Towards hadronic initial states in CoLoRFulNNLO

Gábor Somogyi

MTA-DE Particle Physics Research Group
University of Debrecen

with G. Bevilacqua, V. Del Duca, C. Duhr, A. Kardos, Z. Szőr, Z. Trócsányi and Z. Tulipánt

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Outline

1. Introduction
2. CoLoRFulNNLO
3. Towards processes with hadronic initial states
4. Conclusions

Introduction

Challenge of high precision

LHC has now produced > 3 years of 13 TeV data, $\mathcal{L}_{\rm int} > 150$ fb^{-1}

Excellent machine and detector performance in tough environment

- data taking efficiency $\sim$ 94%, at or above 90% used for physics
- average pile-up $\sim$ 38 in 2017 and 2018

Experimental precision reached

- SM benchmark processes (e.g., W, Z production) measured to 1% exp. precision (important tests of and constraints on theory)
- jets also doing great: total experimental systematic uncertainty in the cross section $\sim$ 6%, at low rapidities ($|y| < 2$)

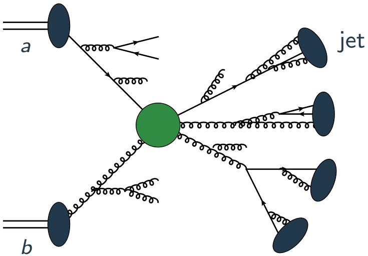
There is lots more data to come

- HL-LHC approved with integrated luminosity goal of 3000 fb^{-1}
- So far, only a fraction of foreseen data registered and analyzed

Must take up the challenge of high precision also on the theory side

QCD at colliders

To fully exploit the physics potential of colliders requires precision, QCD must be understood/modeled as best as feasible



$$d\sigma = \sum_{a,b} \int dx_a \int dx_b \underbrace{f_a(x_a, \mu_F^2)\, f_b(x_b, \mu_F^2)}_{\text{non-pert. PDFs}} \times \underbrace{d\hat{\sigma}_{ab}(x_a, x_b, Q^2, \alpha_S(\mu_R^2))}_{\text{pert. partonic x-sec}} + \mathcal{O}\left((\Lambda/Q)^m\right)$$

- One particular aspect of precision: calculation of exact higher order corrections to physical observables in perturbation theory

Time for NNLO

Mass production of two-loop amplitudes is becoming a reality: need frameworks to handle all of the other parts of the NNLO calculation too

- We must deal with double real and real-virtual kinematic singularities present at intermediate stages of the calculation
- More and more approaches are maturing to form general prescriptions which can be coded into general numerical tools
- Automated NNLO calculations are on the way

CoLoRFulNNLO is such a framework

CoLoRFulNNLO

The NNLO cross section

Aim: compute cross sections at NNLO with arbitrary acceptance cuts (J) in $d = 4$

$$\sigma^{\mathrm{NNLO}}[J] = \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{VV}}_{m} J_{m}$$

- Phase space integrals must be performed numerically
- All three terms are separately IR divergent in $d = 4$ dimensions
- Infrared singularities cancel between real and virtual quantum corrections at the same order in perturbation theory, for sufficiently inclusive (i.e., IR safe) observables (KLN theorem)

How to make this cancellation explicit, so that the various contributions can be computed numerically?

Sources of IR singularities

The NNLO correction to a generic m-jet observable is the sum of three terms

$$\sigma^{\text{NNLO}} = \int_{m+2} \mathrm{d}\sigma^{\text{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\text{RV}}_{m+1} J_{m+1} + \int_{m} \mathrm{d}\sigma^{\text{VV}}_{m} J_{m}$$

Double real

- Tree level squared MEs with $m+2$-parton kinematics
- MEs diverge as one or two partons unresolved
- phase space integral divergent (up to $\mathrm{O}(\epsilon^{-4})$ poles from PS integration in dim. reg.)
- no loops, so no explicit ϵ poles in dim. reg.

Real-virtual

- One-loop squared MEs with $m+1$-parton kinematics
- MEs diverge as one parton unresolved
- phase space integral divergent (up to $\mathrm{O}(\epsilon^{-2})$ poles from PS integration in dim. reg.)
- one loop, explicit ϵ poles up to $\mathrm{O}(\epsilon^{-2})$ from loop integration in dim. reg.

Double virtual

- Two-loop squared MEs with m-parton kinematics
- jet function screens divergences in MEs as partons become unresolved
- phase space integral is finite
- two loops, explicit ϵ poles up to $\mathrm{O}(\epsilon^{-4})$ from loop integration in dim. reg.

CoLoRFulNNLO

CoLoRFulNNLO is built around the idea that the solution should

- Give the exact perturbative result $\Rightarrow$ subtraction
  (no slicing parameter)
- Be well-defined $\Rightarrow$ completely local counterterms with all spin and color correlations
  (no integrals that are finite but undefined in $d = 4$)
- Lead to general and explicit expressions
  (automation, we use color space notation)

It is also advantageous if in addition

- The cancellation of explicit ϵ-poles in virtual contributions is analytic
  ("mathematical rigor")
- The option exists to constrain the subtractions to near the singular regions ($\alpha_{\max}$)
  (efficiency, important check)
- The construction is algorithmic
  (valid at any order in perturbation theory, in principle)

Structure

Use the same framework that was successful at NLO: local subtraction scheme

The NLO correction to some m-jet observable J

$$\sigma^{\mathrm{NLO}}[J] = \int_{m+1} \left[\mathrm{d}\sigma^{\mathrm{R}}_{m+1} J_{m+1} - \mathrm{d}\sigma^{\mathrm{R,A_1}}_{m+1} J_m \right]_{d=4} + \int_m \left[\mathrm{d}\sigma^{\mathrm{V}}_m + \int_1 \mathrm{d}\sigma^{\mathrm{R,A_1}}_{m+1} \right]_{d=4} J_m$$

The NNLO correction is the sum of three pieces

$$\sigma^{\mathrm{NNLO}}[J] = \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_m \mathrm{d}\sigma^{\mathrm{VV}}_m J_m$$

The three contributions are separately IR divergent in $d = 4$

- RR: double and single unresolved real emission
- RV: single unresolved real emission $\oplus$ ϵ-poles from $m+1$ parton one-loop
- VV: ϵ poles from m parton two-loop

For the RR contribution subtractions are needed to regularize single and double unresolved emission

$$\sigma_{m+2}^{\mathrm{NNLO}} = \int_{m+2} \left\{ \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2} J_m - \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} J_{m+1} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}} J_m \right] \right\}_{d=4}$$

- A_1 and A_2 have overlapping singularities $\Rightarrow$ A_{12} is needed to avoid double subtraction

The RV contribution only involves single unresolved emission

$$\sigma_{m+1}^{\mathrm{NNLO}} = \int_{m+1} \left\{ \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right] J_{m+1} - \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right)^{\mathrm{A}_1} \right] J_m \right\}_{d=4}$$

- Notice the integrated A_1 from RR is still singular $\Rightarrow$ subtraction is needed (last term)

The *m*-parton contribution contains the double virtual and integrated subtractions

$$\sigma_{m}^{\mathrm{NNLO}} = \int_{m} \left\{ \mathrm{d}\sigma_{m}^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right)^{\mathrm{A}_1} \right] \right\}_{d=4} J_m$$

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$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_2}_{m+2} J_m - \left[\mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_{12}}_{m+2} J_m \right] \right\}_{d=4}$$

- A_1 and A_2 have overlapping singularities $\Rightarrow$ A_{12} is needed to avoid double subtraction

The RV contribution only involves single unresolved emission

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left[\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} \right] J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV},\mathrm{A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] J_m \right\}_{d=4}$$

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The non-trivial role of $\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}}$

The sum of subtractions, symbolically (r, s can become unresolved)

$$\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2} + \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}} = \sum_{r,s} \left[\mathcal{D}_{rs} + (\mathcal{D}_r + \mathcal{D}_s) - (\mathcal{D}_{\hat{s}}\mathcal{D}_r + \mathcal{D}_{\hat{r}}\mathcal{D}_s)\right]$$

The dual role of A_{12}

- In the double unresolved limits (r, s unresolved), it cancels A_1

$$\mathrm{d}\sigma_{m+2}^{\mathrm{RR}} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2} = \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} - \mathcal{D}_{rs} = \text{“finite”}$$

$$\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}} = (\mathcal{D}_r + \mathcal{D}_s) - (\mathcal{D}_{\hat{s}}\mathcal{D}_r + \mathcal{D}_{\hat{r}}\mathcal{D}_s) = \text{“finite”}$$

- In the single unresolved limits (say, r unresolved), it cancels A_2 and part of A_1

$$\mathrm{d}\sigma_{m+2}^{\mathrm{RR}} - \left(\text{part of } \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1}\right) = \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} - \mathcal{D}_r = \text{“finite”}$$

$$\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2} - \left(\text{part of } \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}}\right) = \mathcal{D}_{rs} - \mathcal{D}_{\hat{s}}\mathcal{D}_r = \text{“finite”}$$

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Repeat what already worked at NLO!

