3}{2}}\Big(1+\frac{15\ T}{8\ m_\pi}+\frac{105\ T^2}{128\ m_\pi^2}\Big)exp\Big(-\frac{m_\pi}{T}\Big)$$



Semileptonic transition of $B \to D_2^*(2460)\ell\overline{\nu}$ in QCD sum rules

$$\mathcal{H}_{eff}^{tree} = \frac{G_F}{\sqrt{2}} V_{bc}\, \bar{c}\gamma_\mu(1-\gamma_5)b\bar{l}\gamma^\mu(1-\gamma_5)\nu$$



Tree-point correlation function in the QCD sum rules:

$$\Pi_{\mu\alpha\beta} = i^2 \int d^4x \int d^4y e^{-ipx} e^{-ip'x} \langle 0 \mid \mathcal{T}[J^{D_2^*}_{\alpha\beta}(y) J^{tr}_{\mu}(0) J^{B^\dagger}(x)] \mid 0 \rangle$$

$$J^{tr}_{\mu}(0) = \overline{c}(0)\gamma_\mu(1-\gamma_5)b(0)$$

$$J^{B}(x) = \overline{u}(x)\gamma_5 b(x)$$

$$J^{D_2^*}_{\alpha\beta}(y) = \frac{i}{2}\left[\bar{u}(y)\gamma_\alpha \overleftrightarrow{\mathcal{D}}_\beta (y)c(y) + \bar{u}(y)\gamma_\beta \overleftrightarrow{\mathcal{D}}_\alpha (y)c(y)\right],$$

On the phenomenological side, the correlation function is obtained inserting two complete sets of intermediate states with the same quantum numbers as the interpolating currents J^B and $J^{D_2^*}$ into Eq. (15). After performing four-integrals over x and y, we get

$$\begin{aligned}
\Pi^{Phy}_{\mu\alpha\beta}(q^2) &= \frac{\langle 0 \mid J^{D_2^*}_{\alpha\beta}(0) \mid D_2^*(p',\epsilon)\rangle\langle D_2^*(p',\epsilon) \mid J^{tr}_{\mu}(0) \mid B(p)\rangle}{(p^2-m_B^2)(p'^2-m^2_{D_2^*(2460)})} \\
&\times \langle B(p) \mid J^{\dagger}_{B}(0) \mid 0\rangle + \cdots, \\
\langle 0 \mid J^{D_2^*}_{\alpha\beta}(0) \mid D_2^*(p',\epsilon)\rangle &= m^3_{D_2^*} f_{D_2^*}\epsilon_{\alpha\beta} \\
\langle B(p) \mid J^{\dagger}_{B}(0) \mid 0\rangle &= -i\frac{f_B m_B^2}{m_u+m_b} \\
\langle D_2^*(p',\epsilon) \mid J^{tr}_{\mu}(0) \mid B(p)\rangle &= h(q^2)\varepsilon_{\mu\nu\lambda\eta}\epsilon^{*\nu\theta}P_\theta P^\lambda q_\eta - iK(q^2)\epsilon^*_{\mu\nu}P^\nu \\
&- i\epsilon^*_{\lambda\eta}P^\lambda P^\eta\left[P_\mu b_+(q^2)+q_\mu b_-(q^2)\right],
\end{aligned}$$

the final representation of the physical side is obtained as:

$$
\begin{aligned}
\Pi^{Phys}_{\mu\alpha\beta}(q^2) &= \frac{f_{D_2^*} f_B m_{D_2^*} m_B^2}{8(m_b+m_u)(p^2-m_B^2)(p'^2-m_{D_2^*}^2)} \\
&\times \Big\{\frac{2}{3}\Big[-\Delta K(q^2)+\Delta' b_-(q^2)\Big] q_\mu g_{\beta\alpha} \\
&+ \frac{2}{3}\Big[(\Delta-4m_{D_2^*}^2)K(q^2)+\Delta' b_+(q^2)\Big] P_\mu g_{\beta\alpha} \\
&+ i(\Delta-4m_{D_2^*}^2)h(q^2)\varepsilon_{\lambda\nu\beta\mu}P_\lambda P_\alpha q_\nu \\
&+ \Delta K(q^2) q_\alpha g_{\beta\mu} + \text{other structure}\Big\} + ...,
\end{aligned}
$$

where Δ and Δ':

$$
\begin{aligned}
\Delta &= m_B^2 + 3m^2_{D_2^*(2460)} - q^2, \\
\Delta' &= m_B^4 - 2m_B^2(m^2_{D_2^*(2460)} + q^2) + (m^2_{D_2^*(2460)} - q^2)^2
\end{aligned}
$$

