Determination of the strong coupling beyond NNLO

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Introduction

The strong coupling

The **strong coupling** α_s is a fundamental parameter of the Standard Model, its value (at a given energy scale) is a basic constant of Nature.

The numerical value of the strong coupling enters essentially all theoretical predictions for LHC processes $\Rightarrow$ the accurate knowledge of this numerical value is important for fully exploiting LHC results.

It is the least precisely known coupling: $\Delta\alpha_s(M_Z)/\alpha_s(M_Z) \sim 1\%$.

Coupling	Symbol	Value	Error (ppb)
fine-structure constant	α_{EM}	$7.2973525693(11) \times 10^{-3}$	0.15
Fermi constant	G_F	$1.1663787(6) \times 10^{-5}$ GeV^{-2}	510
strong coupling	$\alpha_s(M_Z)$	0.1179(10)	8.5×10^6
gravitational constant	G_N	$6.67430(15) \times 10^{-11}$ m^3 kg^{-1} s^{-2}	2.2×10^4

Its value must be extracted by fitting theoretical predictions to measured data $\Rightarrow$ many options depending on the nature of the observable.

The strong coupling: relevance at LHC

The uncertainty in α_s contributes significantly to many QCD predictions, e.g., cross sections for top quark or Higgs boson production.

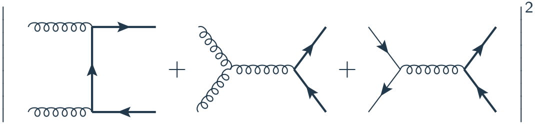
Consider $pp \to t\bar{t}$:

- The total cross section $\sigma_{t\bar{t}}$ has been measured:

$$\sigma_{t\bar{t}} = 803 \pm 2(\text{stat}) \pm 25(\text{syst}) \pm 20(\text{lumi})\ \text{pb}$$

[CMS Collaboration, EPJC 79 (2019) 5, 368]

- The combined uncertainty corresponds to: $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim 4\%$
- But $t\bar{t}$ production is proportional to α_s^2 already at tree level: $\sigma_{t\bar{t}} \propto \alpha_s^2$



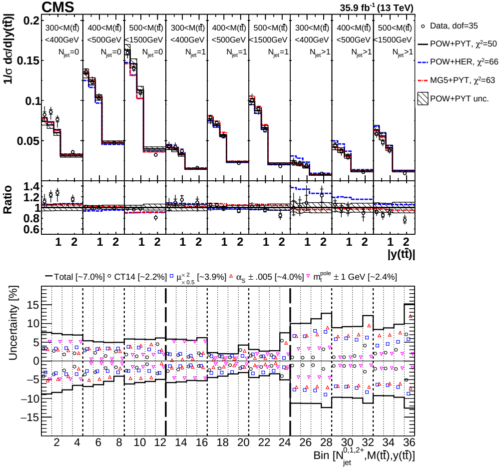
$$\sim g_S^4 = \alpha_s^2$$

- So at a basic level: $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \propto 2\Delta\alpha_s/\alpha_s \sim 2\%$, which is commensurate with the experimental error!

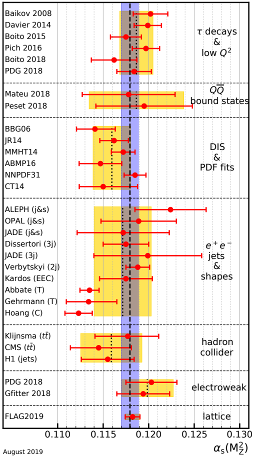
The uncertainty in α_s contributes significantly to many QCD predictions, e.g., cross sections for top quark or Higgs boson production.

Consider $pp \to t\bar{t} + N$ jets:

- CMS has measured normalized triple differential cross sections in N_{jet}, $M(t\bar{t})$ and $y(t\bar{t})$
  [CMS Collaboration, EPJC 80 (2020) 7, 658]
- For bins with the highest precision, i.e., $N_{\text{jet}} = 0$ and $440 < M(t\bar{t}) < 1500$ GeV, they find uncertainties of around 2–3% (stat.) and 3–6% (syst.)
- Total th. uncertainty: $\sim 7\%$, uncertainty from α_s: $\sim 4\%$.
- In many bins, the dominant theoretical uncertainty comes from α_s variation.



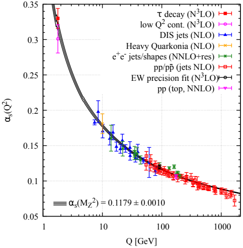
The strong coupling: status



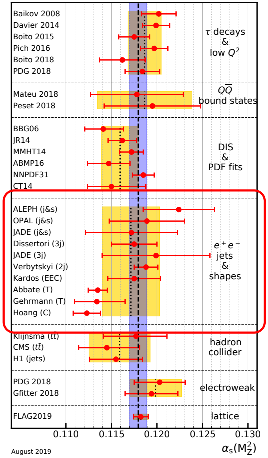
[P. A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 update]

Different approaches

- Many types of observables used to extract the strong coupling over a large energy range of ~ 1 GeV (τ decays) to ~ 1000 GeV (LHC inclusive jets)
- By convention and to facilitate comparison, measurements evolved to $Q = M_Z$, plethora of measurements also checks the predicted running



The strong coupling from e^+e^- annihilation



[P. A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 update]

Why α_s in e^+e^-?

- Electron-positron collisions offer a clean environment for the analysis
- The e^+e^- jets & shapes sub-field alone gives $\sim 2.6\%$ uncertainty due to the large spread between measurements
- **Can $\sim$ 1% precision be achieved**?

Why the differences?

- **Hadronization modeling**: Monte Carlo or analytic
- Perturbative order: fixed order NNLO to N^{3}LO + resummation NLL to N^{3}LL
- Type of observable used: jet rates or event shapes

How best to improve?

The strong coupling from e^+e^- annihilation: issues

The current situation raises some questions:

- No new data foreseen in the near future, so would including **more perturbative orders** (fixed order and/or resummation) improve precision **without any new data**?
- If not, what are the **limiting factors** for precision in future QCD studies?
- What should be done to **eliminate** those factors?

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To address these issues, two state-of-the-art pQCD analyses are presented:

1. An analysis based on the two-jet rate R_2 computed at N^{3}LO+NNLL accuracy. In this analysis Monte Carlo tools are used to obtain hadronization corrections.

   [A. Verbytskyi, A. Banfi, A. Kardos, P. F. Monni, S. Kluth, GS, Z. Szőr, Z. Trócsányi, Z. Tulipánt, G. Zanderighi, JHEP **1908** (2019) 129 [arXiv:1902.08158 [hep-ph]]]

2. An analysis of event shape averages where unknown perturbative corrections beyond NNLO are estimated from data. Hadronization corrections are obtained using both Monte Carlo tools as well as analytic models.

   [A. Kardos, GS, A. Verbytskyi, Eur. Phys. J. C **81** (2021) 4, 292 [arXiv:2009.00281 [hep-ph]]]

The strong coupling from jet rates

Jet rates in electron-positron annihilation

QCD predicts that the hadrons created in electron-positron annihilation are produced in **beams of collimated particles – jets**.

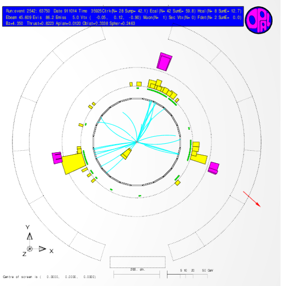
Jet finding algorithms are used to define precisely how many jets are present in an event and which particle belongs to a given jet.

Durham algorithm used in this study. Define a "distance measure" on final-state objects

$$y_{ij} = 2\frac{\min(E_i^2, E_j^2)}{E_{\text{vis}}^2}(1 - \cos\theta_{ij})$$

Jets are defined by the following **recursive algorithm**

1. Find the smallest y_{ij}, suppose this is $\min(y_{ij}) = y_{kl}$.
2. If y_{kl} is greater than a pre-determined cutoff y_{cut} (i.e., $\min(y_{ij}) > y_{\text{cut}}$), then we are done, each final-state object is a jet.
3. If $y_{kl} < y_{\text{cut}}$, then replace the k-th and l-th object by a single new object of momentum $p_k^\mu + p_l^\mu$.