1. Compute relevant IR factorization formulae for squared matrix elements
2. Use those to construct general, explicit, local subtractions
3. Integrate the subtractions once and for all, check cancellation of ϵ-poles
4. Apply to specific processs

Use known ingredients

Collinear and soft factorization of QCD matrix elements at NNLO known

- Tree level 3-parton splitting functions and double soft gg and $q\bar{q}$ currents

[Campbell, Glover 1997; Catani, Grazzini 1998; Del Duca, Frizzo, Maltoni 1999; Kosower 2002]

- One-loop 2-parton splitting functions and soft gluon current

[Bern, Dixon, Dunbar, Kosower 1994; Bern, Del Duca, Kilgore, Schmidt 1998-9; Kosower, Uwer 1999; Catani, Grazzini 2000; Kosower 2003]

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But note

- Unresolved regions in phase space overlap
- Quantities in factorization formulae (z, $k_\perp$) are only well-defined in the strict limit

Defining the subtraction terms – issues

1. Matching of limits to avoid multiple subtraction in overlapping singular regions of phase space. General structure dictated by "sieve principle". E.g., at NLO simply: collinear limit + soft limit − collinear–soft limit.

$$\mathbf{A}_1 = \sum \Big(\mathbf{C} + \mathbf{S} - \mathbf{C} \cap \mathbf{S} \Big)$$

   At NNLO for double radiation we have

$$\mathbf{A}_2 = \sum \Big[\mathbf{C}_3 + \mathbf{C}_{2;2} + \mathbf{CS} + \mathbf{S} - (\mathbf{C}_3 \cap \mathbf{CS} + \mathbf{C}_3 \cap \mathbf{S} + \mathbf{C}_{2;2} \cap \mathbf{CS} + \mathbf{C}_{2;2} \cap \mathbf{S} + \mathbf{CS} \cap \mathbf{S}) + (\mathbf{C}_3 \cap \mathbf{CS} \cap \mathbf{S} + \mathbf{C}_{2;2} \cap \mathbf{CS} \cap \mathbf{S}) \Big]$$

2. Extension of IR factorization formulae over full phase space: define momenta entering factorized matrix elements and momentum fractions in splitting kernels. Requires momentum mappings that respect factorization and delicate structure of cancellations in all limits.

$$\{p\}_{m+1} \xrightarrow{r} \{\tilde{p}\}_m : \quad \mathrm{d}\phi_{m+1}(\{p\}_{m+1}; Q) = \mathrm{d}\phi_m(\{\tilde{p}\}_m; Q)[\mathrm{d}p_{1,m}]$$

$$\{p\}_{m+2} \xrightarrow{r,s} \{\tilde{p}\}_m : \quad \mathrm{d}\phi_{m+2}(\{p\}_{m+2}; Q) = \mathrm{d}\phi_m(\{\tilde{p}\}_m; Q)[\mathrm{d}p_{2,m}]$$

3. Integration of the counterterms over the phase space of unresolved emission.

Defining the subtraction terms – issues

Issues specific to NNLO

1. Matching: since limits do not commute in general, care must be taken to specify the proper ordering.

2. Extension: the A_1 counterterms for single unresolved real emission (unintegrated and integrated) must have universal IR limits, so that A_{12} can be constructed in general. This is (obviously) not guaranteed by QCD factorization.

3. Choosing the counterterms such that integration over the unresolved phase space becomes more straightforward may conflict with the delicate internal cancellations between subtractions. Integrating the counterterms is tedious.

General features of CoLoRFulNNLO

CoLoRFulNNLO: Completely Local subtRactions for Fully differential NNLO

Subtractions built using universal IR limit formulae and exact PS factorization

- Altarelli-Parisi splitting functions, soft currents
- PS factorizations based on momentum mappings that can be generalized to any number of unresolved partons

Completely local in color $\otimes$ spin space, fully differential in phase space

- No need to consider the color decomposition of real emission ME's
- Azimuthal correlations correctly taken into account in gluon splitting
- Can check explicitly that the ratio of the sum of counterterms to the real emission cross section tends to unity in any IR limit

Poles of integrated subtraction terms computed analytically

- Can check pole cancellation in (double) virtual contribution explicitly

Explicit formulae for processes with colorless initial state

- Automation is possible (`MCCSM`)

Towards processes with hadronic initial states

The NNLO cross section with hadronic initial states

Overall structure unchanged, but must include (known) mass factorization counterterms

$$\sigma^{\mathrm{NNLO}}[J] = \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{VV}}_{m} J_{m} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{C}_1}_{m+1} J_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{C}_2}_{m} J_{m}$$

"No new conceptual issues, but lots of tedious details to work out."

Morally true ✔

- IR factorization formulae known from crossing and/or direct computation
- Principles of matching, extension unchanged (only more terms to catalog)

But ✘

- Need new mappings for initial-final collinear limits
- Naive crossing of momentum fractions z and transverse momenta $k_{\perp}$ will not work

Defining the subtraction terms for ISR

When defining the new subtraction terms for ISR, we must keep in mind

- In CoLoRFulNNLO, no sectoring functions are used so each subtraction term is defined and subtracted over the whole phase space.
- Because of the delicate structure of cancellations between various subtraction terms (recall non-trivial role of $\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}}$), special care must be taken to define momentum mappings and momentum fractions that respect the structure of these cancellations.
- E.g., 3-particle momentum fractions must tend to specific 2-particle momentum fractions in appropriate limits.
- Naive crossing does not work for defining 3-particle momentum fractions for initial-final-final collinear splitting.

Defining the subtraction terms for ISR

Momentum fractions from crossing?

- Single collinear ✔

$$z_{i,r} = \frac{p_i \cdot Q}{(p_i + p_r) \cdot Q} \quad \Rightarrow \quad x_{a,r} = \frac{1}{z_{i,r}}\bigg|_{p_i \to -p_a} = 1 - \frac{p_r \cdot Q}{p_a \cdot Q}$$

  It is easy to see that $x_a \in [0, 1]$.

- Triple collinear ✘

$$z_{i,rs} = \frac{p_i \cdot Q}{(p_i + p_r + p_s) \cdot Q} \quad \Rightarrow \quad x_{a,rs} \overset{?}{=} \frac{1}{z_{i,rs}}\bigg|_{p_i \to -p_a} = 1 - \frac{p_r \cdot Q}{p_a \cdot Q} - \frac{p_s \cdot Q}{p_a \cdot Q}$$

  But we find that $x_{a,rs} \notin [0, 1]$! In fact, $x_{a,rs}$ can vanish at "ordinary" points inside the double real phase space.

Momentum fractions for initial-final collinear splitting cannot be defined by naive crossing.

We have tentative definitions for momentum fractions and transverse momenta for all single and double limits, the specific formulae are somewhat elaborate.

Triple collinear subtraction terms for ISR

Consider the $p_a||p_r||p_s$ limit: the factorization formula reads

$$\mathbf{C}_{ars}|\mathcal{M}^{(0)}_{m+2}(\{p_i\}, p_r, p_s; p_a + p_b)|^2 = (8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon})^2 \frac{1}{x_a} \frac{1}{s^2_{ars}} \hat{P}_{f_a f_r f_s}(\{s_{jk}, x_j, k_{\perp,j}\}) \otimes |\mathcal{M}^{(0)}_m(\{p_i\}; x_a p_a + p_b)|^2$$

Define the subtraction term as

$$\mathcal{C}^{IFF}_{ars}(p_r, p_s, \dots; p_a + p_b) \equiv (8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon})^2 \frac{1}{\tilde{x}_a} \frac{1}{s^2_{ars}} \hat{P}_{f_a f_r f_s}(\{s_{jk}, \tilde{x}_j, \tilde{k}_{\perp,j}\}) \otimes |\mathcal{M}^{(0)}_m(\{\tilde{p}_i\}; \tilde{p}_a + \tilde{p}_b)|^2$$

Must specify explicilty

- the set momenta $\{\tilde{p}_i\} \equiv \{\tilde{p}_1, \dots, \tilde{p}_{m+2}\}$ entering the m-parton factorized ME
- the momentum fractions $\tilde{x}_j$ and transverse momenta $\tilde{k}_{\perp,j}$ $(j = a, r, s)$

Triple collinear IFF mapping

The mapping must implement momentum conservation and the mass-shell conditions

$$\tilde{p}_a^\mu = \xi_a p_a^\mu$$
$$\tilde{p}_b^\mu = p_b^\mu$$
$$\tilde{p}_i^\mu = \Lambda(K,\tilde{K})^\mu{}_\nu p_i^\nu$$

where $\Lambda(K,\tilde{K})^\mu{}_\nu$ is a proper Lorentz transformation which takes $\tilde{K}^\mu$ into K^μ, where

$$K^\mu = p_a^\mu + p_b^\mu - p_r^\mu - p_s^\mu \quad \text{and} \quad \tilde{K}^\mu = \tilde{p}_a^\mu + \tilde{p}_b^\mu$$