On the QCD side, the correlation function is calculated by expanding the time ordering product of the B and $D_2^*(2460)$ mesons' currents and the transition current via operator product expansion (OPE) in deep Euclidean region where the short (perturbative) and long distance (nonperturbative) contributions are separated. By inserting the previously represented currents into Eq. (2) and after contracting out all quark fields applying the Wick's theorem, we obtain:

$$
\begin{aligned}
\Pi_{\mu\alpha\beta}^{QCD}(q^2) &= \frac{-i^3}{4}\int d^4x \int d^4y e^{-ip\cdot x} e^{ip'\cdot y} \\
&\times \left\{ Tr\left[S_u^{ik}(x-y)\gamma_\alpha \overleftrightarrow{\mathcal{D}}_\beta (y) S_c^{ij}(y)\gamma_\mu(1-\gamma_5) S_b(-x)^{jk}\gamma_5 \right] \right. \\
&+ \left. \left[\beta \leftrightarrow \alpha\right] \right\}.
\end{aligned}
$$

$$
\begin{aligned}
\Pi^{QCD}_{\mu\alpha\beta}(q^2) &= \left(\Pi_1^{pert}(q^2) + \Pi_1^{nonpert}(q^2)\right) q_\alpha g_{\beta\mu} \\
&+ \left(\Pi_2^{pert}(q^2) + \Pi_2^{nonpert}(q^2)\right) q_\mu g_{\beta\alpha} \\
&+ \left(\Pi_3^{pert}(q^2) + \Pi_3^{nonpert}(q^2)\right) P_\mu g_{\beta\alpha} \\
&+ \left(\Pi_4^{pert}(q^2) + \Pi_4^{nonpert}(q^2)\right) \varepsilon_{\lambda\nu\beta\mu} P_\lambda P_\alpha q_\nu \\
&+ \; other \;\; structures.
\end{aligned}
$$

$$\Pi_1^{pert}(q^2) = \int ds \int ds' \frac{\rho_1(s, s', q^2)}{(s - p^2)(s' - p'^2)},$$

$$\begin{aligned}
\rho_1(s, s', q^2) &= \int_0^1 dx \int_0^{1-x} dy \Bigg\{ \frac{1}{64\pi^2(x+y-1)^3} \\
&\times \Big[m_b(x+y-1)^3(8x^2 - 8y^2 + 6x - 6y - 6) \\
&+ 3m_c\Big(8x^5 + 6x^4(4y-3) \\
&- 6x(y-1)^2(3+2y+4y^2) - 2(2+3y+4y^2) \\
&\times (y-1)^3 + 2x^3(1 - 18y + 8y^2) \\
&+ x^2(22 - 5y - 16y^3)\Big)\Big] \Bigg\},
\end{aligned}$$

$$
\begin{aligned}
\Pi_1^{nonpert} &= \Big\{ \frac{m_b^4 + 4m_b^2m_c^2 + 2m_b^2(m_c^2 - q^2) + (m_c^2 - q^2)^2}{64r^2r'^2} \\
&+ \frac{m_b^2m_c^2(m_b^2 + m_c^2 - q^2)}{32r^2r'^3} - \frac{m_b^2 + 4m_bm_c + m_c^2 - q^2}{64rr'^2} \\
&+ \frac{m_b^3m_c + m_b^2m_c^2 + 2m_bm_c^3 + m_c^4 - m_c^2q^2}{32rr'^3} \\
&+ \frac{m_b^4 + 2m_b^3m_c + m_b^2m_c^2 - m_b^2q^2}{32r^3r'} + \frac{m_c^2}{32r'^3} - \frac{1}{32r'^2} + \frac{1}{32r^2} \\
&+ \frac{3m_b^2 + 2m_bm_c + 3m_c^2 - 3q^2}{64r^2r'} + \frac{m_b^2}{32r^3} - \frac{1}{32rr'} \Big\} m_0^2\langle\bar{u}u\rangle \\
&- \Big(\frac{m_b^2 + 2m_bm_c + m_c^2 - q^2}{16rr'} + \frac{1}{16r} + \frac{1}{16r'}\Big)\langle\bar{u}u\rangle,
\end{aligned}
$$

In order to suppress the contributions of the higher states and continuum, we apply double Borel transformation with respect to the initial and final momenta squared using

$$\widehat{B} \qquad \frac{1}{(p^2 - m_b^2)^m} \frac{1}{(p'^2 - m_c^2)^n} \to \frac{(-1)^{m+n}}{\Gamma[m]\Gamma[n]} e^{-m_b^2/M^2} e^{-m_c^2/M'^2} \frac{1}{(M^2)^{m-1}(M'^2)^{n-1}}$$

quark-hadron duality:

$$\rho^{higher\ \ states}(s,s') = \rho^{OPE}(s,s')\theta(s-s_0)\theta(s'-s_0').$$

$$
\begin{aligned}
b_{-} &= \frac{-12(m_b+m_u)}{f_B f_{D_2^*} m_B^2 m_{D_2^*}\left(m_B^4+(m_{D_2^*}^2-q^2)^2-2m_B^2(m_{D_2^*}^2+q^2)\right)} e^{\frac{m_B^2}{M^2}} e^{\frac{m_{D_2^*}^2}{M'^2}} \\
&\Bigg\{ \int_{(m_b+m_u)^2}^{s_0^2} ds \int_{(m_c+m_u)^2}^{s_0'^2} ds' \int_0^1 dx \int_0^{1-x} dy e^{\frac{-s}{M^2}} e^{\frac{-s'}{M'^2}} \\
&\Bigg[\frac{1}{128\pi^4(x+y-1)^3}\Big(m_b(x+y-1)^3(3-6x-2x^2+6y+2y^2 \\
&- 3m_c\Big(6x^4(y-1)-3x(y-1)^2(1+2y^2)(y-1)^3(1+2y^2) \\
&+ x^3(5-12y+4y^2)+x^2(1+4y-4y^3)+2x^5\Big)\Big)\Bigg]\Theta[L(s,s',q^2 \\
&- e^{\frac{-m_B^2}{M^2}} e^{\frac{-m_{D_2^*}^2}{M'^2}} \frac{f_B f_{D_2^*} m_B^2 m_{D_2^*}(m_B^2+3m_{D_2^*}^2+q^2)}{12(m_b+m_u)} K(q^2)\Bigg\}
\end{aligned}
$$