Jet rates in electron-positron annihilation

Jet rates: R_n is the fraction of n-jet events for a given y_{cut}

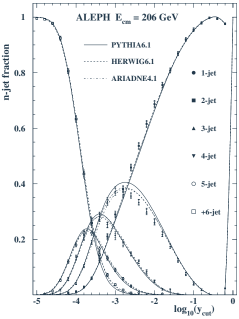
$$R_n(y_{\text{cut}}) = \frac{\sigma_{n-\text{jet}}(y_{\text{cut}})}{\sigma_{\text{tot}}}$$

General behavior easy to understand

- if y_{cut} is large, there are many steps of combining objects $\Rightarrow$ few jets
- if y_{cut} is small, there are few steps of combining objects $\Rightarrow$ many jets
- $\sum_n R_n = 1$

Good candidate for α_s measurement

- High perturbative accuracy, especially for R_2
- Lots of precise data from LEP (and PETRA)
- Jet rates are known to be less sensitive to hadronization corrections than event shapes
- R_3 was used multiple times in the past to extract $\alpha_s(M_Z)$



[ALEPH Coll., Eur. Phys. J. **C35**, 457 (2004)]

Jet rates analysis: components

- Data from LEP and PETRA + new OPAL measurements used to build correlation model for older measurements.
- Fixed-order perturbative predictions + some b-quark mass corrections
- Resummation + matching
- Non-perturbative corrections from state-of-the-art MC event generators + Lund and cluster hadronization models

Data

Combined analysis using 20+ datasets from 4 collaborations

The data covers a **wide range of cms energies**: $\sqrt{s} = 35 - 207\,\text{GeV}$

Experiment	Data $\sqrt{s}$, (average), GeV	MC $\sqrt{s}$, GeV	Events
OPAL	91.2(91.2)	91.2	1508031
OPAL	189.0(189.0)	189	3300
OPAL	183.0(183.0)	183	1082
OPAL	172.0(172.0)	172	224
OPAL	161.0(161.0)	161	281
OPAL	130.0 — 136.0(133.0)	133	630
L3	201.5 — 209.1(206.2)	206	4146
L3	199.2 — 203.8(200.2)	200	2456
L3	191.4 — 196.0(194.4)	194	2403
L3	188.4 — 189.9(188.6)	189	4479
L3	180.8 — 184.2(182.8)	183	1500
L3	161.2 — 164.7(161.3)	161	424
L3	135.9 — 140.1(136.1)	136	414
L3	129.9 — 130.4(130.1)	130	556
JADE	43.4 — 44.3(43.7)	44	4110
JADE	34.5 — 35.5(34.9)	35	29514
ALEPH	91.2(91.2)	91.2	3600000
ALEPH	206.0(206.0)	206	3578
ALEPH	189.0(189.0)	189	3578
ALEPH	183.0(183.0)	183	1319
ALEPH	172.0(172.0)	172	257
ALEPH	161.0(161.0)	161	319
ALEPH	133.0(133.0)	133	806

Data selection:

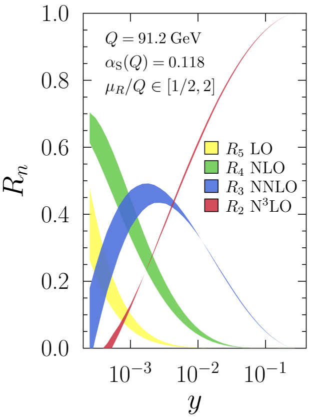
- measurements with both charged and neutral final state particles
- corrected for detector effects
- corrected for QED initial state radiation
- no overlap with other samples
- sufficient precision
- sufficient information on dataset available

Fixed-order predictions for jet rates

Fixed-order predictions up to and including α_s^3 corrections known for some time

[Gehrmann-De Ridder et al., Phys. Rev. Lett. **100** (2008) 172001, Weinzierl, Phys. Rev. Lett. **101** (2008) 162001]

$$R_n(y) = \delta_{2,n} + \frac{\alpha_s(Q)}{2\pi} A_n(y) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 B_n(y) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^3 C_n(y) + \mathcal{O}(\alpha_s^4)$$



- R_3 computed at **NNLO** accuracy using CoLoRFulNNLO ⇒ obtain R_2 at **N^{3}LO**

  [Del Duca et al., Phys. Rev. **D94** (2016) no.7, 074019]
- very good numerical precision and stability
- b-mass corrections from `Zbb4`: note only **NLO** for R_3 ⇒ **NNLO** for R_2

  [Nason, Oleari, Phys. Lett. **B407**, 57 (1997)]
- mass effects included at distribution level, e.g.

$$R_2(y) = (1 - r_b) R_2^{\mathrm{N^3LO}}(y)_{m_b=0} + r_b R_2^{\mathrm{NNLO}}(y)_{m_b \neq 0}$$

where r_b is the fraction of b-quark events

$$r_b = \frac{\sigma_{m_b\neq 0}(e^+e^- \to b\bar{b})}{\sigma_{m_b\neq 0}(e^+e^- \to \text{hardons})}$$

Resummation

Fixed order diverges in the limit of $y \to 0$ as $\sim \alpha_s^n \ln^{2n-1} y$, i.e., the fixed-order coefficients at n-th order include terms $\{\ln^k y\}_{k=1}^{2n-1}$.

For small y the logarithms become large, $\alpha_s^n \ln^{2n-1} y \sim 1$, invalidating the use of fixed-order perturbation theory.

Logarithmically enhanced terms must be resummed to all orders to obtain a description appropriate in the $y \to 0$ limit. Resummation can be systematically improved by resumming more towers of logs: leading logs (LL), next-to-leading logs (NLL), etc.

$$
\begin{array}{llllllllll}
R_2(y) \sim \Big\{ 1 & & & & & & & & & \text{LO} \\
+ \alpha_s \Big[& \log y & + & 1 & & & & & \Big] & \text{NLO} \\
+ \alpha_s^2 \Big[& \log^3 y & + & \log^2 y & + & \log y & + & 1 & \Big] & \text{NNLO} \\
+ \alpha_s^3 \Big[& \log^5 y & + & \log^4 y & + & \log^3 y & + & \log^2 y & \ldots \Big] \Big\} & \text{N}^3\text{LO} \\
 & \vdots & & & & & & & & \\
 & \text{LL} & & \text{NLL} & & \text{NNLL} & & & &
\end{array}
$$

Combining fixed-order and resummed predictions

Fixed-order and resummed calculations are complementary to each other: they describe data over different kinematical ranges

In order to obtain predictions over a wide kinematical range, the two computations must be combined without double counting ("matching")

Matched predictions at $N^3LO+NNLL$:

- all terms from first four rows (N^3LO)
- in addition, first three terms from all rows (NNLL)
- must take care to count the first three terms of the first four rows only once

$$
\begin{array}{llllllllll}
R_2(y) \sim \Big\{ 1 & & & & & & & & & \text{LO} \\
+\, \alpha_s \Big[& \log y & + & 1 & & & & & \Big] & \text{NLO} \\
+\, \alpha_s^2 \Big[& \log^3 y & + & \log^2 y & + & \log y & + & 1 & \Big] & \text{NNLO} \\
+\, \alpha_s^3 \Big[& \log^5 y & + & \log^4 y & + & \log^3 y & + & \log^2 y & \ldots \Big]\Big\} & \text{N}^3\text{LO} \\
& \vdots & & & & & & & & \\
& \text{LL} & & \text{NLL} & & \text{NNLL} & & & &
\end{array}
$$

Resummed predictions for jet rates

Resummed predictions for R_2 at **NNLL** accuracy have been computed more recently

[Banfi et al., Phys. Rev. Lett. **117** (2016) 172001]

$$R_2(y) = e^{-R_{\mathrm{NNLL}}(y)} \left[\left(1 + \frac{\alpha_s(Q)}{2\pi} H^{(1)} + \frac{\alpha_s(Q\sqrt{y})}{2\pi} C_{\mathrm{hc}}^{(1)} \right) \mathcal{F}_{\mathrm{NLL}}(y) + \frac{\alpha_s(Q)}{2\pi} \delta\mathcal{F}_{\mathrm{NNLL}}(y) \right]$$

- resummation performed with the `ARES` program
- matching to fixed-order: $\log R$ scheme
- counting of logs (NNLL) here refers to logs in $\ln R_2$

In contrast, resummed predictions for R_3 have a much lower logarithmic accuracy

- more colored emitters
- state-of-the-art resummation includes only $\mathcal{O}(\alpha_s^n L^{2n})$ and $\mathcal{O}(\alpha_s^n L^{2n-1})$ terms in R_3 (note different logarithmic counting)
- in this analysis, no resummation for R_3 is performed

$\Downarrow$

Main focus on N^{3}LO+NNLL for R_2, but also simultaneous analysis with NNLO for R_3

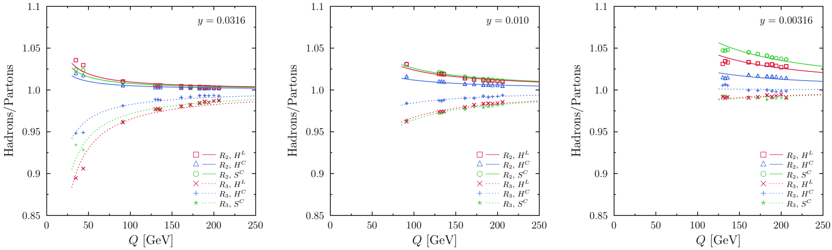
Effects associated with the parton-to-hadron transition cannot be computed in perturbation theory and must be estimated by other means.