Requiring $K^2 = \tilde{K}^2$ fixes ξ_a (note p_a, p_b, p_r and p_s are assumed massless),

$$\xi_a = \frac{(p_a + p_b - p_r - p_s)^2}{(p_a + p_b)^2} = 1 - \frac{p_r \cdot Q}{p_a \cdot Q} - \frac{p_s \cdot Q}{p_a \cdot Q} + \frac{2 p_r \cdot p_s}{Q^2}\,,$$

with $Q = p_a + p_b$

Triple collinear IFF momentum fractions

We want to define $\tilde{x}_a\,, \tilde{x}_r\,, \tilde{x}_s$ such that

- $\tilde{x}_a\,, \tilde{x}_r\,, \tilde{x}_s \in [0,1]$ over the full phase space
- $\tilde{x}_a + \tilde{x}_r + \tilde{x}_s = 1$, i.e., they sum to one
- have the correct behavior in the single unresolved limits (e.g., $\mathbf{C}_{ar}$, $\mathbf{C}_{rs}$, etc.) to match iterated single unresolved subtractions (recall role of A_{12})

One option:

$$\tilde{x}_a = \xi_a = 1 - y_{rQ} - y_{sQ} + y_{rs}\,, \qquad \tilde{x}_r = y_{rQ} - y_{rs}\frac{y_{ar}}{y_{a(rs)}}\,, \qquad \tilde{x}_s = y_{sQ} - y_{rs}\frac{y_{as}}{y_{a(rs)}}$$

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notice terms not predicted by crossing

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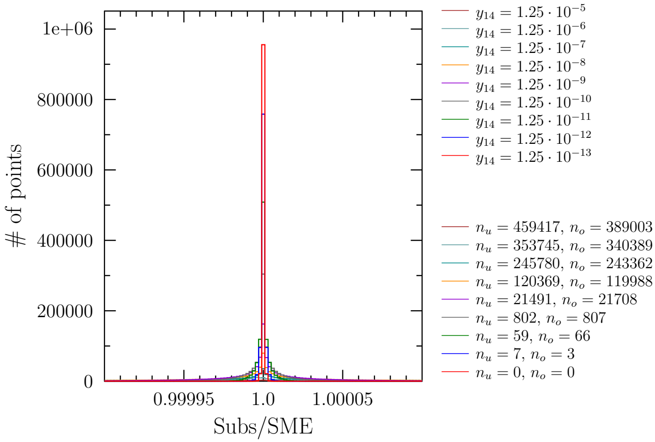
notice terms not predicted by crossing

Transverse momenta:

$$\tilde{k}_{\perp,a} = -\tilde{k}_{\perp,r} - \tilde{k}_{\perp,s}, \qquad \tilde{k}_{\perp,r} = p_{r,\perp}, \qquad \tilde{k}_{\perp,s} = p_{s,\perp}$$

also not given by crossing

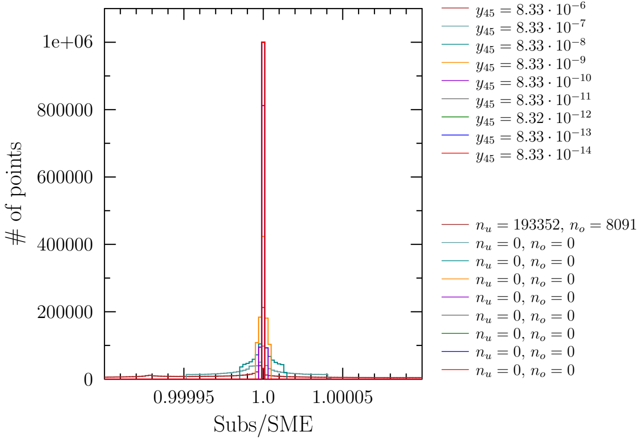
Check that the ratio of the double real emission matrix element to the the sum of all subtractions tends to one for all IR limits. E.g., $u(p_1) + \bar{d}(p_2) \rightarrow W^-(p_3) + g(p_4) + g(p_5)$



$\mathbf{C}_{14}$ limit

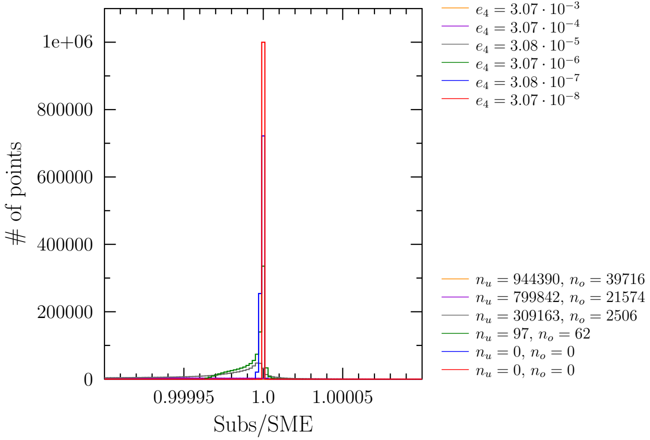
Does it work?

Check that the ratio of the double real emission matrix element to the the sum of all subtractions tends to one for all IR limits. E.g., $u(p_1) + \bar{d}(p_2) \to W^-(p_3) + g(p_4) + g(p_5)$



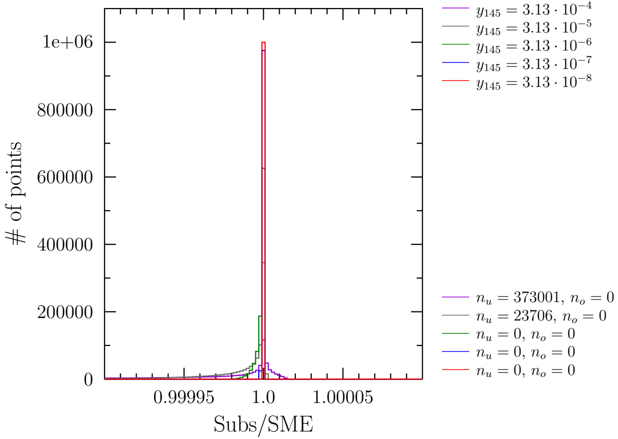
$\mathbf{C}_{45}$ limit

Check that the ratio of the double real emission matrix element to the the sum of all subtractions tends to one for all IR limits. E.g., $u(p_1) + \bar{d}(p_2) \to W^-(p_3) + g(p_4) + g(p_5)$



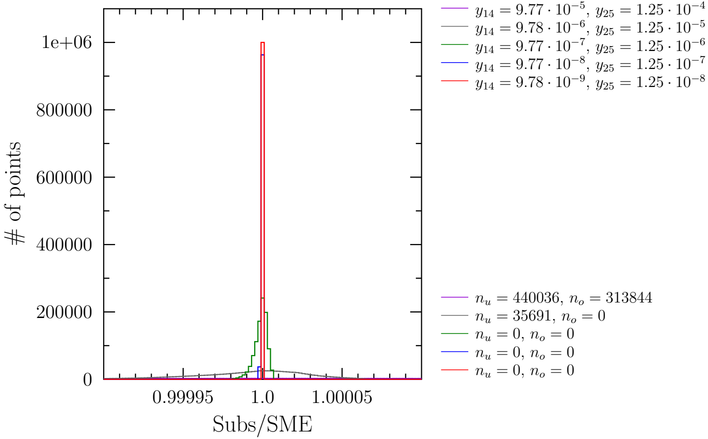
$\mathbf{S}_4$ limit

Check that the ratio of the double real emission matrix element to the the sum of all subtractions tends to one for all IR limits. E.g., $u(p_1) + \bar{d}(p_2) \to W^-(p_3) + g(p_4) + g(p_5)$



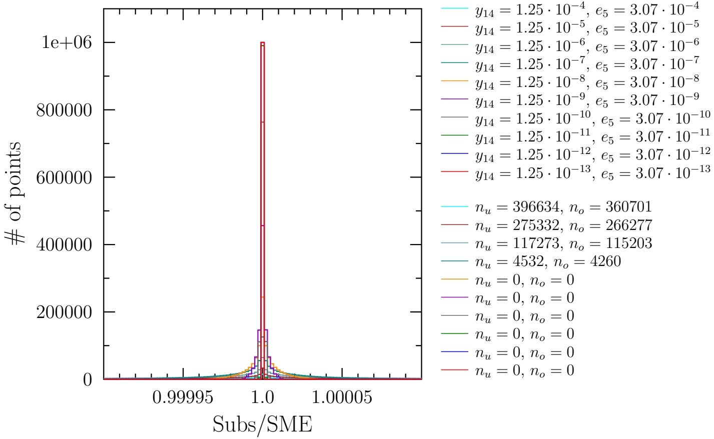
$\mathbf{C}_{145}$ limit

Check that the ratio of the double real emission matrix element to the the sum of all subtractions tends to one for all IR limits. E.g., $u(p_1) + \bar{d}(p_2) \to W^-(p_3) + g(p_4) + g(p_5)$