The sum rules for the form factors contain four auxiliary parameters:

$$31\ GeV^2 \leq s_0 \leq 35\ GeV^2$$

$$7\ GeV^2 \leq s'_0 \leq 9\ GeV^2$$

$$10 GeV^2 \leq M^2 \leq 20 GeV^2$$

$$5 GeV^2 \leq M'^2 \leq 15 GeV^2$$

$$
\begin{aligned}
\frac{d\Gamma}{dq^2} &= \frac{\lambda(m_B^2, m_{D_2^*}^2, q^2)}{4m_{D_2^*}^2}\Big(\frac{q^2-m_\ell^2}{q^2}\Big)^2\frac{\sqrt{\lambda(m_B^2, m_{D_2^*}^2, q^2)}G_F^2V_{cb}^2}{384m_B^3\pi^3} \\
& \Big\{\frac{1}{2q^2}\Big[3m_\ell^2\lambda(m_B^2, m_{D_2^*}^2, q^2)[V_0(q^2)]^2 \\
&+ (m_\ell^2+2q^2)\Big|\frac{1}{2m_{D_2^*}}\Big[(m_B^2-m_{D_2^*}^2-q^2)(m_B-m_{D_2^*})V_1(q^2) \\
&- \frac{\lambda(m_B^2, m_{D_2^*}^2, q^2)}{m_B-m_{D_2^*}}V_2(q^2)\Big]\Big|^2\Big]+\frac{2}{3}(m_\ell^2+2q^2)\lambda(m_B^2, m_{D_2^*}^2, q^2) \\
&\times \Big[\Big|\frac{A(q^2)}{m_B-m_{D_2^*}}-\frac{(m_B-m_{D_2^*})V_1(q^2)}{\sqrt{\lambda(m_B^2, m_{D_2^*}^2, q^2)}}\Big|^2 \ldots\ldots\Big]\Big\},
\end{aligned}
$$

where

$$
\begin{aligned}
A(q^2) &= -(m_B - m_{D_2^*})h(q^2), \\
V_1(q^2) &= -\frac{k(q^2)}{m_B - m_{D_2^*}}, \\
V_2(q^2) &= (m_B - m_{D_2^*})b_+(q^2), \\
V_0(q^2) &= \frac{m_B - m_{D_2^*}}{2m_{D_2^*}} V_1(q^2) - \frac{m_B + m_{D_2^*}}{2m_{D_2^*}} V_2(q^2) - \frac{q^2}{2m_{D_2^*}} b_-(q^2)
\end{aligned}
$$

Our calculations show that the form factors are truncated at $q^2 \simeq 5GeV^2$. In order to estimate the decay width of the $B \to D_2^*(2460)\ell\overline{\nu}$ transition, we have to obtain their fit functions in the whole physical region, $m_\ell^2 \leq q^2 \leq (m_B - m_{D_2^*})^2$.

$$f(q^2) = f_0 \exp[c_1 \frac{q^2}{m_{fit}^2} + c_2 \left(\frac{q^2}{m_{fit}^2}\right)^2]$$

	f_0	c_1	c_2	m^2_{fit}
$K(q^2)$	0.76658	0.76658	0.40712	27.8784
$b_-(q^2)$	$0.00705\ GeV^{-2}$	0.18308	11.46764	27.8784
$b_+(q^2)$	$-0.02879\ GeV^{-2}$	1.33671	24.18371	27.8784
$h(q^2)$	$-0.00917\ GeV^{-2}$	1.32918	1.1963	27.8784

Table : Parameters appearing in the fit function of the form factors.

	$\Gamma(GeV)$	Br
$B \to D_2^*(2460)\tau\overline{\nu}_\tau$	7.11×10^{-17}	0.18×10^{-3}
$B \to D_2^*(2460)\mu\overline{\nu}_\mu$	3.96×10^{-16}	0.99×10^{-3}
$B \to D_2^*(2460)e\overline{\nu}_e$	3.98×10^{-16}	1.01×10^{-3}

Conclusions

- The orders of branching fractions show that this transition can be detected at LHCb for all lepton channels.
- Considering the recent experimental progress especially at LHC we hope we will have experimental data on the branching fraction of the semileptonic $B \to D_2^*(2460)\ell\overline{\nu}$ transition in near future, comparison of which with the results of our work can give more information about the nature and internal structure of the $D_2^*(2460)$ tensor meson.

MASS AND LEPTONIC DECAY CONSTANT OF Z_C(3900)

The $Z_c^{\pm}$ states discovered by BESIII in the process $e^+e^- \to \pi^+\pi^- J/\psi$ were observed by the Belle collaboration in 2013.

Their existence were also confirmed on the basis of the CLEO-c data analysis.

The $Z_c^{\pm} \to J/\psi\pi^{\pm}$ decays demonstrate that $Z_c^{\pm}$ are tetraquark states with constituents $c\bar{c}u\bar{d}$ and $c\bar{c}d\bar{u}$.