In this analysis: obtained using state-of-the-art MC event generators: $e^+e^- \to jjjjj$ merged samples with massive b-quarks

- **Default setup "H^L": `Herwig7.1.4` for $e^+e^- \to 2, 3, 4, 5$ jets, 2 and 3 jets at NLO using `MadGraph5` and `OpenLoops` + Lund fragmentation model**
- Setup for hadronization systematics "H^C": `Herwig7.1.4` for $e^+e^- \to 2, 3, 4, 5$ jets, 2 and 3 jets at NLO using `MadGraph5` and `OpenLoops` + cluster fragmentation model
- Setup for cross-checks "S^C": `Sherpa2.2.6` for $e^+e^- \to 2, 3, 4, 5$ jets, 2 jets at NLO using `AMEGIC`, `COMIX` and `OpenLoops` + cluster fragmentation model

Hadronization correction factors

Hadronization correction factors for several values of y_{cut} for R_2 and R_3



- Correlations between R_2 and R_3 taken into account.
- Simultaneous corrections for R_2 and R_3 preserve physical constraints

$$R_{n,\text{hadrons}} \geq 0\,, \qquad R_{2,\text{hadrons}} + R_{3,\text{hadrons}} + R_{\geq 4,\text{hadrons}} = 1$$

Fit procedure

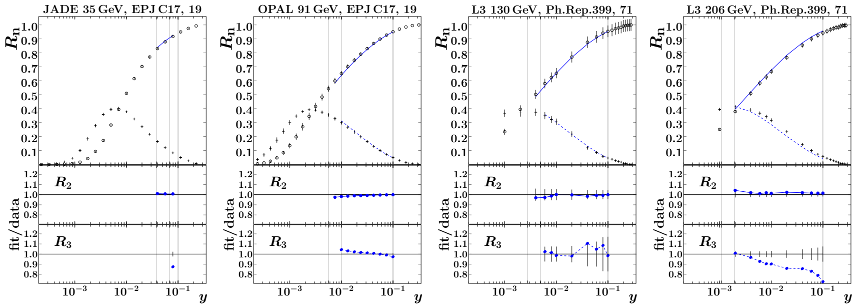
To find the optimal value of α_s, `MINUIT2` is used to minimize

$$\chi^2(\alpha_s) = \sum_{\text{data set}} \chi^2(\alpha_s)_{\text{data set}}$$

where $\chi^2(\alpha_s)$ are computed separately for each data set

$$\chi^2(\alpha_s) = \vec{r}\, V^{-1} \vec{r}^{\,T}\,, \qquad \vec{r} = (\vec{D} - \vec{P}(\alpha_s))$$

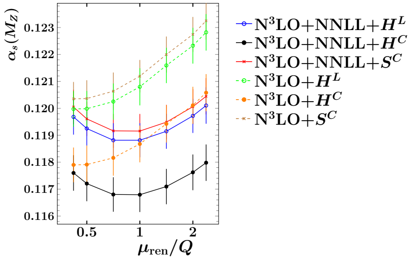
- $\vec{D}$: vector of data points
- $\vec{P}(\alpha_s)$: vector of theoretical predictions
- V: covariance matrix for $\vec{D}$ (statistical correlations estimated from MC generated samples, systematic correlations modeled to mimic patters observed in OPAL data)

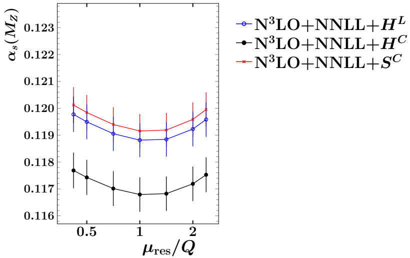


Central result and fit range selection

- avoid regions where theoretical predictions or hadronization model are unreliable
- Q^2-dependent fit range: $[-2.25 + \mathcal{L}, -1]$ for R_2 and $[-2 + \mathcal{L}, -1]$ for R_3 (if used), where $\mathcal{L} = \ln \frac{M_Z^2}{Q^2}$
- note separate fit ranges for R_2 and R_3 (if used)
- smallest $\chi^2/ndof$, low sensitivity to fit range

Fits: systematics and uncertainties





Estimate the uncertainty by

- varying the renormalization scale $\mu_{\text{ren}} \in [Q/2, 2Q]$: (*ren.*)
- varying the resummation scale $\mu_{\text{res}} \in [Q/2, 2Q]$: (*res.*)
- varying the hadronization model H^L vs. H^C: (*hadr.*)
- fit uncertainty is obtained from the $\chi^2 + 1$ criterion as implemented in `MINUIT2`: (*exp.*)

Notice much reduced renormalization scale uncertainty when NNLL resummation for R_2 is included

Results: R_2

Extraction of $\alpha_s(M_Z)$ from the two-jet rate R_2 measured over a wide range of cms energies in e^+e^- collisions has been performed at N^{3}LO+NNLL accuracy for the first time:

$$\alpha_s(M_Z) = 0.11881 \pm 0.00063(exp.) \pm 0.00101(hadr.) \pm 0.00045(ren.) \pm 0.00034(res.)$$
$$\alpha_s(M_Z) = 0.11881 \pm 0.00131(comb.)$$

- main source of uncertainty: **hadronization modeling**
- uncertainty from scale variation is considerably smaller than from hadronization
- experimental uncertainty comparable to perturbative one

Inclusion of **NNLL resummation crucial** for reducing perturbative uncertainty

Results: $R_2 + R_3$

Combined fit of R_2 at N^{3}LO+NNLL and R_3 at NNLO, taking into account for the first time the correlation between the observables gives:

$$\alpha_s(M_Z) = 0.11989 \pm 0.00045(exp.) \pm 0.00098(hadr.) \pm 0.00046(ren.) \pm 0.00017(res.)$$
$$\alpha_s(M_Z) = 0.11989 \pm 0.00118(comb.)$$

- result is fully compatible with R_2-only fit
- formally more precise than a fit based on R_2 alone,
- but much more sensitive to fit range selection

An accurate resummation of R_3 could potentially reduce the sensitivity to fit range selection and lead to an even more precise determination of $\alpha_s(M_Z)$

The strong coupling from jet rates: final result

The following value of $\alpha_s(M_Z)$ was obtained in the analysis

$$\boldsymbol{\alpha_s(M_Z) = 0.11881 \pm 0.00063\ (exp.) \pm 0.00101\ (hadr.) \pm 0.00045\ (ren.) \pm 0.00034\ (res.)}$$
$$\boldsymbol{\alpha_s(M_Z) = 0.11881 \pm 0.00131\ (comb.)}$$

- The result agrees with the world average $\alpha_s(M_Z)_{\mathrm{PDG2020}} = 0.1179 \pm 0.0010$ and has an uncertainty that is of the same size
- The presented result is the most precise in its subclass [Salam, arXiv:1712.05165v2]

Determination	Data and procedure
0.1175 ± 0.0025	ALEPH 3-jet rate (NNLO+MChad)
0.1199 ± 0.0059	JADE 3-jet rate (NNLO+NLL+MChad)
0.1224 ± 0.0039	ALEPH event shapes (NNLO+NLL+MChad)
0.1172 ± 0.0051	JADE event shapes (NNLO+NLL+MChad)
0.1189 ± 0.0041	OPAL event shapes (NNLO+NLL+MChad)
$0.1164^{+0.0028}_{-0.0026}$	Thrust (NNLO+NLL+anlhad)
$0.1134^{+0.0031}_{-0.0025}$	Thrust (NNLO+NNLL+anlhad)
0.1135 ± 0.0011	Thrust (SCET NNLO+N^{3}LL+anlhad)
0.1123 ± 0.0015	C-parameter (SCET NNLO+N^{3}LL+anlhad)

So what is the issue?

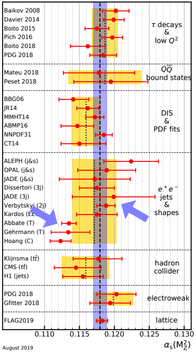
The main source of uncertainty is due to hadronization modeling and differences with other precise determinations remain.

Monte Carlo simulations used to obtain hadronization corrections, but

- the parton level of the MC simulation is not equivalent to a fixed-order calculation
- the tuning of the shower/hadronization models performed using theoretical predictions with lower perturbative accuracy

Sizeable difference with other precise determinations, e.g. those based on thrust (T). Basic differences

- NNLO vs. N^{3}LO perturbative accuracy
- Monte Calo vs. analytic hadronization models



Can we examine the role of higher orders and hadronization models in a single analysis?

The strong coupling from event shape averages

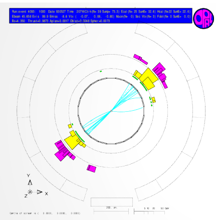
Event shapes

To address this question, we perform a state-of-the-art perturbative QCD analysis of event shape averages.

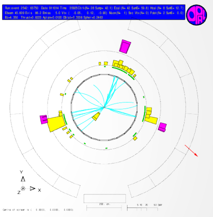
Event shapes associate a single number to the entire event, describing some specific aspect of the global event topology.