$\mathbf{C}_{14;25}$ limit

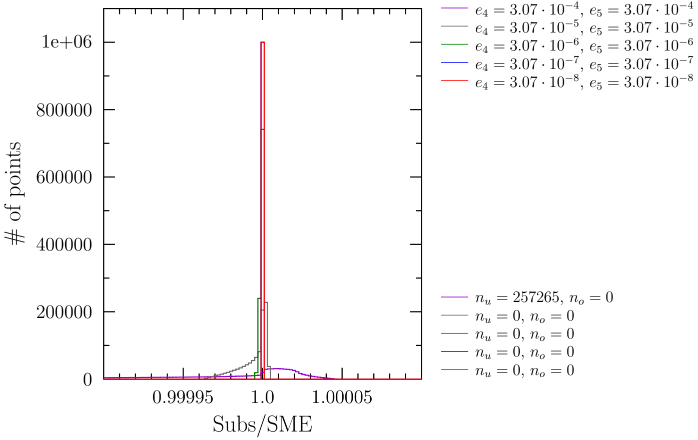
Check that the ratio of the double real emission matrix element to the the sum of all subtractions tends to one for all IR limits. E.g., $u(p_1) + \bar{d}(p_2) \to W^-(p_3) + g(p_4) + g(p_5)$



$\mathbb{C}\mathbb{S}_{14;5}$ limit

Does it work?

Check that the ratio of the double real emission matrix element to the the sum of all subtractions tends to one for all IR limits. E.g., $u(p_1) + \bar{d}(p_2) \to W^-(p_3) + g(p_4) + g(p_5)$



$\mathbf{S}_{45}$ limit

Does it work?

Subtractions work as designed in all limits, so try to integrate

- Every partonic MC calculation has a cutoff parameter: e.g., minimal two-particle invariants allowed

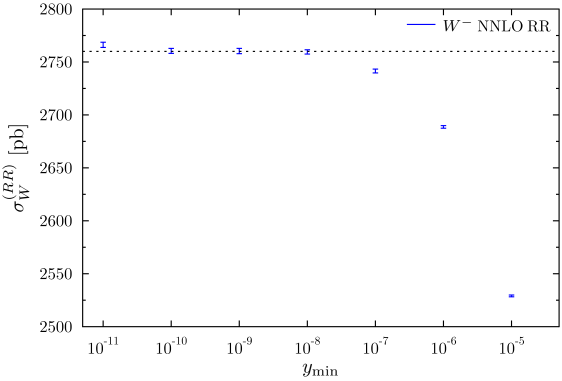
$$y_{ij} = \frac{(p_i + p_j)^2}{\hat{s}} > y_{\min}\,, \quad \forall\, i,j$$

- This is not a slicing parameter, but a technical cutoff parameter.
- It is necessitated by floating point arithmetics.
- The minimal possible choice of $y_{\min}$ depends on the floating point number representation used.
- The dependence of physical quantities on $y_{\min}$ should cancel as $y_{\min} \to 0$ if the subtraction terms are correct.

Does it work?

Subtractions work as designed in all limits, so try to integrate

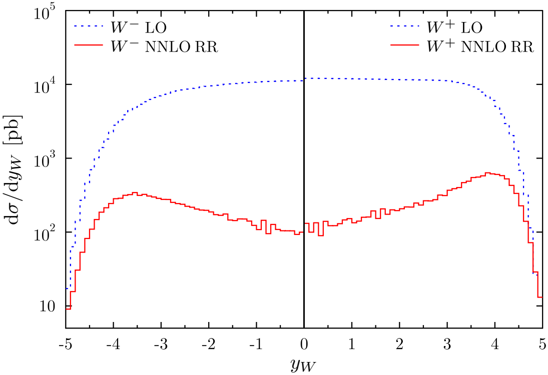
- Cutoff dependence of subtracted RR contribution to total cross section for $pp \to W^-$ (using double precision)



Does it work?

Subtractions work as designed in all limits, so try to integrate

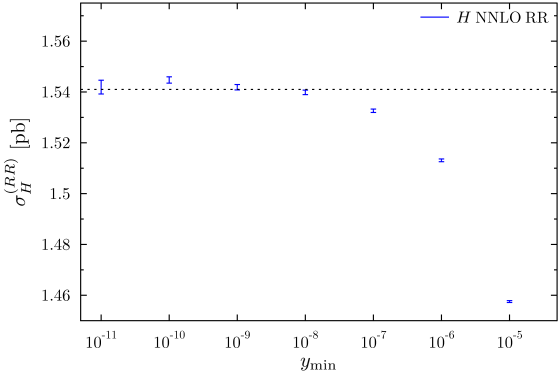
- Subtracted RR contribution to rapidity distribution of the W in $pp \to W^{\pm}$ (nonphysical, does not include RV and VV)



Does it work?

Subtractions work as designed in all limits, so try to integrate

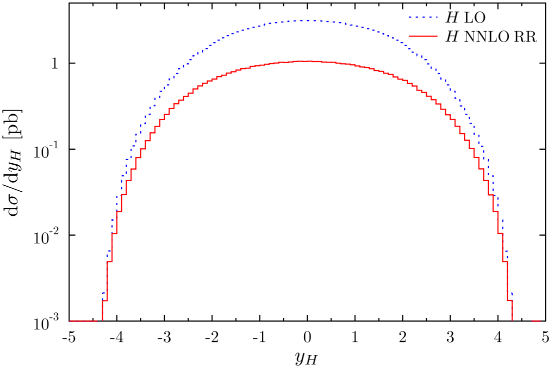
- Cutoff dependence of subtracted RR contribution to total cross section for $gg \to H$ (using double precision)



Does it work?

Subtractions work as designed in all limits, so try to integrate

- Subtracted RR contribution to rapidity distribution of the H in $gg \to H$ (nonphysical, does not include RV and VV)



Integrating the triple collinear subtraction

Momentum mapping used to define $\mathcal{C}_{ars}^{IFF}$ leads to phase space convolution of the form

$$\mathrm{d}\phi_{m+2}(\{p_i\}, p_r, p_r; p_a + p_b) = \int_{\xi_{\min}}^{\xi_{\max}} d\xi \, \mathrm{d}\phi_m(\{\tilde{p}_i\}; \xi p_a + p_b) \frac{Q^2}{2\pi} \mathrm{d}\phi_3(p_r, p_s, P; Q)$$

- momentum P is massive with $P^2 = \xi Q^2$

The subtraction term is a product (in spin space) of

- the factorized matrix element depending on $\{\tilde{p}_i\}$
- a singular factor $\frac{1}{\tilde{x}_a} \frac{1}{s_{ars}^2} \hat{P}_{f_a f_r f_s}$, to be integrated over $\mathrm{d}\phi_3(p_r, p_s, P; Q)$

Can compute once and for all the integral over unresolved partons

$$\int_2 \mathcal{C}_{ars}^{IFF} = (8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon})^2 \int_{\xi_{\min}}^{\xi_{\max}} d\xi \left[\int \mathrm{d}\phi_3(p_r, p_s, P; Q) \frac{1}{\tilde{x}_a} \frac{1}{s_{ars}^2} \hat{P}_{f_a f_r f_s} \right] \otimes |\mathcal{M}_m^{(0)}(\{\tilde{p}_i\}; \xi p_a + p_b)|^2$$

Basic integrals

We must evaluate

$$\int_{\xi_{\min}}^{\xi_{\max}} d\xi \left[\int \mathrm{d}\phi_3(p_r, p_s, P; Q) \frac{1}{\tilde{x}_a} \frac{1}{s_{ars}^2} \hat{P}_{f_a f_r f_s} \right]$$

- Use $\tilde{x}_a = \xi$, integrate over the three-parton phase space first
- $\tilde{x}_a$ dependence of $\hat{P}_{f_a f_r f_s}$ is simple: all terms contain just a power of $\tilde{x}_a$ and/or $(1 - \tilde{x}_a)$
- First step: compute (with $\tilde{x}_a$ fixed)

$$\int \mathrm{d}\phi_3(p_r, p_s, P; Q) \frac{1}{s_{ars}^2} \hat{P}_{f_a f_r f_s}$$

After decomposing and using $r \leftrightarrow s$ symmetry, we find

$$\int \mathrm{d}\phi_3(p_r, p_s, P; Q) \frac{1}{s_{ars}^2} \hat{P}_{f_a f_r f_s} = \sum_{j,k,l,p,q} c_{ars}^{(j,k,l,p,q)} \int \mathrm{d}\phi_3(p_r, p_s, P; Q) \frac{1}{s_{ars}^2} \frac{1}{s_{ar}^j s_{as}^k s_{rs}^l \tilde{x}_r^p \tilde{x}_s^q}$$

with $\{j,k,l,p,q\} = \Big\{ \underbrace{\{1,1,-2,1,0\}}_{1}, \ldots, \underbrace{\{-2,0,2,-2,0\}}_{55} \Big\}$