In order to calculate the mass and decay constant of the Z_c^+ state in the framework of QCD sum rules, we start from the two-point correlation function

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{iq\cdot x}\langle 0|\mathcal{T}\{J_\mu^{Z_c}(x)J_\nu^{Z_c\dagger}(0)\}|0\rangle,$$

$$J^{PC} = 1^{+-}$$

$$\begin{aligned} J_\nu^{Z_c}(x) &= \frac{i\epsilon\tilde{\epsilon}}{\sqrt{2}}\left\{\left[u_a^T(x)C\gamma_5 c_b(x)\right]\left[\overline{d}_d(x)\gamma_\nu C\overline{c}_e^T(x)\right]\right. \\ &\left. - \left[u_a^T(x)C\gamma_\nu c_b(x)\right]\left[\overline{d}_d(x)\gamma_5 C\overline{c}_e^T(x)\right]\right\}. \end{aligned}$$

Physical Side:

In order to derive QCD sum rule expression we first calculate the correlation function in terms of the physical degrees of freedom. Performing integral over *x* in Eq. (1), we get

$$\Pi_{\mu\nu}^{\mathrm{Phys}}(q) = \frac{\langle 0|J_\mu^{Z_c}|Z_c(q)\rangle\langle Z_c(q)|J_\nu^{Z_c\dagger}|0\rangle}{m_{Z_c}^2 - q^2} + \dots$$

$$\langle 0|J_\mu^{Z_c}|Z_c(q)\rangle = f_{Z_c} m_{Z_c} \varepsilon_\mu,$$

$$\Pi_{\mu\nu}^{\mathrm{Phys}}(q) = \frac{m_{Z_c}^2 f_{Z_c}^2}{m_{Z_c}^2 - q^2}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_{Z_c}^2}\right) + \dots$$

$$\mathcal{B}_{q^2}\Pi_{\mu\nu}^{\mathrm{Phys}}(q) = m_{Z_c}^2 f_{Z_c}^2 e^{-m_{Z_c}^2/M^2}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_{Z_c}^2}\right) + \dots$$

QCD Side:

The same function in QCD side, $\Pi^{\mathrm{QCD}}{}_{\mu\nu}(q)$, has to be determined employing of the quark-gluon degrees of freedom. To this end, we contract the heavy and light quark fields and find

$$\Pi_{\mu\nu}^{\mathrm{QCD}}(q) = -\frac{i}{2}\int d^4x e^{iqx}\epsilon\tilde{\epsilon}\epsilon'\tilde{\epsilon}'\Big\{\mathrm{Tr}\Big[\gamma_5\widetilde{S}_u^{aa'}(x)$$
$$\times\gamma_5 S_c^{bb'}(x)\Big]\,\mathrm{Tr}\Big[\gamma_\mu\widetilde{S}_c^{e'e}(-x)\gamma_\nu S_d^{d'd}(-x)\Big]$$
$$-\mathrm{Tr}\Big[\gamma_\mu\widetilde{S}_c^{e'e}(-x)\gamma_5 S_d^{d'd}(-x)\Big]\,\mathrm{Tr}\Big[\gamma_\nu\widetilde{S}_u^{aa'}(x)$$
$$\times\,\gamma_5 S_c^{bb'}(x)\Big] - \mathrm{Tr}\Big[\gamma_5\widetilde{S}_u^{a'a}(x)\gamma_\mu S_c^{b'b}(x)\Big]$$
$$\times\mathrm{Tr}\Big[\gamma_5\widetilde{S}_c^{e'e}(-x)\gamma_\nu S_d^{d'd}(-x)\Big] + \mathrm{Tr}\Big[\gamma_\nu\widetilde{S}_u^{aa'}(x)$$
$$\times\,\gamma_\mu S_c^{bb'}(x)\Big]\,\mathrm{Tr}\Big[\gamma_5\widetilde{S}_c^{e'e}(-x)\gamma_5 S_d^{d'd}(-x)\Big]\Big\},$$

where

$$\widetilde{S}_{c(q)}^{ij}(x) = C S_{c(q)}^{ij\mathrm{T}}(x) C$$

the heavy-quark propagator $S_c^{ij}(x)$:

$$S_c^{ij}(x) = i\int \frac{d^4k}{(2\pi)^4} e^{-ikx} \left[\frac{\delta_{ij}\left(\not{k} + m_c\right)}{k^2 - m_c^2} - \frac{gG_{ij}^{\alpha\beta}}{4} \frac{\sigma_{\alpha\beta}\left(\not{k} + m_c\right) + \left(\not{k} + m_c\right)\sigma_{\alpha\beta}}{(k^2 - m_c^2)^2} + \frac{g^2}{12} G_{\alpha\beta}^{A} G^{A\alpha\beta} \delta_{ij} m_c \frac{k^2 + m_c\not{k}}{(k^2 - m_c^2)^4} + \ldots \right].$$

The light-quark propagator:

$$S_q^{ij}(x) = i\frac{\not{x}}{2\pi^2 x^4}\delta_{ij} - \frac{m_q}{4\pi^2 x^2}\delta_{ij} - \frac{\langle q\overline{q}\rangle}{12} \times \left(1 - i\frac{m_q}{4}\not{x}\right)\delta_{ij} - \frac{x^2}{192} m_0^2 \langle q\overline{q}\rangle \left(1 - i\frac{m_q}{6}\not{x}\right)\delta_{ij} - i\frac{gG_{ij}^{\alpha\beta}}{32\pi^2 x^2}\left[\sigma_{\alpha\beta}\not{x} + \not{x}\sigma_{\alpha\beta}\right] + \ldots.$$