Thrust (T):

- Definition: $T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \right)$
- Generally $1/2 \leq T \leq 1$, where $T = 1/2$ for spherically symmetric events and $T \to 1$ in the dijet limit ("pencil-like" event)



$T \sim 1$



$T \sim 1/2$

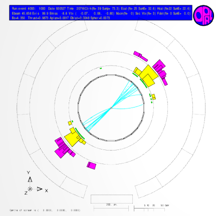
Event shapes

To address this question, we perform a state-of-the-art perturbative QCD analysis of event shape averages.

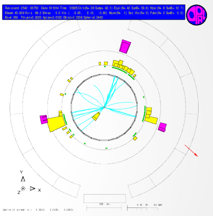
Event shapes associate a single number to the entire event, describing some specific aspect of the global event topology.

***C*-parameter** (C):

- Definition: $C = 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1)$, λ_i are eigenvalues of $\Theta^{\rho\sigma} = \frac{1}{\sum_i |\vec{p}_i|} \sum_i \frac{\vec{p}_i^{\,\rho}\, \vec{p}_i^{\,\sigma}}{|\vec{p}_i|}$
- Generally $0 \leq C \leq 1$, where $C = 1$ for spherically symmetric events and $C \to 0$ in the dijet limit ("pencil-like" event). For planar events $0 \leq C \leq 3/4$.



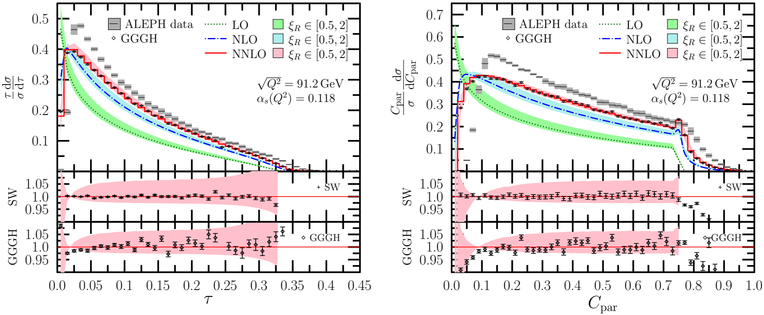
$C \sim 0$



$C \sim 1$

Three-jet event shapes (i.e., those that have a non-trivial distribution already with three final-state momenta) can be computed to **NNLO** accuracy in pQCD (note $\tau = 1 - T$).

[Gehrmann-De Ridder et al., JHEP **0712** (2007) 094,
Weinzierl, JHEP **0906** (2009) 041,
Del Duca et al., Phys. Rev. D **94** (2016) 7, 074019]



Clearly **event shape moments** can also be computed at the same accuracy.

Event shapes analysis: components

- Data form 20+ datasets with a wide range of energies: focus on thrust (T) and the C-parameter (C).
- Estimation of unknown N^3LO perturbative QCD coefficients from data (hence the focus on event shape averages $\Rightarrow$ small number of coefficients to fit).
- Hadronization corrections obtained from both Monte Carlo tools as well as analytic models extended to N^3LO for the first time.

Data

Combined analysis using 20+ datasets and a **wide range of energies**: $\sqrt{s} = 29$–206 GeV

	Measured		Used	
Source	Observables	Points, $\sqrt{s}$ range (GeV)	Observables	Points, $\sqrt{s}$ range (GeV)
ALEPH	$\langle(1-T)^1\rangle$	1,[133]	$\langle(1-T)^1\rangle$	1,[133]
ALEPH	$\langle(1-T)^1\rangle$	1,[91]	$\langle(1-T)^1\rangle$	1,[91]
ALEPH	$\langle(1-T)^1\rangle$	9,[91, 206]	$\langle(1-T)^1\rangle$	9,[91, 206]
AMY	$\langle(1-T)^1\rangle$	1,[55]	$\langle(1-T)^1\rangle$	1,[55]
DELPHI	$\langle(1-T)^{1,2,3}\rangle$	15,[91, 183]	$\langle(1-T)^1\rangle$	5,[91, 183]
DELPHI	$\langle(1-T)^1\rangle$	15,[45, 202]	$\langle(1-T)^1\rangle$	11,[45, 202]
HRS	$\langle(1-T)^1\rangle$	1,[29]	$\langle(1-T)^1\rangle$	1,[29]
JADE	$\langle(1-T)^{1,2,3,4,5}\rangle$	30,[14, 43]	$\langle(1-T)^1\rangle$	4,[34, 43]
L3	$\langle(1-T)^1\rangle$	1,[91]	$\langle(1-T)^1\rangle$	1,[91]
L3	$\langle(1-T)^{1,2}\rangle$	30,[41, 206]	$\langle(1-T)^1\rangle$	15,[41, 206]
MARK	$\langle(1-T)^1\rangle$	1,[89]	$\langle(1-T)^1\rangle$	1,[89]
MARK	$\langle(1-T)^1\rangle$	1,[29]	$\langle(1-T)^1\rangle$	1,[29]
MARKII	$\langle(1-T)^1\rangle$	1,[89]	$\langle(1-T)^1\rangle$	1,[89]
OPAL	$\langle(1-T)^{1,2,3,4,5}\rangle$	60,[91, 206]	$\langle(1-T)^1\rangle$	12,[91, 206]
TASSO	$\langle(1-T)^1\rangle$	4,[14, 44]	$\langle(1-T)^1\rangle$	2,[35, 44]
ALEPH	$\langle C^1\rangle$	1,[91]	$\langle C^1\rangle$	1,[91]
DELPHI	$\langle C^1\rangle$	15,[45, 202]	$\langle C^1\rangle$	11,[45, 202]
DELPHI	$\langle C^{1,2,3}\rangle$	12,[133, 183]	$\langle C^1\rangle$	4,[133, 183]
JADE	$\langle C^{1,2,3,4,5}\rangle$	30,[14, 43]	$\langle C^1\rangle$	4,[34, 43]
L3	$\langle C^1\rangle$	1,[91]	$\langle C^1\rangle$	1,[91]
L3	$\langle C^{1,2}\rangle$	18,[130, 206]	$\langle C^1\rangle$	9,[130, 206]
OPAL	$\langle C^{1,2,3,4,5}\rangle$	60,[91, 206]	$\langle C^1\rangle$	12,[91, 206]

Fixed-order predictions for event shape moments

The n-th moment of an event shape O is defined by

$$\langle O^n \rangle = \frac{1}{\sigma_{\text{tot}}} \int_{O_{\min}}^{O_{\max}} O^n \frac{\mathrm{d}\sigma(O)}{\mathrm{d}O} \mathrm{d}O$$

Fixed-order predictions up to and including α_s^4 terms read

$$\langle O^n \rangle = \frac{\alpha_s(Q)}{2\pi} A^{\langle O^n \rangle} + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 B^{\langle O^n \rangle} + \left(\frac{\alpha_s(Q)}{2\pi}\right)^3 C^{\langle O^n \rangle} + \left(\frac{\alpha_s(Q)}{2\pi}\right)^4 D^{\langle O^n \rangle} + \mathcal{O}(\alpha_s^5)$$

- First three coefficients ($A^{\langle O^n \rangle}$, $B^{\langle O^n \rangle}$ and $C^{\langle O^n \rangle}$) have been known for some time

  [Gehrmann-De Ridder et al., JHEP **05** (2009) 106, Weinzierl, Phys. Rev. D **80** (2009) 094018]
- Recomputed for this study with **CoLoRFulNNLO** ⇒ **very good numerical precision**

  [Del Duca et al., Phys. Rev. D **94** (2016) no.7, 074019]
- b-mass corrections from Zbb4: note only **NLO**

  [Nason, Oleari, Phys. Lett. B **407**, 57 (1997)]

$$A^{\langle O^n \rangle} = (1 - r_b(Q)) A^{\langle O^n \rangle}_{m_b=0} + r_b(Q) A^{\langle O^n \rangle}_{m_b\neq 0}$$

$$B^{\langle O^n \rangle} = (1 - r_b(Q)) B^{\langle O^n \rangle}_{m_b=0} + r_b(Q) B^{\langle O^n \rangle}_{m_b\neq 0}$$

where r_b is the fraction of b-quark events

$$r_b(Q) = \frac{\sigma_{m_b\neq 0}(e^+e^- \to b\bar{b})}{\sigma_{m_b\neq 0}(e^+e^- \to \text{hadrons})}$$

Event shape averages: predictions at NNLO and beyond

We focus on averages of the C-parameter $\langle C^1 \rangle$ and one minus thrust $\langle (1-T)^1 \rangle$

- abundance of available measurements (see above)
- avoid correlations between various moments (not reported by most measurements)

Fixed-order predictions at scale $Q = m_Z$ for the perturbative coefficients [normalized to the leading order cross section $\sigma_0(e^+e^- \to \text{hadrons})$]

Coefficient	This work	Analytic	GGGH	SW
$A_0^{\langle (1-T)^1 \rangle}$	2.1034(1)	2.10347	2.1035	2.10344(3)
$B_0^{\langle (1-T)^1 \rangle}$	44.995(1)		44.999(2)	44.999(5)
$C_0^{\langle (1-T)^1 \rangle}$	979.6(6)		867(21)	1100(30)
$A_0^{\langle C^1 \rangle}$	8.6332(5)	8.63789	8.6379	8.6378(1)
$B_0^{\langle C^1 \rangle}$	172.834(5)	172.859	172.778(7)	172.8(3)
$C_0^{\langle C^1 \rangle}$	3525(3)		3212(89)	4200(100)

[Gehrmann-De Ridder et al., JHEP **05** (2009) 106 (GGGH), Weinzierl, Phys. Rev. D **80** (2009) 094018] (SW)

We extract $D^{\langle (1-T)^1 \rangle}$ and $D^{\langle C^1 \rangle}$ from data together with $\alpha_s(M_Z)$ in the analysis.