Basic integrals

We must evaluate

$$\int_{\xi_{\min}}^{\xi_{\max}} d\xi \left[\int \mathrm{d}\phi_3(p_r, p_s, P; Q) \frac{1}{\tilde{x}_a} \frac{1}{s_{ars}^2} \hat{P}_{f_a f_r f_s} \right]$$

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with $\{j, k, l, p, q\} = \Big\{ \underbrace{\{1, 1, -2, 1, 0\}}_{1}, \ldots, \underbrace{\{-2, 0, 2, -2, 0\}}_{55} \Big\}$

Solving the integrals by direct integration

A possible strategy: direct integration

1. choose explicit phase space parametrization of phase space
2. write the parametric integral representation in chosen variables
3. resolve ϵ poles by sector decomposition
4. pole coefficients are finite multidimensional parametric integrals
5. evaluate the parametric integrals numerically or analytically if feasible

Status:

- derived two separate explicit parametrizations of phase space based on different variables (useful check)
- in one, we can "solve" angular integrals in terms of hypergeometric (${}_2F_1$, Appell F_1) functions (reduce dimensionality of integral)
- accurate numerical integration is feasible
- analytic integration of at least some poles is feasible

Direct integration: an example

Consider the integral (of mass dimension zero)

$$I(\xi,\epsilon) = \frac{Q^2}{V_3}\int \mathrm{d}\phi_3(p_r, p_s, P; Q)\,\frac{1}{s_{ars}^2}\,\frac{s_{as}}{s_{rs}\tilde{x}_r}$$

It is easy to show that the integral can only depend on ξ and we obtain e.g.,

$$I(\xi=0.2,\epsilon) = -\frac{0.833333}{\epsilon^3} + \frac{3.67679}{\epsilon^2} - \frac{10.4127}{\epsilon} - 2.10664 + 8.35941\epsilon + \mathrm{O}(\epsilon^2)$$

$$I(\xi=0.5,\epsilon) = -\frac{1.33333}{\epsilon^3} + \frac{2.76531}{\epsilon^2} - \frac{0.375613}{\epsilon} - 5.68314 + 21.2348\epsilon + \mathrm{O}(\epsilon^2)$$

$$I(\xi=0.8,\epsilon) = -\frac{3.33333}{\epsilon^3} - \frac{6.08727}{\epsilon^2} + \frac{4.93674}{\epsilon} + 27.9761 + 102.051\epsilon + \mathrm{O}(\epsilon^2)$$

- relative accuracy is 10^{-6} on $\mathrm{O}(\epsilon)$ part and 10^{-7} or better on rest
- timing per point $\leq$ 15s on a single core (only 3d numerical integral)

Can also compute first two poles analytically from sector decomposition representation

$$I(\xi,\epsilon) = -\frac{2}{3(1-\xi)\epsilon^3} + \frac{9 + 8\ln(1-\xi) - \ln\xi}{3(1-\xi)\epsilon^2} + \mathrm{O}(\epsilon^{-1})$$

$$\int \mathrm{d}\phi_3(p_r, p_s, P; Q) \frac{1}{s_{ars}^2} \frac{1}{s_{ar}^j s_{as}^k s_{rs}^l \tilde{x}_r^p \tilde{x}_s^q}$$

A possible strategy: reverse unitarity

1. rewrite δ-functions in the phase space measure as (differences of) propagators, i.e., phase space integrals $\Rightarrow$ loop integrals
2. perform IBP reduction to identify a set of master integrals
3. evaluate the master integrals e.g., by the method of differential equations

Status:

- when no $\tilde{x}_r$ or $\tilde{x}_s$ is involved (i.e., $p = q = 0$) we find only two MIs which can be evaluated in terms of ${}_2F_1$ functions ✔
- the appearance of $\tilde{x}_r$ or $\tilde{x}_s$ in the numerator (i.e., $p < 0$ or $q < 0$) causes no issues: all denominators are still of the standard $1/(p^2 \pm m^2)$ or $1/(p \cdot q \pm m^2)$ type ✔
- when we have $1/\tilde{x}_r$ or $1/\tilde{x}_s$, denominators quadratic in scalar products involving loop momenta appear, e.g., $1/[(p_r \cdot Q)(p_a \cdot p_r + p_a \cdot p_s) - (p_r \cdot p_s)(p_a \cdot p_r)]$ ✘

From basic integrals to integrated counterterms

Recall that we must finally evaluate (note $\xi_{\min} = \left(\sum_i m_i\right)^2 / Q^2$ and $\xi_{\max} = 1$)

$$\int_{\xi_{\min}}^{\xi_{\max}} d\xi \left[\int \mathrm{d}\phi_3(p_r, p_s, P; Q) \frac{1}{\tilde{x}_a} \frac{1}{s_{ars}^2} \hat{P}_{f_a f_r f_s}\right]$$

So far, discussed only computing

$$I(\xi, \epsilon) = \int \mathrm{d}\phi_3(p_r, p_s, P; Q) \frac{1}{\tilde{x}_a} \frac{1}{s_{ars}^2} \hat{P}_{f_a f_r f_s}$$

The ξ dependence of $I(\xi, \epsilon)$ must be interpreted with care: the $\epsilon \to 0$ limit must be taken uniformly in ξ. Hence, $I(\xi, \epsilon)$ must be interpreted as a ξ-distribution whose coefficients contain poles in ϵ.

$$I(\xi, \epsilon) = [I(\xi, \epsilon)]_+ + \delta(1 - \xi) \int_{\xi_{\min}}^{\xi_{\max}} d\xi' I(\xi', \epsilon)$$

- need to know the all-order (in ϵ) behavior of $I(\xi, \epsilon)$ around $\xi = 1$
- in particular the fixed-order ϵ-expansion of $I(\xi, \epsilon)$ is not quite enough

Status

Extension to hadronic initial states on the way

- Subtraction terms for double real radiation defined for generic processes
- Tested convergence of regularized double real part in simplest processes
- Subtraction terms for real-virtual radiation tentatively defined for generic processes

TODO:

- More testing of double real and real-virtual subtractions
- Subtraction terms for mass factorization counterterms (NLO complexity)
- Some integrals done, but many more to do
- Can we use reverse unitarity with non-standard propagators? Note similarity to the analytic computation of energy-energy correlation at NLO by Dixon et al.

[Dixon, Luo, Shtabovenko, Yang, Zhu 2018]

Conclusions

Conclusions

Amazing progress in fixed order calculations in the past decade

- Automation of NLO
- Mass production of two-loop amplitudes is becoming a reality
- Approaches to NNLO are maturing into general frameworks

CoLoRFulNNLO method: Completely Local subtRactions for Fully differential NNLO

- Construction of subtraction terms based on IR limit formulae
- Analytic integration of subtraction terms feasible with modern techniques
- Good numerical convergence and stability for $e^+e^- \to$ hadrons

Extension to hadronic initial states on the way

- Defined subtraction terms for regularizing infrared singularities in double real radiation for generic processes
- Cancellation of kinematic singularities and stability in double real radiation demonstrated for W and Higgs production
- Main remaining challenge: integration of subtraction terms

Thank you for your attention!

Extra material

Non-commuting IR limits

The symbolic operators $\mathbf{C}_{ir}$ and $\mathbf{S}_r$ denote taking the single collinear and single soft limits

- Collinear: $p_i || p_r$ $\left(p_i \to z_i p_{ir} + k_\perp + \mathcal{O}(k_\perp^2),\ p_r \to z_r p_{ir} - k_\perp + \mathcal{O}(k_\perp^2)\right)$

$$\mathbf{C}_{ir} |\mathcal{M}^{(0)}_{m+2}(p_i, p_r, \ldots)|^2 = 8\pi\alpha_\mathrm{s}\mu^{2\epsilon} \frac{1}{s_{ir}} \hat{P}_{f_i f_r}(z_i, z_r, k_\perp; \epsilon) \otimes |\mathcal{M}^{(0)}_{m+1}(p_{ir}, \ldots)|^2$$

- Soft: $p_r \to 0$

$$\mathbf{S}_r |\mathcal{M}^{(0)}_{m+2}(p_r, \ldots)|^2 = -8\pi\alpha_\mathrm{s}\mu^{2\epsilon} \sum_{j,k} \frac{s_{jk}}{s_{jr} s_{kr}} |\mathcal{M}^{(0)}_{m+1,(i,k)}(\cancel{p_r}, \ldots)|^2$$

In order to avoid double subtraction when p_r is both soft and collinear to another momentum p_i, we need to remove the "collinear-soft" contribution.

However, the soft and collinear limits do not commute at the level of factorization formulae.