The correlation function $\Pi^{\mathrm{QCD}}{}_{\mu\nu}(q)$ has also the following decomposition over the Lorentz structures:

$$\Pi_{\mu\nu}^{\mathrm{QCD}}(q) = \Pi^{\mathrm{QCD}}(q^2) g_{\mu\nu} + \widetilde{\Pi}^{\mathrm{QCD}}(q^2) q_\mu q_\nu .$$

The QCD sum rule expression for the mass and decay constant can be derived after choosing the same structures in both $\Pi^{\mathrm{Phys}}{}_{\mu\nu}(q)$ and $\Pi^{\mathrm{QCD}}{}_{\mu\nu}(q)$. We choose to work with the term $\sim q_\mu q_\nu$ and invariant function $\Pi e^{\mathrm{QCD}}(q^2)$, which can be represented as the dispersion integral

$$\widetilde{\Pi}^{\mathrm{QCD}}(q^2) = \int_{4m_c^2}^{\infty} \frac{\rho^{\mathrm{QCD}}(s)}{s-q^2} + \ldots,$$

$$\rho^{\mathrm{pert}}(s) = \frac{1}{384\pi^6} \int_0^1 dz \int_0^{1-z} dw \frac{r^8}{(w-1)} \left\{ wz^2 \left[swz(w+z-1) - m_c^2 \frac{w+z}{r} \right]^2 \right.$$
$$\left. \times \left[m_c^2 \frac{w+z}{r} (4w(w-1) + 3z(w-1) + 3z^2) - swz(w+z-1)(3z^2 + (w-1)(7w+3z)) \right] \right\} \theta(L),$$

$$\rho_3(s) = \frac{1}{16\pi^4} \int_0^1 dz \int_0^{1-z} dw m_c r^5 wz(w+z-1) \left[\langle u\overline{u} \rangle w + \langle d\overline{d} \rangle z \right] \left[m_c^2 \frac{w+z}{r} - swz(w+z-1) \right] \theta(L),$$

....

Applying the Borel transformation on the variable q^2 to the invariant amplitude $\Pi_{\mu\nu}^{Phys}$ equating the obtained expression with the relevant part of $B_{q^2}\Pi_{\mu\nu}^{Phys}(q)$, and subtracting the continuum contribution, we finally obtain the required sum rule. Thus, the mass of the Z_c state can be evaluated from the sum rule:

$$m_{Z_c}^2 = \frac{\int_{4m_c^2}^{s_0} ds s \rho^{\mathrm{QCD}}(s) e^{-s/M^2}}{\int_{4m_c^2}^{s_0} ds \rho(s) e^{-s/M^2}},$$

whereas to extract the numerical value of the decay constant f_{Z_c} we employ the formula

$$f_{Z_c}^2 e^{-m_{Z_c}^2/M^2} = \int_{4m_c^2}^{s_0} ds \rho^{\mathrm{QCD}}(s) e^{-s/M^2}$$

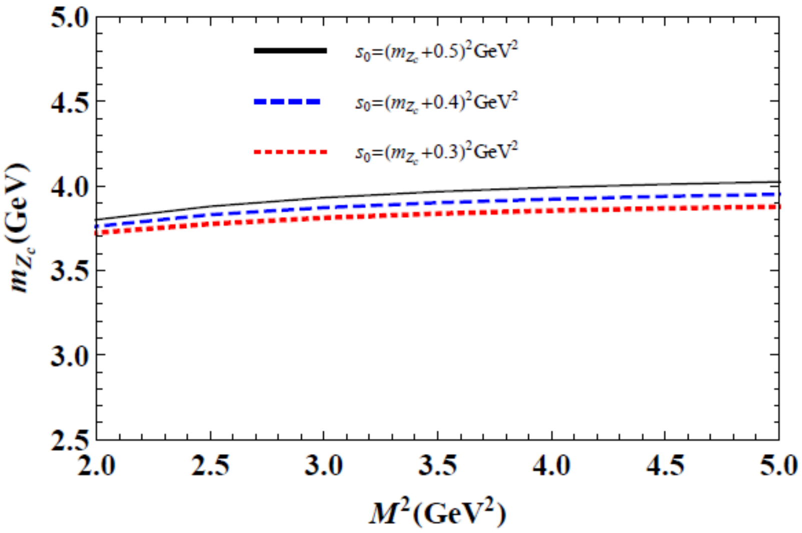


FIG. 2: The mass m_{Z_c} as a function of the Borel parameter M^2 for different values of s_0.

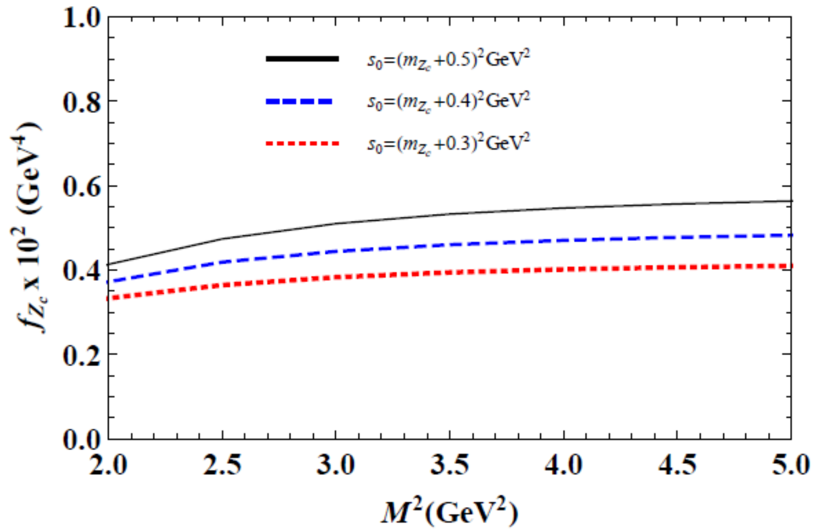


FIG. 3: The decay constant f_{Z_c} vs Borel parameter M^2. The values of the parameter s_0 are shown in the figure.