Event shape averages: predictions at NNLO and beyond

Importantly, the main point of extracting the N^{3}LO coefficients $D^{\langle (1-T)^1 \rangle}$ and $D^{\langle C^1 \rangle}$ from data is **not** to get an accurate determination of these quantities.

Rather, it is to model them as best as possible in order to be able to **assess the impact** of including terms beyond NNLO in the extraction of the strong coupling in the absence of an actual calculation of those terms.

Modeling of non-perturbative corrections

The modeling of non-perturbative corrections is essential to perform a meaningful comparison of predictions with data.

To basic approaches

1. **Monte Carlo (MC) hadronization**: extract hadronization corrections from Monte Carlo simulations.

   Issue: the parton level of an MC simulation is not equivalent to a fixed-order calculation + issue of tuning.

2. **Analytic hadronization**: use analytic models to describe the effects of hadronization on observables.

   Issue: systematics are difficult to control.

$$\Downarrow$$

Apply both approaches and examine the impact of the choice on the extracted value of the strong coupling.

Monte Carlo hadronization

Hadronization corrections obtained using state-of-the-art MC event generators: $e^+e^- \to Z/\gamma \to 2,3,4,5$ parton processes generated using `MadGraph5` and `OpenLoops`, 2-parton final state at NLO.

To study hadronization systematics, we employ different setups (similar to jet rates analysis):

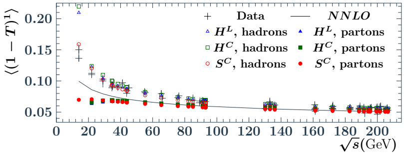
- **Default setup "H^L": `Herwig7.2.0` with Lund fragmentation model**
- Setup for systematics "H^C": `Herwig7.2.0` with cluster fragmentation model
- Setup for cross-checks "S^C": `Sherpa2.2.8` with cluster fragmentation model

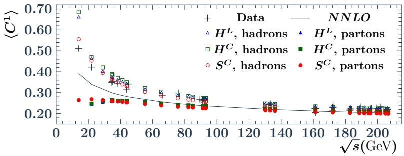
Hadronization corrections are ratios of observables calculated from MC generated events at hadron and parton levels.

To account for the presence of a shower cut-off scale $Q_0 \approx \mathcal{O}(1\,\text{GeV})$ in MC generators, predictions were computed with several values of Q_0 and extrapolated to $Q_0 \to 0$ GeV.

$$\langle O^n \rangle_{\text{corrected}} = \langle O^n \rangle_{\text{theory}} \times \frac{\langle O^n \rangle_{\text{MC hadrons},\ Q_0=0\ \text{GeV}}}{\langle O^n \rangle_{\text{MC partons},\ Q_0=0\ \text{GeV}}}$$

Data and predictions by MC event generators extrapolated to $Q_0 \to 0$ GeV.





- Hadron and parton level MC predictions provide **reasonable descriptions** of data and NNLO theory for wide range of energy
- **Non-physical** behaviour of MC parton level results for small $\sqrt{s}$: $\langle O^n\rangle$ increases with energy

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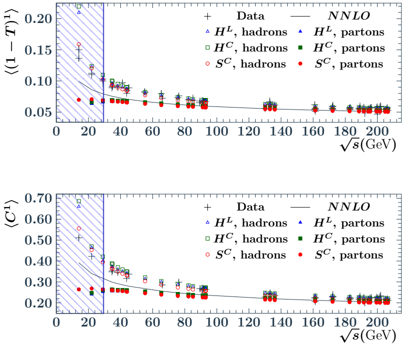

- Hadron and parton level MC predictions provide **reasonable descriptions** of data and NNLO theory for wide range of energy
- **Non-physical** behaviour of MC parton level results for small $\sqrt{s}$: $\langle O^n\rangle$ increases with energy

$\Downarrow$

- **Exclude measurements** with $\sqrt{s} < 29$ GeV
- Weaker criterion than requiring that MC matches data well, but retains as much data as possible

Dispersive model of analytic hadronization corrections for event shapes: hadronization corrections simply shift the perturbative event shape distributions

$$\frac{\mathrm{d}\sigma_{\text{hadrons}}(O)}{\mathrm{d}O} = \frac{\mathrm{d}\sigma_{\text{partons}}(O - a_O\mathcal{P})}{\mathrm{d}O}$$

- the a_O are observable-specific **constants**, e.g., $a_{1-T} = 2$ and $a_C = 3\pi$
- the power correction $\mathcal{P}$ is **universal**

$$\mathcal{P}(\alpha_s, Q, \alpha_0) = \frac{4C_F}{\pi^2}\mathcal{M} \times \frac{\mu_I}{Q} \times \left\{\alpha_0(\mu_I) - \alpha_S + \mathcal{O}(\alpha_s^2)\right\}$$

where $\mathcal{M}$ is the so-called Milan factor and α_0 is a non-perturbative model parameter

Under these **assumptions**, we find that non-perturbative corrections simply shift the perturbative event shape averages

$$\langle O^1\rangle_{\text{hadrons}} = \langle O^1\rangle_{\text{partons}} + a_O\mathcal{P}$$

Issues with the dispersive model

Recently, both of these assumptions have been challenged

- the a_O are observable-specific **constants**

  Issue: a_O have been computed in the two-jet limit, but they actually depend on the value of O. This dependence is a source of uncertainty in α_s extractions based on event shapes and analytic hadronization models that has not been accounted for so far and may be responsible for some of the tension between recent α_s determinations.

  [Luisoni, Monni, Salam, Eur. Phys. J. C **81** (2021) 2, 158, Caola et al., arXiv:2108.08897 [hep-ph]]

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  Issue: non-inclusive corrections, e.g., those parametrized by the Milan factor $\mathcal{M}$ may not in fact be universal beyond NLO

In this analysis we take the pragmatic viewpoint that this approach nevertheless provides a reasonable model for non-perturbative corrections. However, the applicability of the dispersive model should be investigated.

Computing the power correction

We must compute $\mathcal{P}$ at $\mathcal{O}(\alpha_s^4)$ accuracy. Ingredients of the computation are

- The running of the strong coupling in the $\overline{\text{MS}}$ scheme
- The relation between the effective soft coupling in the Catani–Marchesini–Webber (CMW) scheme α_s^{CMW} and the strong coupling defined in the $\overline{\text{MS}}$ scheme α_s

$$\alpha_S^{CMW} = \alpha_S \left[1 + \frac{\alpha_S}{2\pi} K + \left(\frac{\alpha_S}{2\pi}\right)^2 L + \left(\frac{\alpha_S}{2\pi}\right)^3 M + \mathcal{O}(\alpha_S^4)\right]$$

- K, L and M are in principle computable constants
- K is simply the one-loop cusp anomalous dimension
- L and M can be computed once the effective soft coupling is explicitly defined $\Rightarrow$ several proposals in the literature beyond NLL, so L and M are "scheme-dependent"

Effective soft coupling schemes

The Catani–Marchesini–Webber soft coupling at NLL (α_s is the strong coupling in the $\overline{\text{MS}}$ scheme, $C_q = C_F,\ C_g = C_A$)

$$\mathcal{A}_i^{CMW}(\alpha_s) = C_i \frac{\alpha_s^{CMW}}{\pi} = C_i \frac{\alpha_s^{CMW}}{\pi}\left(1 + \frac{\alpha_s}{2\pi} K\right)$$

Proposals for definitions beyond NLL

$$\mathcal{A}_{T,i}(\alpha_s) = \frac{1}{2}\mu^2 \int_0^\infty dm_T^2 dk_T^2 \delta(\mu^2 - k_T^2) w_i(k)$$
$$\mathcal{A}_{0,i}(\alpha_s) = \frac{1}{2}\mu^2 \int_0^\infty dm_T^2 dk_T^2 \delta(\mu^2 - m_T^2) w_i(k)$$

where $w_i(k)$ is called the web function, it gives the "probability" of correlated emission (including the corresponding virtual corrections) of an arbitrary number of soft partons with total momentum k.