Non-commuting IR limits

Consider the soft limit of the collinear formula: $\mathbf{S}_r\mathbf{C}_{ir}$

- Momentum fractions:

$$\mathbf{S}_r z_i \to 1, \qquad \mathbf{S}_r z_r \to 0$$

- Altarelli-Parisi splitting kernels: e.g., for $q \to qg$ splitting $(z_i + z_r = 1)$

$$P_{qg}(z_i, z_r; \epsilon) = C_{\mathrm{F}}\left[\frac{1+z_i^2}{1-z_i} - \epsilon(1-z_i)\right] \quad \Rightarrow \quad \mathbf{S}_r P_{qg}(z_i, z_r; \epsilon) \to \frac{2}{z_r} C_{\mathrm{F}}$$

and in general

$$\mathbf{S}_r P_{f_i f_r}(z_i, z_r, k_\perp; \epsilon) \to \frac{2}{z_r} T_{ir}^2$$

- Soft-collinear limit

$$\mathbf{S}_r\mathbf{C}_{ir}|\mathcal{M}_{m+2}^{(0)}(p_i, p_r, \ldots)|^2 = 8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon}\frac{1}{s_{ir}}\frac{2}{z_r}\boldsymbol{T}_{ir}^2|\mathcal{M}_{m+1}^{(0)}(p_i, \ldots)|^2$$

Non-commuting IR limits

Consider the collinear limit of the soft formula: $\mathbf{C}_{ir}\mathbf{S}_r$

- Two-particle invariants

$$\mathbf{C}_{ir} s_{il} \to z_i s_{(ir)l}, \qquad \mathbf{C}_{ir} s_{lr} \to z_r s_{(ir)l}, \qquad l = j, k$$

- Eikonal factor

$$\mathbf{C}_{ir} \sum_{j,k} \frac{s_{jk}}{s_{jr} s_{kr}} \boldsymbol{T}_j \boldsymbol{T}_k = \mathbf{C}_{ir} \sum_{k} \frac{2 s_{ik}}{s_{ir} s_{kr}} \boldsymbol{T}_i \boldsymbol{T}_k \to \sum_{k} \frac{2}{s_{ir}} \frac{z_i}{z_r} \boldsymbol{T}_i \boldsymbol{T}_k = -\frac{2}{s_{ir}} \frac{z_i}{z_r} \boldsymbol{T}_i^2$$

- Collinear-soft limit

$$\mathbf{C}_{ir}\mathbf{S}_r |\mathcal{M}_{m+2}^{(0)}(p_i, p_r, \ldots)|^2 = 8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon} \frac{1}{s_{ir}} \frac{2z_i}{z_r} \boldsymbol{T}_i^2 |\mathcal{M}_{m+1}^{(0)}(p_i, \ldots)|^2$$

Non-commuting IR limits

Hence limits do not commute: $\mathbf{S}_r\mathbf{C}_{ir} \neq \mathbf{C}_{ir}\mathbf{S}_r$

$$\mathbf{S}_r\mathbf{C}_{ir}|\mathcal{M}^{(0)}_{m+2}|^2 \propto \frac{1}{s_{ir}}\frac{2}{z_r}\boldsymbol{T}^2_{ir}|\mathcal{M}^{(0)}_{m+1}|^2 \qquad \text{but} \qquad \mathbf{C}_{ir}\mathbf{S}_r|\mathcal{M}^{(0)}_{m+2}|^2 \propto \frac{1}{s_{ir}}\frac{2z_i}{z_r}\boldsymbol{T}^2_{i}|\mathcal{M}^{(0)}_{m+1}|^2$$

- Reason: soft operators send some momentum fractions to one: $\mathbf{S}_r z_i \to 1$
- Note: no explicit phasespace parametrization, so no specific parameter controls the approach to limits

Which ordering to use?

- $\mathbf{S}_r\mathbf{C}_{ir}$ will not work in the collinear limit

$$\mathbf{S}_r\left(\mathbf{C}_{ir} - \mathbf{S}_r\mathbf{C}_{ir}\right)|\mathcal{M}^{(0)}_{m+2}|^2 = 0 \qquad \text{but} \qquad \mathbf{C}_{ir}\left(\mathbf{S}_r - \mathbf{S}_r\mathbf{C}_{ir}\right)|\mathcal{M}^{(0)}_{m+2}|^2 \neq 0$$

- $\mathbf{C}_{ir}\mathbf{S}_r$ will work in both limits

$$\mathbf{S}_r\left(\mathbf{C}_{ir} - \mathbf{C}_{ir}\mathbf{S}_r\right)|\mathcal{M}^{(0)}_{m+2}|^2 = 0 \qquad \text{but} \qquad \mathbf{C}_{ir}\left(\mathbf{S}_r - \mathbf{C}_{ir}\mathbf{S}_r\right)|\mathcal{M}^{(0)}_{m+2}|^2 = 0$$

This phenomenon arises also in double unresolved limits. In general, limits must be ordered form “more soft” to “less soft”.

Universal limits for subtraction terms

The existence of universal IR limits of approximate cross sections is (clearly) not guaranteed by QCD factorizataion.

- We do not specify which momenta can become unresolved, hence the single unresolved subtraction terms must themselves have universal IR limits
- In the real-virtual contribution, these terms appear in integrated form, and these forms again must have universal IR limits
- These are non-trivial constraints, since the (unintegrated and integrated) single soft factorization formula involves color-correlated matrix elements

$$\mathcal{S}_r^{(0,0)} \propto \sum_{i,k} \frac{s_{ik}}{s_{ir} s_{kr}} \langle \mathcal{M}_{m+1}^{(0)} | \boldsymbol{T}_i \boldsymbol{T}_k | \mathcal{M}_{m+1}^{(0)} \rangle$$

- In, say, the $p_j || p_s$ limit only the sum

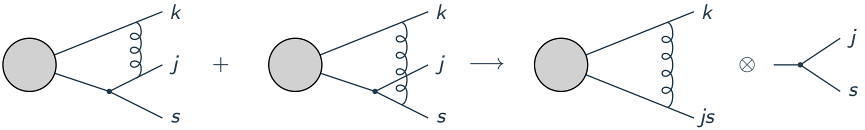
$$\langle \mathcal{M}_{m+1}^{(0)} | \boldsymbol{T}_j \boldsymbol{T}_k | \mathcal{M}_{m+1}^{(0)} \rangle + \langle \mathcal{M}_{m+1}^{(0)} | \boldsymbol{T}_s \boldsymbol{T}_k | \mathcal{M}_{m+1}^{(0)} \rangle$$

factorizes, due to soft gluon coherence, but not the two pieces separately

In, say, the $p_j||p_s$ limit only the sum

$$\langle \mathcal{M}^{(0)}_{m+1} | \boldsymbol{T}_j \boldsymbol{T}_k | \mathcal{M}^{(0)}_{m+1} \rangle + \langle \mathcal{M}^{(0)}_{m+1} | \boldsymbol{T}_s \boldsymbol{T}_k | \mathcal{M}^{(0)}_{m+1} \rangle$$

factorizes, due to soft gluon coherence, but not the two pieces separately



Universal limits for subtraction terms

Then we must make sure that in any collinear limit (for any i and r), the two appropriate terms from the soft formula actually go to the same limit

- The eikonal factors are homogeneous in p_j and p_s, so they go to the same limit (note no partial fraction decomposition)

$$\mathbf{C}_{js} \frac{s_{jk}}{s_{jr} s_{kr}} = \frac{z_j s_{(js)k}}{z_j s_{(js)r} s_{kr}} = \frac{s_{(js)k}}{s_{(js)r} s_{kr}} \qquad \text{and} \qquad \mathbf{C}_{js} \frac{s_{sk}}{s_{rs} s_{kr}} = \frac{z_s s_{(js)k}}{z_s s_{(js)r} s_{kr}} = \frac{s_{(js)k}}{s_{(js)r} s_{kr}}$$

- But we must also have that the mapped momenta that appear in the factorized matrix elements in

$$\langle \mathcal{M}^{(0)}_{m+1} | \boldsymbol{T}_j \boldsymbol{T}_k | \mathcal{M}^{(0)}_{m+1} \rangle \qquad \text{and} \qquad \langle \mathcal{M}^{(0)}_{m+1} | \boldsymbol{T}_s \boldsymbol{T}_k | \mathcal{M}^{(0)}_{m+1} \rangle$$

  also go to the same limit.

- Constrains the soft momentum mapping. A trivial way of satisfying this constraint is to use the same mapped momenta in all terms in the soft formula $\Leftrightarrow$ dipole picture.