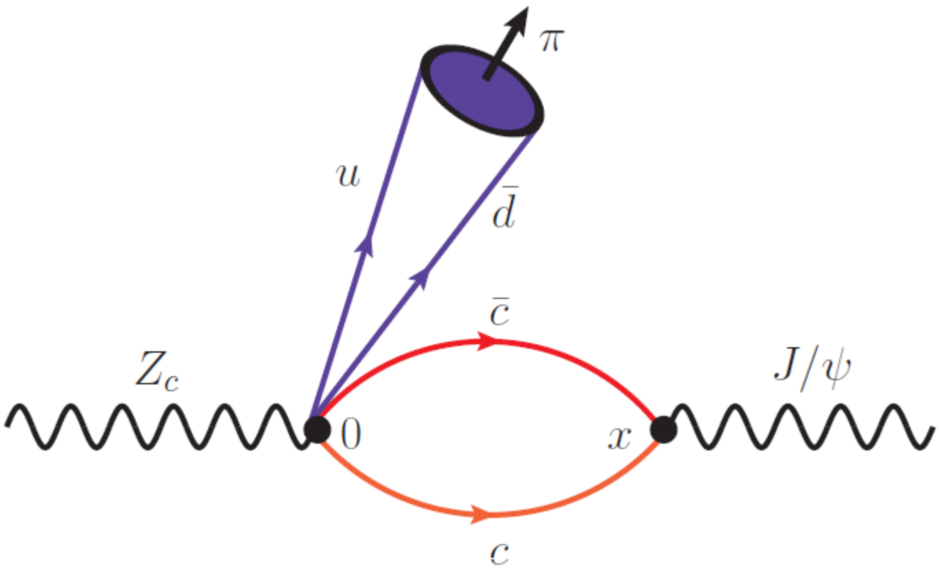
$$(3.9+0.3)^2 \text{ GeV}^2 \leq s_0 \leq (3.9+0.5)^2 \text{ GeV}^2$$

	m_{Z_c} (MeV)
BESIII [26]	3899 ± 6
Belle [27]	3895 ± 8
Present Work	3900 ± 210
Z. Wang, T. Huang [32]	3910^{+110}_{-90}

TABLE II: Experimental data and theoretical predictions for the mass of Z_c state.

$$f_{Z_c} = (0.46 \pm 0.03) \times 10^{-2} \text{ GeV}^4$$

THE $Z_c J/\psi\pi$ VERTEX



We start our analysis from the vertex $Z_c J/\psi\pi$ aiming to calculate $g_{Z_c J/\psi\pi}$: we consider the correlation function:

$$\Pi_{\mu\nu}(p,q) = i\int d^4x e^{ipx}\langle\pi(q)|\mathcal{T}\{J_\mu^{J/\psi}(x)J_\nu^{Z_c\dagger}(0)\}|0\rangle,$$

where

$$J_\mu^{J/\psi}(x) = \overline{c}_i(x)\gamma_\mu c_i(x),$$

$$J_\nu^{Z_c}(x) = \frac{i\epsilon\tilde{\epsilon}}{\sqrt{2}}\{[u_a^T(x)C\gamma_5 c_b(x)]\ [\overline{d}_d(x)\gamma_\nu C\overline{c}_e^T(x)] - [u_a^T(x)C\gamma_\nu c_b(x)]\ [\overline{d}_d(x)\gamma_5 C\overline{c}_e^T(x)]\}.$$

THE $Z_c J/\psi\pi$ VERTEX

To derive the sum rules for the coupling, we calculate $\Pi_{\mu\nu}(p,q)$ in terms of the physical degrees of freedom.

$$\Pi_{\mu\nu}^{\mathrm{Phys}}(p,q) = \frac{\langle 0|J_\mu^{J/\psi}|J/\psi(p)\rangle}{p^2 - m_{J/\psi}^2}\langle J/\psi(p)\,\pi(q)|Z_c(p')\rangle \times \frac{\langle Z_c(p')|J_\nu^{Z_c\dagger}|0\rangle}{p'^2 - m_{Z_c}^2} + \ldots .$$

We introduce the matrix elements

$$\langle 0|J_\mu^{J/\psi}|J/\psi(p)\rangle = f_{J/\psi} m_{J/\psi}\varepsilon_\mu,$$
$$\langle Z_c(p')|J_\nu^{Z_c\dagger}|0\rangle = f_{Z_c} m_{Z_c}\varepsilon_\nu'^*$$
$$\langle J/\psi(p)\,\pi(q)|Z_c(p')\rangle = [(p\cdot p')(\varepsilon^*\cdot\varepsilon') - (p\cdot\varepsilon')(p'\cdot\varepsilon^*)]\, g_{Z_cJ/\psi\pi},$$