[Catani, De Florian and Grazzini, Eur. Phys. J. C **79**, 685 (2019),
Banfi, El-Menoufi and P.F. Monni JHEP **01**, 083 (2019)]

Effective soft coupling schemes

Given these definitions, the expansion of α_s^{CMW} in terms of α_s, and hence L and M, can in principle be computed (note in each scheme K is the one-loop cusp anomalous dimension)

$$(\alpha_s^{CMW})_{\text{scheme}} = \alpha_s \left[1 + \frac{\alpha_s}{2\pi}K + \left(\frac{\alpha_s}{2\pi}\right)^2 L_{\text{scheme}} + \left(\frac{\alpha_s}{2\pi}\right)^3 M_{\text{scheme}} + \mathcal{O}(\alpha_s^4)\right]$$

- A^0 scheme: L and M computed from $\mathcal{A}_{0,i}$
- A^T scheme: L computed from $\mathcal{A}_{T,i}$, but the complete expression for M is missing in this scheme, hence we set $M_T = M_0$
- A^{cusp} scheme: L and M are simply set equal to the two-and three-loop cusp anomalous dimensions (for ease of comparison with existing results)

The power correction

The power correction at $\mathcal{O}(\alpha_s^4)$ accuracy reads

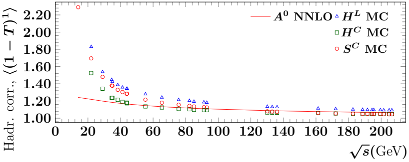
$$\mathcal{P}(\alpha_S, Q, \alpha_0) = \frac{4C_F}{\pi^2}\,\mathcal{M} \times \frac{\mu_I}{Q} \times \Bigg\{ \alpha_0(\mu_I) - \Bigg[\alpha_S(\mu_R) + \left(K + \beta_0 \left(1 + \ln \frac{\mu_R}{\mu_I} \right) \right) \frac{\alpha_S^2(\mu_R)}{2\pi}$$
$$+ \left(2L + (4\beta_0 (\beta_0 + K) + \beta_1) \left(1 + \ln \frac{\mu_R}{\mu_I} \right) + 2\beta_0^2 \ln^2 \frac{\mu_R}{\mu_I} \right) \frac{\alpha_S^3(\mu_R)}{8\pi^2}$$
$$+ \Bigg(4M + (2\beta_0 (12\beta_0(\beta_0 + K) + 5\beta_1) + \beta_2 + 4\beta_1 K + 12\beta_0 L) \left(1 + \ln \frac{\mu_R}{\mu_I} \right)$$
$$+ \beta_0(12\beta_0(\beta_0 + K) + 5\beta_1) \ln^2 \frac{\mu_R}{\mu_I} + 4\beta_0^3 \ln^3 \frac{\mu_R}{\mu_I} \Bigg) \frac{\alpha_S^4(\mu_R)}{32\pi^3} \Bigg] \Bigg\}$$

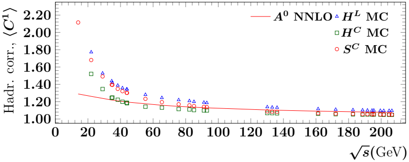
- $\mathcal{M}$ is the so-called Milan factor with estimated value $\mathcal{M}_{\text{est.}} \pm \delta\mathcal{M}_{\text{est.}} = 1.49 \pm 0.30$.
- μ_I is the scale where the perturbative and non-perturbative couplings are matched. Following the usual choice, we set $\mu_I = 2\,\text{GeV}$.
- $\alpha_0(\mu_I)$ corresponds to the first moment of the effective soft coupling below the scale μ_I and is a **non-perturbative parameter** of the model

$$\alpha_0(\mu_I) = \frac{1}{\mu_I} \int_0^{\mu_I} \mathrm{d}\mu\, \alpha_s^{CMW}(\mu)$$

Hadronization correction factors

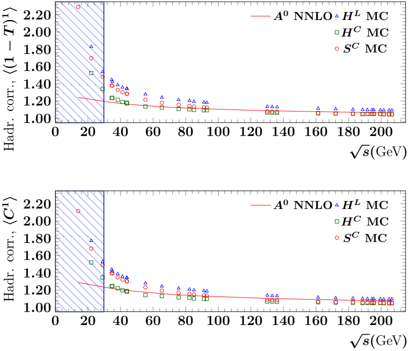
Ratios of hadron-level to parton-level predictions





- Analytic hadronization: the result for the A^0 scheme is shown, but the different schemes give very similar results.

Ratios of hadron-level to parton-level predictions



- Analytic hadronization: the result for the A^0 scheme is shown, but the different schemes give very similar results.
- Recall measurements with $\sqrt{s} < 29$ GeV are excluded.
- Weaker criterion than requiring that sub-leading power corrections are small.
- Serves to highlight the discrepancies between MC and analytic models where hadronization effects are most pronounced (low energies).

Fit procedure

Values of α_s determined using optimization procedures in `MINUIT2`

$$\chi^2(\alpha_S) = \sum_{i}^{\text{all data sets}} \chi_i^2(\alpha_S)$$

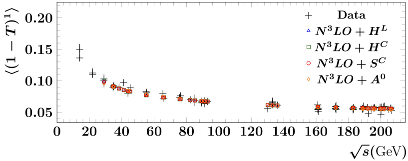
where $\chi_i^2(\alpha_S)$ for data set i is

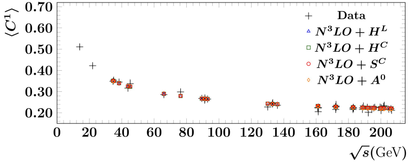
$$\chi_i^2(\alpha_S) = (\vec{D} - \vec{P}(\alpha_S))V^{-1}(\vec{D} - \vec{P}(\alpha_S))^T$$

- $\vec{D}$: vector of data points
- $\vec{P}(\alpha_S)$: vector of calculated predictions
- V: the covariance matrix of $\vec{D}$ (diagonal, stat. and syst. uncertainties added in quadrature for every measurement)

Fit results

Results of the fits at N^{3}LO vs. data. In addition to $\alpha_s(M_Z)$, we fit also

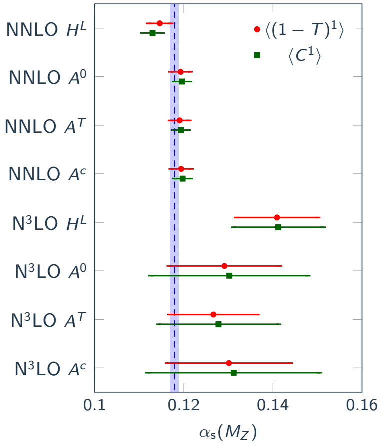




- the $\mathcal{O}(\alpha_s^4)$ perturbative coefficient $D^{\langle O^n\rangle}$ (in N^{3}LO fits)
- the non-perturbative parameter $\alpha_0(2\,\text{GeV})$ (when using the analytic hadronization model)
- the Milan factor $\mathcal{M}$, in order to include the uncertainty on its theoretical value consistently (constrained fit)
- since the dependence on analytic hadronization scheme is mild so only the result for the A^0 scheme is shown

Results: $\alpha_s(M_Z)$

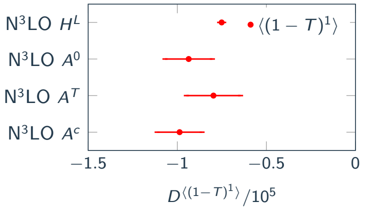
The extractions of $\alpha_s(M_Z)$ from $\langle(1-T)^1\rangle$ and $\langle C^1\rangle$ data

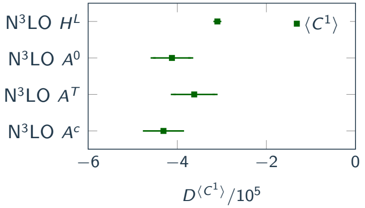


- Good agreement between fits to $\langle(1-T)^1\rangle$ and $\langle C^1\rangle$ data both at NNLO and N^3LO $\Rightarrow$ internal consistency of extraction procedure
- Analytic hadronization scheme-dependence is mild.
- **Large discrepancy between results obtained with MC and analytic hadronization models** both at NNLO and N^3LO $\Rightarrow$ suggests that the discrepancy has a fundamental origin and would hold even with exact N^3LO predictions.
- **Better understanding of hadronization is key.**

Results: $D^{\langle O^n \rangle}$

The extractions of the $\mathcal{O}(\alpha_s^4)$ perturbative coefficients $D^{\langle (1-T)^1 \rangle}$ and $D^{\langle C^1 \rangle}$ from data

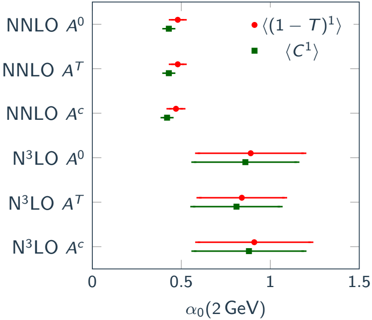




- Extracted values of the perturbative coefficients show reasonable agreement for both observables between fits using MC and analytic hadronization models $\Rightarrow$ demonstrates the viability of extracting higher-order coefficients from data.
- The amount and consistency of current data is an issue, would need large amounts of consistent data, e.g., from FCC-*ee* or CEPC.
- Precise high-energy data would be especially valuable.