Energy-energy correlation

Energy weighted distribution of angles χ between particles

$$\frac{1}{\sigma_{\rm tot}}\frac{{\rm d}\Sigma(\chi)}{{\rm d}\cos\chi} \equiv \frac{1}{\sigma_{\rm tot}}\int\sum_{i,j}\frac{E_i E_j}{Q^2}{\rm d}\sigma_{e^+e^-\to\, ij+X}\,\delta(\cos\chi-\cos\theta_{ij})$$

Was measured extensively at LEP and predecessors

Accurate theory predictions available

- NNLO fixed order from CoLoRFulNNLO
- NNLL resummation in back-to-back region [de Florian, Grazzini 2005]

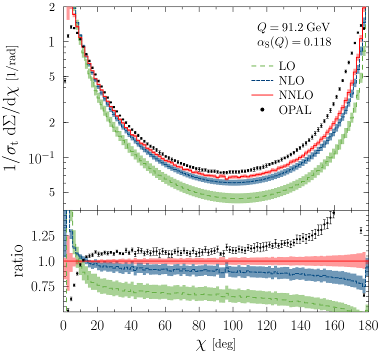
Potential for yapa (yet another precision $\alpha_{\rm s}(M_Z)$)

EEC data

- EEC one of the oldest event shapes [Basham, Brown, Ellis, Love 1978]
- However, no measurements after LEP1...
- Transverse EEC in multijet events used successfully at LHC to determine α_s at NLO [ATLAS coll., Eur. Phys. J. C77 (2017) 872, Phys. Lett. B750 (2015) 427-447]

Experiment	$\sqrt{s}$, GeV, data	$\sqrt{s}$, GeV, MC	Events
SLD	91.2(91.2)	91.2	60000
OPAL	91.2(91.2)	91.2	336247
OPAL	91.2(91.2)	91.2	128032
L3	91.2(91.2)	91.2	169700
DELPHI	91.2(91.2)	91.2	120600
TOPAZ	59.0 – 60.0(59.5)	59.5	540
TOPAZ	52.0 – 55.0(53.3)	53.3	745
TASSO	38.4 – 46.8(43.5)	43.5	6434
TASSO	32.0 – 35.2(34.0)	34.0	52118
PLUTO	34.6(34.6)	34.0	6964
JADE	29.0 – 36.0(34.0)	34.0	12719
CELLO	34.0(34.0)	34.0	2600
MARKII	29.0(29.0)	29.0	5024
MARKII	29.0(29.0)	29.0	13829
MAC	29.0(29.0)	29.0	65000
TASSO	21.0 – 23.0(22.0)	22.0	1913
JADE	22.0(22.0)	22.0	1399
CELLO	22.0(22.0)	22.0	2000
TASSO	12.4 – 14.4(14.0)	14.0	2704
JADE	14.0(14.0)	14.0	2112

- NLO correction is large as judged by scale variation ⇒ must go to NNLO
- Higher order predictions improve agreement with data
- Fixed order prediction diverges in the forward and back-to-back regions ⇒ resummation is required
- Sizeable deviations from data even at NNLO ⇒ must take into account hadronization corrections



[Tulipánt, Kardos, GS, Eur. Phys. J. C 77 (2017) no.11, 749]

Fixed order diverges in the back-to-back limit as $\sim \alpha_s^n \ln^{2n-1} y$ where $y = \cos^2(\chi/2)$

Resummation known up to NNLL accuracy (and N^{3}LL is on the way using SCET)

[de Florian, Grazzini 2005; Moult, Zhu 2018]

$$\left[\frac{1}{\sigma_{\rm tot}}\frac{d\Sigma(\chi)}{d\cos\chi}\right]_{\rm (res.)} = \frac{Q^2}{8} H(\alpha_s) \int_0^\infty db\, J_0(b\, Q\sqrt{y}) S(Q,b)$$

The log-enhanced terms are collected in the Sudakov form factor

$$S(Q,b) = \exp\left\{ - \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left[A(\alpha_s(q^2)) \ln\frac{Q^2}{q^2} + B(\alpha_s(q^2)) \right] \right\}$$

The $A(\alpha_s)$, $B(\alpha_s)$ and $H(\alpha_s)$ functions can be computed pertrubatively

$$A(\alpha_s) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^n A^{(n)}, \quad B(\alpha_s) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^n B^{(n)}, \quad H(\alpha_s) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^n H^{(n)}$$

Hadronization corrections

Point-by-point multiplicative correction factors were derived using modern MC tools

- `Sherpa2.2.4` for $e^+e^- \to 2, 3, 4, 5$ jets, 2 jets at NLO using `AMEGIC`, `COMIX` and `GoSam`, Lund (S^L) or cluster (S^C) hadronization
- `Herwig7.1.1` for $e^+e^- \to 2, 3, 4, 5$ jets, 2 jets at NLO using `MadGraph5` and `GoSam`, cluster (H^M) hadronization only

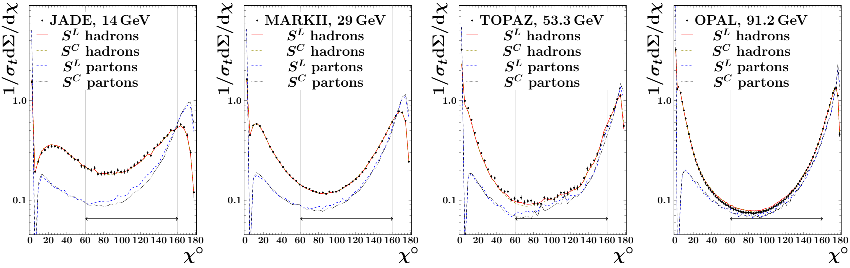
Hadronization corrections are ratios of hadron to parton level distributions in the MCs

Simulated samples were reweighted to data at hadron level on an event-by-event basis to assure a better description of data ("poor man's tuning")

Simultaneously allows for the estimation of the missing statistical correlations of data points

Hadronization corrections

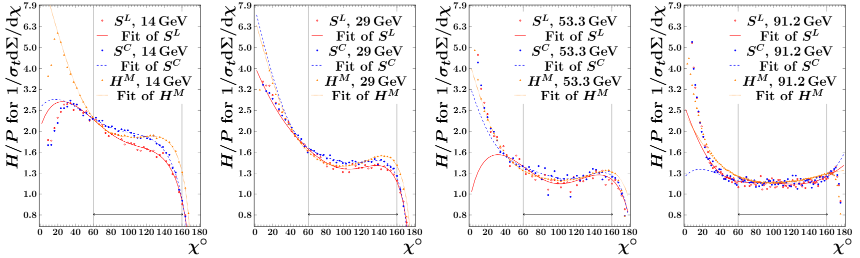
MC predictions at parton and hadron level after reweighting



[Kardos, Kluth, GS, Tulipánt, Verbytskyi
Eur. Phys. J. C 78 (2018) no.6, 498]

- Hadronization corrections decrease as $\sim 1/Q$, O(10)% at 91.2 GeV

Hadron/parton ratios after reweighting at hadron level

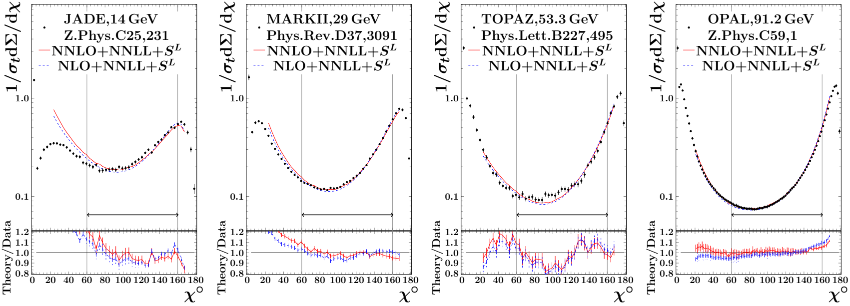


[Kardos, Kluth, GS, Tulipánt, Verbytskyi
Eur. Phys. J. C 78 (2018) no.6, 498]

- Hadronization corrections are parametrized using smooth functions to tame statistical fluctuations (the parametrization is valid only in the fit range)

Fits to data

Fits to data of NNLO+NNLL and NLO+NNLL predictions in the S^L setup



[Kardos, Kluth, GS, Tulipánt, Verbytskyi
Eur. Phys. J. C 78 (2018) no.6, 498]

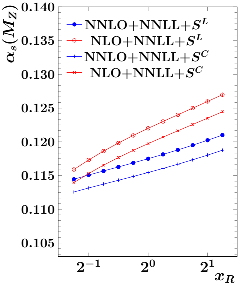
- Fit range $[60^\circ, 160^\circ]$, chosen to avoid regions where the theoretical prediction or hadronization corrections become unreliable
- The result is insensitive to a $\pm 5^\circ$ change in fit range

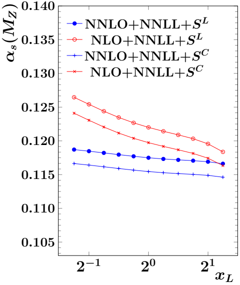
Fit uncertainties

Estimated the uncertainty by

- Varying the renormalization scale $x_R = \mu_R/Q \in [1/2, 2]$: (*ren.*)
- Varying the resummation scale $x_L \in [1/2, 2]$: (*res.*)
- Varying the hadronization model S^L vs. S^C: (*hadr.*)
- Considering the fit uncertainty from the $\chi^2 + 1$ criterion as implemented in `MINUIT2`: (*exp.*)

Notice reduced slope at NNLO+NNLL





Result

Main result from global fit at NNLO+NNLL with S^L setup

$$\alpha_s(M_Z) = 0.11750 \pm 0.00018(\mathit{exp.}) \pm 0.00102(\mathit{hadr.}) \pm 0.00257(\mathit{ren.}) \pm 0.00078(\mathit{res.})$$
$$\alpha_s(M_Z) = 0.11750 \pm 0.00287(\mathit{comb.})$$

Note using NLO+NNLL only (i.e., no NNLO), we find

$$\alpha_s(M_Z) = 0.12200 \pm 0.00023(\mathit{exp.}) \pm 0.00113(\mathit{hadr.}) \pm 0.00433(\mathit{ren.}) \pm 0.00293(\mathit{res.})$$
$$\alpha_s(M_Z) = 0.12200 \pm 0.00535(\mathit{comb.})$$

Inclusion of NNLO corrections crucial in reducing uncertainty: factor of 1/2!