Having used these matrix elements we can rewrite the correlation function as

$$\Pi_{\mu\nu}^{\mathrm{Phys}}(p,q) = \frac{f_{J/\psi} f_{Z_c} m_{Z_c} m_{J/\psi} g_{Z_cJ/\psi\pi}}{\left(p'^2 - m_{Z_c}^2\right)\left(p^2 - m_{J/\psi}^2\right)} \times \left(\frac{m_{Z_c}^2 + m_{J/\psi}^2}{2} g_{\mu\nu} - p'_\mu p_\nu\right) + \ldots$$
$$= \Pi_\pi^{\mathrm{Phys}}(p^2,(p+q)^2) g_{\mu\nu} + \widetilde{\Pi}_\pi^{\mathrm{Phys}}(p^2,(p+q)^2) p'_\mu p_\nu$$

THE $Z_c J/\psi\pi$ VERTEX

Now we need to calculate the correlation function in terms of the quark-gluon degrees of freedom and find the QCD side of the sum rules. Having contracted heavy quarks fields we get

$$\Pi_{\mu\nu}^{\mathrm{QCD}}(p,q) = \int d^4x e^{ipx} \frac{\epsilon\widetilde{\epsilon}}{\sqrt{2}} \Big[\gamma_5 \widetilde{S}_c^{ib}(x)\gamma_\mu \times \widetilde{S}_c^{ei}(-x)\gamma_\nu + \gamma_\nu \widetilde{S}_c^{ib}(x)\gamma_\mu \widetilde{S}_c^{ei}(-x)\gamma_5\Big]_{\alpha\beta} \times \langle \pi(q)|\overline{u}_\alpha^a(0) d_\beta^d(0)|0\rangle,$$

……………………..

…………

$$g_{Z_cJ/\psi\pi} = \frac{2}{f_{J/\psi} f_{Z_c} m_{Z_c} m_{J/\psi}(m_{Z_c}^2 + m_{J/\psi}^2)} \times \left(1 - M^2 \frac{d}{dM^2}\right) M^2 \times \int_{4m_c^2}^{s_0} ds e^{(m_{Z_c}^2 + m_{J/\psi}^2 - 2s)/2M^2} \rho_\pi^{\mathrm{QCD}}(s).$$

$$\rho_\pi^{\mathrm{QCD}}(s) = \frac{f_\pi \mu_\pi (s + 2m_c^2)\sqrt{s(s-4m_c^2)}}{12\sqrt{2}\pi^2 s}.$$

THE $Z_c J/\psi\pi$ VERTEX

The width of the decay $Z_c \to J/\psi\pi$

$$\Gamma\left(Z_c \to J/\psi\pi\right) = \frac{g^2_{Z_c J/\psi\pi} m^2_{J/\psi}}{24\pi} \lambda\left(m_{Z_c},\ m_{J/\psi}, m_\pi\right) \times \left[3 + \frac{2\lambda^2\left(m_{Z_c},\ m_{J/\psi}, m_\pi\right)}{m^2_{J/\psi}}\right],$$

where

$$\lambda(a,\ b,\ c) = \frac{\sqrt{a^4 + b^4 + c^4 - 2\left(a^2b^2 + a^2c^2 + b^2c^2\right)}}{2a}.$$

THE $Z_c J/\psi\pi$ VERTEX

Numerical results:

$$g_{Z_c J/\psi\pi} = (0.39 \pm 0.06)\ \text{GeV}^{-1}.$$

$$\Gamma(Z_c \to J/\psi\pi) = (41.9 \pm 9.4)\ \text{MeV}$$

$$g_{Z_c \eta_c \rho} = (1.39 \pm 0.15)\ \text{GeV}^{-1},$$

$$\Gamma(Z_c \to \eta_c\rho) = (23.8 \pm 4.9)\ \text{MeV}.$$

Considering the transitions $Z_c \to J/\psi\pi$ and $Z_c \to \eta_c\rho$ as dominant channels we obtain for the total width of Z_c approximately

$$\Gamma_{Z_c} = 65.7 \pm 10.6\ \text{MeV},$$

It is in a good consistency with:

$\Gamma_{Z_c} = 63 \pm 35$ MeV — Z. Q. Liu *et al.* [Belle Collaboration], Phys. Rev. Lett. **110**, 252002 (2013).

$\Gamma_{Z_c} = 63.0 \pm 18.1$ MeV — J. M. Dias, F. S. Navarra, M. Nielsen and C. M. Zanetti, Phys. Rev. D **88**, 016004 (2013).

$\Gamma_{Z_c} = 46 \pm 22$ MeV — M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **110**, 252001 (2013).

arXiv:1601.03847

X(5568)???

24 February 2016, the D0 Collaboration reported the observation of a narrow structure $X(5568)$ in the decay chain $X(5568) \to B_s^0\pi^\pm$, $B_s^0 \to J/\psi\varphi$, $J/\psi \to \mu^+\mu^-$, $\varphi \to K^+K^-$ (V. M. Abazov *et al.* [D0 Collaboration], arXiv:1602.07588 [hep-ex]) based on $p\bar{p}$ collision data at s = 1.96 TeV collected at the Fermilab Tevatron collider.

This is the first observation of a hadronic state with four quarks of different flavors.

Namely, from the observed decay channel $X_b(5568) \to B_s^0\pi^\pm$ it is not difficult to conclude that the state $X_b(5568)$ consists of *b, s, u, d* quarks.