Results: $\alpha_0(2\,\mathrm{GeV})$

The extractions of the non-perturbative parameter $\alpha_0(2\,\mathrm{GeV})$ from $\langle(1-T)^1\rangle$ and $\langle C^1\rangle$ data



- Recall this parameter is scheme-dependent, so its values in different schemes should not be directly compared. Nevertheless, **the choice of scheme has only a small numerical impact**.
- Values extracted from $\langle(1-T)^1\rangle$ and $\langle C^1\rangle$ data agree well with each other both at NNLO and N^{3}LO.
- Rather large uncertainties at N^{3}LO primarily due to insufficient amount and quality of data as well as the extraction method itself.

What did we learn?

The aim of the analysis was to examine the role higher order corrections and the choice of hadronization models play in a single analysis.

In particular, we wanted to assess the factors that will determine the precision of QCD analyses of e^+e^- data once theoretical predictions at $\mathcal{O}(\alpha_s^4)$ accuracy become available.

To do this, we have performed an extraction of $\alpha_s(M_Z)$ from the averages of event shapes $\langle(1-T)^1\rangle$ and $\langle C^1\rangle$.

- Using **NNLO theory** and **analytic hadronization models**, the obtained results are consistent with the last world average $\alpha_s(M_Z)_{\mathrm{PDG2020}} = 0.1179 \pm 0.0010$.
- We considered a method of extracting $\alpha_s(M_Z)$ at N^{3}LO by estimating the missing $\mathcal{O}(\alpha_s^4)$ perturbative coefficient from data. The values of $\alpha_s(M_Z)$ obtained in this way are compatible with the last world average, within somewhat large uncertainties, e.g.,

$$\alpha_s(M_Z)^{N^3LO+A^0} = 0.12911 \pm 0.00177(exp.) \pm 0.0123(scale)$$

- Both MC and analytic hadronization models were used, the latter being extended to $\mathcal{O}(\alpha_s^4)$ for the first time.
- The comparison of results obtained with MC and analytic hadronization suggests that **future extractions** of $\alpha_s(M_Z)$ **will be strongly affected by the modeling of hadronization effects**.

Lessons

Improving perturbative predictions

Improving the perturbative predictions is clearly important.

More N's, more legs

- beyond NNLO for 3-jet rate/event shapes
- beyond 3-jet rate/event shapes at NNLO
- improved logarithmic accuracy
  for 3-jet rate/event shapes

Mass effects, mixed EW×QCD corrections

- mass corrections (finite m_b) beyond NLO
- mixed EW×QCD corrections

Improving perturbative predictions

Improving the perturbative predictions is clearly important.

More N's, more legs

• beyond NNLO for 3-jet rate/event shapes	$\Rightarrow$	ultimate precision, but may be of limited use by itself
• beyond 3-jet rate/event shapes at NNLO	$\Rightarrow$	would be nice. . .
• improved logarithmic accuracy for 3-jet rate/event shapes	$\Rightarrow$	**already within reach!**

Mass effects, mixed EW×QCD corrections

• mass corrections (finite m_b) beyond NLO	$\Rightarrow$	more relevant
• mixed EW×QCD corrections	$\Rightarrow$	more relevant

Improving perturbative predictions

Improving the perturbative predictions is clearly important.

More N's, more legs

- beyond NNLO for 3-jet rate/event shapes ⇒ ultimate precision, but may be of limited use by itself
- beyond 3-jet rate/event shapes at NNLO ⇒ would be nice...
- improved logarithmic accuracy for 3-jet rate/event shapes ⇒ **already within reach!**

Mass effects, mixed EW×QCD corrections

- mass corrections (finite m_b) beyond NLO ⇒ more relevant
- mixed EW×QCD corrections ⇒ more relevant

However

- the necessary 2- and 3-loop matrix elements are presently **not known**, however this is a very active area, so **expect new results**
- using those matrix elements to compute **physical observables** is a **separate issue** in itself (definitely beyond NNLO), new ideas may be needed

The role of hadronization corrections

But the **elephant in the room**: hadronization modeling

- discrepancies between results obtained with MC and analytic hadronization models will likely remain in place even after including exact higher order perturbative corrections beyond NNLO
- naively going to higher energies helps: hadr. corr. $\sim 1/Q$, however...
- the energy of foreseen machines (FCC-*ee*, CEPC) is not orders of magnitude larger than LEP
- moreover, going up in energy there is non-trivial interplay between smaller hadronization corrections but larger background and much smaller luminosity

The role of hadronization corrections

Bottom line: **need better MC's + hadronization models/calibration** in e^+e^-

In a perfect world

- Parton showers with NNLL logarithmic accuracy matched to NNLO
- Hadronization models calibrated from scratch with many different observables, since current models were tuned using MC's with lower accuracy

Alternatively

- Need a (much) more refined analytical understanding of non-perturbative corrections, for recent advances see e.g.,

  [Luisoni, Monni, Salam, Eur. Phys. J. C **81** (2021) 2, 158, Caola et al., arXiv:2108.08897 [hep-ph]]

- Look for better observables with smaller hadronization corrections, e.g., groomed event shapes

  [Baron, Marzani, Theeuwes, JHEP **08** (2018) 105, Kardos, Larkoski, Trócsányi, Phys. Lett. B **809** (2020) 135704]

Conclusions

So where do we stand?

- No new data foreseen in the near future, so would including **more perturbative orders** (fixed order and/or resummation) improve precision **without any new data**?

  Not by itself. More perturbative orders alone are not likely to dramatically improve the precision of strong coupling extractions from existing data.

- If not, what are the **limiting factors** for precision in future QCD studies?

  Main limiting factors are: systematics related to the estimation of **hadronization corrections** as well as the quality and consistency of current data.

- What should be done to **eliminate** those factors?

  In addition to advancing the perturbative predictions, we **must seriously refine our understanding/modeling of non-perturbative effects**. This would be aided greatly by dedicated low-energy (below the Z-peak) measurements at future e^+e^- facilities.

Thank you for your attention!

Backup slides

Hadronization corrections: simultaneous corrections for R_2 and R_3

Challenge: simultaneous corrections for R_2 and R_3

- hadronization corrections derived on a bin-by-bin basis, $R_{n,\mathrm{hadron}} = R_{n,\mathrm{parton}} f_n(y)$, $n = 2, 3, 4, \ldots$ can violate physical constraints: $0 \leq R_n \leq 1$ and $\sum_n R_n = 1$

Solution:

- introduce ξ_1 and ξ_2 such that at parton level $R_{2,\mathrm{parton}} + R_{3,\mathrm{parton}} + R_{\geq 4,\mathrm{parton}} = 1$

$$R_{2,\mathrm{parton}} = \cos^2 \xi_1 \,, \qquad R_{3,\mathrm{parton}} = \sin^2 \xi_1 \cos^2 \xi_2 \,, \qquad R_{\geq 4,\mathrm{parton}} = \sin^2 \xi_1 \sin^2 \xi_2 \,,$$

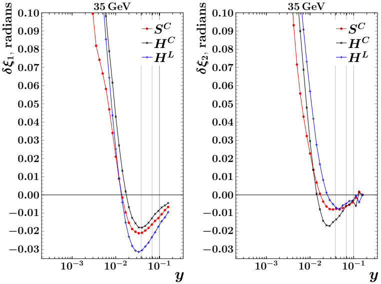
- similarly at hadron level, set

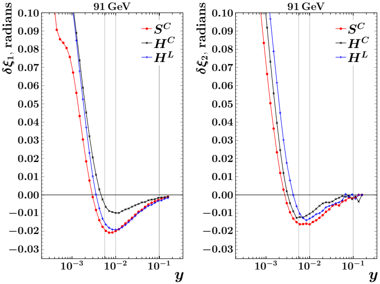
$$R_{2,\mathrm{hadron}} = \cos^2(\xi_1 + \delta\xi_1) \,, \qquad R_{3,\mathrm{hadron}} = \sin^2(\xi_1 + \delta\xi_1) \cos^2(\xi_2 + \delta\xi_2) \,,$$
$$R_{\geq 4,\mathrm{hadron}} = \sin^2(\xi_1 + \delta\xi_1) \sin^2(\xi_2 + \delta\xi_2)$$

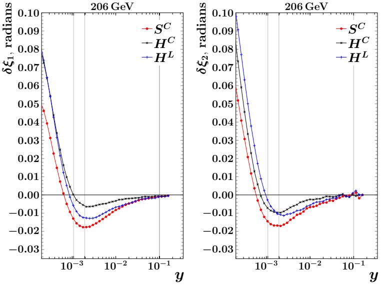
- the functions $\delta\xi_1(y)$ and $\delta\xi_2(y)$ account for hadronization corrections and are extracted from the MC samples