The result is consistent with the world average ($\alpha_s(M_Z) = 0.1175 \pm 0.0029$ vs. 0.1181 ± 0.0011) and competitive with other precision event shapes ($1 - T$, C, etc.)

VH production with $H \to b\bar{b}$ decay at the LHC

Motivations

- Associated VH production is most sensitive production mode to search for $H \to b\bar{b}$
  - leptons, missing E_{T} to trigger
  - high p_{T} V to suppress backgrounds
- Unique opportunity to study both the Higgs boson coupling to vector bosons and down-type quarks
- $H \to b\bar{b}$ has the largest branching ratio (58%) for $m_H = 125$ GeV
- Drives the uncertainty of the total Higgs boson width

Theory: narrow width approximation very accurate ($\Gamma_H \ll m_H$), so need fully differential calculations for production and decay

- VH production with leptonic V decays known in NNLO QCD (using q_{T} subtraction)

[Ferrera, Grazzini, Tramontano 2011]

- $H \to b\bar{b}$ known in NNLO QCD (using sector decomposition and CoLoRFulNNLO)

[Anastasiou, Herzog, Lazopoulos 2012;
Del Duca, Duhr, GS, Tramontano Z. Trócsányi 2015]

$VH(b\bar{b})$ in full NNLO QCD

Consider $pp \to VH + X \to l_1 l_2 b\bar{b} + X$ in the narrow width approximation

$$d\sigma_{pp\to VH\to Vb\bar{b}} = d\sigma_{pp\to VH} \times \frac{d\Gamma_{H\to b\bar{b}}}{\Gamma_H} = \left[\sum_{k=0}^{\infty} d\sigma^{(k)}_{pp\to VH}\right] \times \left[\frac{\sum_{k=0}^{\infty} d\Gamma^{(k)}_{H\to b\bar{b}}}{\sum_{k=0}^{\infty} \Gamma^{(k)}_{H\to b\bar{b}}}\right] \times \mathrm{Br}(H\to b\bar{b})$$

For full NNLO, expand up to second order

$$\begin{aligned} d\sigma^{\mathrm{NNLO}}_{pp\to VH\to Vb\bar{b}} = \Bigg[& d\sigma^{(0)}_{pp\to VH} \times \frac{d\Gamma^{(0)}_{H\to b\bar{b}} + d\Gamma^{(1)}_{H\to b\bar{b}} + d\Gamma^{(2)}_{H\to b\bar{b}}}{\Gamma^{(0)}_{H\to b\bar{b}} + \Gamma^{(1)}_{H\to b\bar{b}} + \Gamma^{(2)}_{H\to b\bar{b}}} \\ & + d\sigma^{(1)}_{pp\to VH} \times \frac{d\Gamma^{(0)}_{H\to b\bar{b}} + d\Gamma^{(1)}_{H\to b\bar{b}}}{\Gamma^{(0)}_{H\to b\bar{b}} + \Gamma^{(1)}_{H\to b\bar{b}}} \\ & + d\sigma^{(2)}_{pp\to VH} \times \frac{d\Gamma^{(0)}_{H\to b\bar{b}}}{\Gamma^{(0)}_{H\to b\bar{b}}} \Bigg] \times \mathrm{Br}(H\to b\bar{b}) \end{aligned}$$

$VH(b\bar{b})$ in full NNLO QCD

Previous partial NNLO calculations did not consider NNLO corrections in decay

[Ferrera, Grazzini, Tramontano 2014-5
Campbell, Ellis, Williams 2016]

$$d\sigma^{\mathrm{NNLO(prod)+NLO(dec)}}_{pp\to VH\to Vb\bar{b}} = \left[d\sigma^{(0)}_{pp\to VH} \times \frac{d\Gamma^{(0)}_{H\to b\bar{b}} + d\Gamma^{(1)}_{H\to b\bar{b}}}{\Gamma^{(0)}_{H\to b\bar{b}} + \Gamma^{(1)}_{H\to b\bar{b}}} + \left(d\sigma^{(1)}_{pp\to VH} + d\sigma^{(2)}_{pp\to VH} \right) \times \frac{d\Gamma^{(0)}_{H\to b\bar{b}}}{\Gamma^{(0)}_{H\to b\bar{b}}} \right] \times \mathrm{Br}(H\to b\bar{b})$$

New: include NNLO contributions in decay and the combination of NLO contributions for production and decay

Results: cross sections

Kinematical selection cuts

$pp \to W^+H + X \to l\nu_l b\bar{b} + X$

- $p_T^l > 15$ GeV, $|\eta_l| < 2.5$
- $E_T^{miss} > 30$ GeV
- $p_T^W > 150$ GeV
- at least two b-jets with $p_T^b > 25$ GeV and $|\eta_b| < 2.5$

$pp \to ZH + X \to \nu\nu b\bar{b} + X$

- $E_T^{miss} > 150$ GeV
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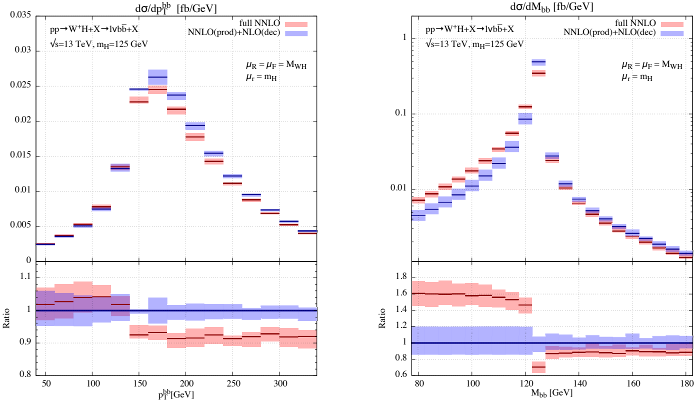
- at least two b-jets with $p_T^b > 25$ GeV and $|\eta_b| < 2.5$

Cross section predictions at the LHC with $\sqrt{s} = 13$ TeV

σ (fb)	NNLO(prod)+NLO(dec)	full NNLO
$pp \to W^+H + X \to l\nu_l b\bar{b} + X$	$3.94^{+1\%}_{-1.5\%}$	$3.70^{+1.5\%}_{-1.5\%}$
$pp \to ZH + X \to \nu\nu b\bar{b} + X$	$8.65^{+4.5\%}_{-3.5\%}$	$8.24^{+4.5\%}_{-3.5\%}$

- Cross sections reduced by $\sim$ 5–6% at full NNLO wrt. NNLO(prod)+NLO(dec)
- Uncertainties correspond to scale variation

Transverse momentum and invariant mass of leading b-jet pair: $W^+H(b\bar{b})$

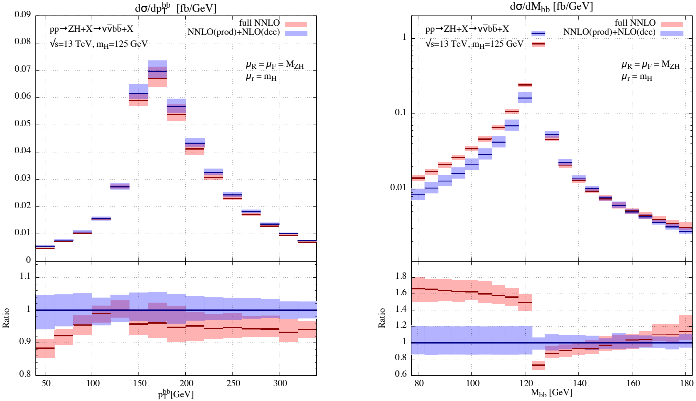


[Ferrera, GS, Tramontano Phys. Lett. B 780 (2018) 346-351]

- Contributions included in full NNLO produce important effects on the shapes: -8% – $+5\%$ corrections in $p_T^{b\bar{b}}$, -30% – $+60\%$ corrections in $M_{b\bar{b}}$!

Results: distributions

Transverse momentum and invariant mass of leading b-jet pair: $ZH(b\bar{b})$



[Ferrera, GS, Tramontano Phys. Lett. B 780 (2018) 346-351]

- Contributions included in full NNLO produce important effects on the shapes: -10% – -5% corrections in $p_T^{b\bar{b}}$, -30% – $+70\%$ corrections in $M_{b\bar{b}}$!