The assigned quantum numbers for the X_b state are $J^{PC} = 0^{++}$, its mass extracted from the experiment is equal to $m_X = 5567.8 \pm 2.9(stat)^{+0.9}_{-1.9}(syst)MeV$, and the decay width was estimated as $\Gamma = 21.9 \pm 6.4(stat)^{+5.0}_{-2.5}(syst)MeV$.

Thus, within the diquark-antidiquark model the X_b may be described as $[bu][\bar{d}\bar{s}]$, $[bd][\bar{s}\bar{u}]$, $[su][\bar{b}\bar{d}]$ or $[sd][\bar{b}\bar{u}]$ bound state.

Alternatively, it may be considered as a molecule composed of B and K mesons.

X(5568)???

To calculate the mass and decay constant of the X_b state in the framework of QCD sum rules, we start from the two-point correlation function

$$\Pi(p) = i \int d^4x e^{ip\cdot x} \langle 0|\mathcal{T}\{J^{X_b}(x) J^{X_b\dagger}(0)\}|0\rangle.$$

for $J^{PC} = 0^{++}$, interpolating current:

$$J^{X_b}(x) = \varepsilon^{ijk}\varepsilon^{imn}\left[s^j(x) C\gamma_\mu u^k(x)\right]\left[\overline{b}^m(x)\gamma^\mu C\overline{d}^n(x)\right]$$

Tetraquarks

$$J^{X_b}(x) = \left[\overline{d}^a(x)\gamma_5 s^a(x)\right]\left[\overline{b}^b(x)\gamma_5 u^b(x)\right]$$

Molecule

Physical Side:

$$\Pi^{\mathrm{Phys}}(p) = \frac{\langle 0|J^{X_b}|X_b(p)\rangle\langle X_b(p)|J^{X_b\dagger}|0\rangle}{m_{X_b}^2 - p^2} + \ldots$$

$$\langle 0|J^{X_b}|X_b(p)\rangle = f_{X_b} m_{X_b}$$

$$\Pi^{\mathrm{Phys}}(p) = \frac{m_{X_b}^2 f_{X_b}^2}{m_{X_b}^2 - p^2} + \ldots \Rightarrow \mathcal{B}_{p^2}\Pi^{\mathrm{Phys}}(p) = m_{X_b}^2 f_{X b}^2 e^{-m_{X_b}^2/M^2} + \ldots$$

X(5568)???

QCD Side:

$$\Pi^{\mathrm{QCD}}(p) = i\int d^4x e^{ipx} \varepsilon^{ijk}\varepsilon^{imn}\varepsilon^{i'j'k'}\varepsilon^{i'm'n'}$$
$$\times \mathrm{Tr}\left[\gamma_\mu \widetilde{S}_d^{n'n}(-x)\gamma_\nu S_b^{m'm}(-x)\right]$$
$$\times \mathrm{Tr}\left[\gamma^\nu \widetilde{S}_s^{jj'}(x)\gamma^\mu S_u^{kk'}(x)\right].$$

Tetraquarks

$$\Pi^{\mathrm{QCD}}(p) = i\int d^4x e^{ipx}\mathrm{Tr}\left[\gamma_5 S_s^{aa'}(x)\gamma_5 S_d^{a'a}(-x)\right]$$
$$\times \mathrm{Tr}\left[\gamma_5 S_u^{bb'}(x)\gamma_5 S_b^{b'b}(-x)\right],$$

Molecule

As a result:

$$\boxed{m_{X_b}^2 = \frac{\int_{(m_b+m_s)^2}^{s_0} ds s \rho^{\mathrm{QCD}}(s) e^{-s/M^2}}{\int_{(m_b+m_s)^2}^{s_0} ds \rho^{\mathrm{QCD}}(s) e^{-s/M^2}},}$$

$$\boxed{f_{X_b}^2 m_{X_b}^2 e^{-m_{X_b}^2/M^2} = \int_{(m_b+m_s)^2}^{s_0} ds \rho^{\mathrm{QCD}}(s) e^{-s/M^2}}$$

X(5568)???

	$J^{X_b}_{\mathrm{Di}}$	$J^{X_b}_{\mathrm{Mol}}$
m_{X_b}	(5584 ± 137) MeV	(5764 ± 142) MeV
f_{X_b}	$(0.24 \pm 0.02) \cdot 10^{-2}$ GeV^4	$(0.17 \pm 0.02) \cdot 10^{-2}$ GeV^4
$g_{X_b B_s \pi}$	(0.63 ± 0.14) GeV^{-1}	(0.50 ± 0.12) GeV^{-1}
Γ	(24.5 ± 8.2) MeV	(34.4 ± 12.2) MeV

TABLE II: The sum rule predictions for the parameters of the X_b state evaluated using the diquark-antidiquark (the left column) and molecule (the right column) models.

arXiv:1603.02708

arXiv:1602.08642

arXiv:1603.01471

arXiv:1603.00290

27 February 2016 is the important day!!!!

arXiv:1602.08711

$$\begin{aligned} M_X &= (5.57 \pm 0.12)\ \mathrm{GeV}\,, \\ \lambda_X &= (6.7 \pm 1.6) \times 10^{-3}\,\mathrm{GeV}^5 \end{aligned}$$

arXiv:1602.08916

$$m_{X,1^+} = 5.59 \pm 0.15\ \mathrm{GeV}\,.$$

arXiv:1602.09041

$$m_X = (5.58 \pm 0.17)\,\mathrm{GeV},$$

arXiv:1602.08806

THANKS...
Kösz...
Teşekkürler...