This approach clearly preserves physical constraints

Hadronization corrections: $\delta\xi_1(y)$ and $\delta\xi_2(y)$

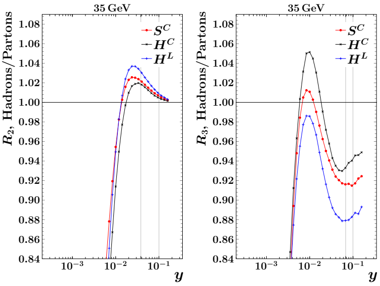


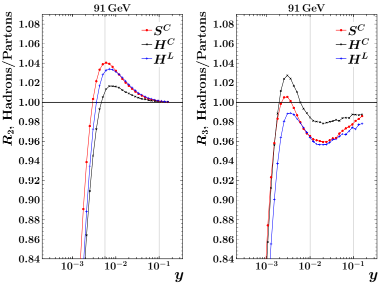


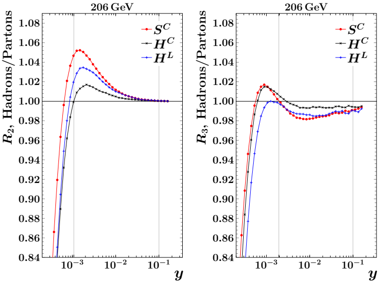


- to avoid binning effects, the hadronization corrections are parametrized with smooth functions
- vertical lines show the fit ranges for the reference fits of R_2 and R_3

Hadronization corrections: hadron to parton ratios







- to avoid binning effects, the hadronization corrections are parametrized with smooth functions
- vertical lines show the fit ranges for the reference fits of R_2 and R_3

R_2 fits

Fit of $\alpha_s(M_Z)$ from experimental data for R_2 obtained using N^3LO and $N^3LO+NNLL$ predictions for R_2. The reported uncertainty comes from `MINUIT2`

Fit ranges, log y Hadronization	N^3LO $\chi^2/ndof$	$N^3LO+NNLL$ $\chi^2/ndof$
$[-1.75+\mathcal{L},-1]$ S^C	0.12121 ± 0.00095 $20/86 = 0.24$	0.11849 ± 0.00092 $20/86 = 0.24$
$[-2+\mathcal{L},-1]$ S^C	0.12114 ± 0.00081 $26/100 = 0.26$	0.11864 ± 0.00075 $26/100 = 0.26$
$[-2.25+\mathcal{L},-1]$ S^C	0.12119 ± 0.00060 $44/150 = 0.29$	0.11916 ± 0.00063 $44/150 = 0.29$
$[-2.5+\mathcal{L},-1]$ S^C	0.12217 ± 0.00052 $89/180 = 0.50$	0.12075 ± 0.00055 $107/180 = 0.59$
$[-1.75+\mathcal{L},-1]$ H^C	0.11957 ± 0.00098 $22/86 = 0.26$	0.11698 ± 0.00093 $22/86 = 0.25$
$[-2+\mathcal{L},-1]$ H^C	0.11923 ± 0.00079 $29/100 = 0.29$	0.11687 ± 0.00076 $28/100 = 0.28$
$[-2.25+\mathcal{L},-1]$ H^C	0.11868 ± 0.00068 $43/150 = 0.28$	0.11679 ± 0.00064 $40/150 = 0.27$
$[-2.5+\mathcal{L},-1]$ H^C	0.11849 ± 0.00050 $58/180 = 0.32$	0.11723 ± 0.00053 $58/180 = 0.32$
$[-1.75+\mathcal{L},-1]$ H^L	0.12171 ± 0.00109 $21/86 = 0.25$	0.11897 ± 0.00092 $21/86 = 0.24$
$[-2+\mathcal{L},-1]$ H^L	0.12144 ± 0.00078 $28/100 = 0.28$	0.11893 ± 0.00075 $26/100 = 0.26$
$[-2.25+\mathcal{L},-1]$ H^L	0.12080 ± 0.00069 $43/150 = 0.28$	0.11881 ± 0.00063 $39/150 = 0.26$
$[-2.5+\mathcal{L},-1]$ H^L	0.12024 ± 0.00051 $57/180 = 0.32$	0.11897 ± 0.00053 $52/180 = 0.29$

$R_2 + R_3$ fits

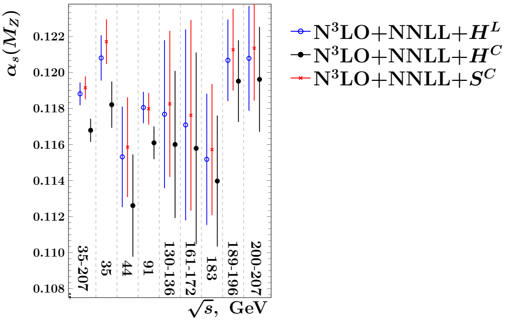
Simultaneous fit of $\alpha_s(M_Z)$ from experimental data for R_2 and R_3 obtained using N^3LO and N^3LO+NNLL predictions for R_2 and NNLO predictions for R_3. The reported uncertainty comes from `MINUIT2`

Fit ranges, $\log y$ Hadronization	N^3LO $\chi^2/ndof$	N^3LO+NNLL $\chi^2/ndof$
$[-1.75+\mathcal{L},-1][-1.5+\mathcal{L},-1]$ S^C	0.12195 ± 0.00072 $120/143 = 0.84$	0.12078 ± 0.00066 $140/143 = 0.98$
$[-2+\mathcal{L},-1][-1.75+\mathcal{L},-1]$ S^C	0.12163 ± 0.00061 $153/187 = 0.82$	0.12065 ± 0.00056 $176/187 = 0.94$
$[-2.25+\mathcal{L},-1][-2+\mathcal{L},-1]$ S^C	0.12075 ± 0.00044 $208/251 = 0.83$	0.11994 ± 0.00041 $222/251 = 0.88$
$[-2.5+\mathcal{L},-1][-2.25+\mathcal{L},-1]$ S^C	0.12143 ± 0.00043 $321/331 = 0.97$	0.12089 ± 0.00044 $336/331 = 1.01$
$[-1.75+\mathcal{L},-1][-1.5+\mathcal{L},-1]$ H^C	0.12068 ± 0.00073 $126/143 = 0.88$	0.11956 ± 0.00066 $147/143 = 1.03$
$[-2+\mathcal{L},-1][-1.75+\mathcal{L},-1]$ H^C	0.12006 ± 0.00061 $163/187 = 0.87$	0.11913 ± 0.00054 $188/187 = 1.01$
$[-2.25+\mathcal{L},-1][-2+\mathcal{L},-1]$ H^C	0.11869 ± 0.00043 $221/251 = 0.88$	0.11793 ± 0.00043 $238/251 = 0.95$
$[-2.5+\mathcal{L},-1][-2.25+\mathcal{L},-1]$ H^C	0.11845 ± 0.00045 $302/331 = 0.91$	0.11799 ± 0.00047 $310/331 = 0.94$
$[-1.75+\mathcal{L},-1][-1.5+\mathcal{L},-1]$ H^L	0.12248 ± 0.00068 $121/143 = 0.85$	0.12129 ± 0.00063 $141/143 = 0.99$
$[-2+\mathcal{L},-1][-1.75+\mathcal{L},-1]$ H^L	0.12211 ± 0.00057 $155/187 = 0.83$	0.12110 ± 0.00053 $180/187 = 0.96$
$[-2.25+\mathcal{L},-1][-2+\mathcal{L},-1]$ H^L	0.12071 ± 0.00044 $209/251 = 0.83$	0.11989 ± 0.00045 $227/251 = 0.90$
$[-2.5+\mathcal{L},-1][-2.25+\mathcal{L},-1]$ H^L	0.12041 ± 0.00044 $266/331 = 0.80$	0.11990 ± 0.00044 $278/331 = 0.84$

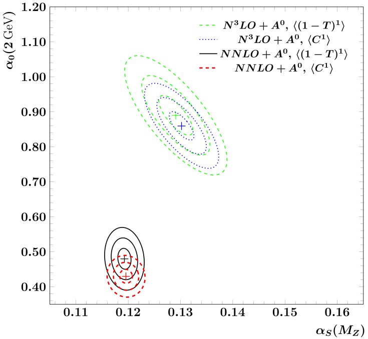
Consistency tests

Several consistency tests performed

- simultaneous fit of $R_2 + R_3$ (see above)
- separate R_3 fit
- variation of χ^2 definition
- change of fit ranges
- multiplicative hadronization corrections
- `Sherpa` MC hadronization S^C
- stability across $\sqrt{s}$ (see below)
- exclusion of data with $\sqrt{s} < M_Z$



Correlations between $\alpha_s(M_Z)$ and $\alpha_0(2\,\text{GeV})$



- contours correspond to 1-, 2- and 3 standard deviations obtained in the fit
- systematic uncertainties not included