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Transition from straight to fractal cracks due to projectile penetration

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Abstract. We present a theoretical study of the fracture of two-dimensional disc-shaped samples due to projectile penetration focusing on the geometrical structure of the crack pattern. The penetration of a cone is simulated into a plate of circular shape using a discrete element model of heterogeneous brittle materials varying the speed of penetration in a broad range. As the cone penetrates a destroyed zone is created from which cracks run to the external boundary of the plate. Computer simulations revealed that in the low speed limit of loading two cracks are generated with nearly straight shape. Increasing the penetration speed the crack pattern remains regular, however, both the number of cracks and their fractal dimension increases. High speed penetration gives rise to a crack network such that the sample gets fragmented into a large number of pieces. We give a quantitative analysis of the evolution of the system from simple cracking through fractal cracks to fragmentation with a connected crack network.

Introduction

Fracture processes of heterogeneous brittle materials have a high technological relevance and still present interesting scientific challenges. Besides materials' features the evolution and final scenario of fracture processes strongly depend on the speed of loading, as well. Investigation mainly focus on the limiting cases of very slow and very fast external driving, however, the transition in between is poorly understood. During the last 10 years a large amount of experimental investigation has been carried out to understand the breakup process of brittle materials induced by projectile penetration varying the speed of loading in a broad range from nearly quasi-static to highly dynamic conditions. Recently, Kadono *et al.* carried out experiments by shooting a projectile on a glass plate analyzing both the geometrical structure of the crack pattern and the probability distribution of the mass of fragments. He pointed out that the crack pattern has fractal character with fractal a dimension D_f = 1.1-1.6 increasing with the loading velocity [1]. Under different boundary conditions Davydova *et al.* obtained a self-similar crack ensemble where the fractal dimension proved to be higher D_f = 1.59-1.83 [2]. These experiments also indicated that the formation of cracks can be described by the balance of elastic and fracture energy [3]. They also suggested that the random patterns spontaneously evolve toward symmetry. We consider this problem in the framework of a discrete element model of heterogeneous brittle materials. Simulating the penetration of a cone in a disc-shaped sample we present a quantitative analysis of the final breaking scenarios emerging at different loading speeds and of the pattern of cracks along which the specimen falls apart. Our simulations results are in a good agreement with recent experimental findings.

Two dimensional discrete element model of projectile penetration

For the numerical investigation of the problem we use a discrete element model of brittle materials with a heterogeneous microstructure in two dimensions. In the model the material is discretized in terms of convex polygons which represent grains having three continuous degrees of freedom, i.e. the center of mass coordinates and a rotation angle. The initial configuration of polygons is obtained the Voronoi tessellation of a rectangular region. In order to represent the mechanical features of the model material a repulsive contact is introduced between polygons: polygons can overlap each other giving rise to a repulsive force proportional to the overlap area [4-6]. Cohesion is captured such that neighboring polygons are connected by elastic beams which can exert forces and torques on the particles. Beams can be stretched, compressed, bent and sheared as the particles move in the two dimensional plane. Crack formation is represented in such a way that beams break when they get overstressed. Breaking of a beam can be caused by stretching and bending which are coupled in the breaking criterion

$$\left(\frac{\varepsilon_{ij}}{\varepsilon_{th}}\right)^2 + \frac{\max(|\Theta_1|,|\Theta_2|)}{\Theta_{th}} > 1, \tag{1}$$

In the above equation, ε denotes the axial strain of the beam and θ_1 and θ_2 denote the bending angles at the two beam ends [4-6]. The two breaking threshold ε_{th} and θ_{th} control the relative importance of the two breaking modes, *i.e.* increasing a threshold reduces the effect of the corresponding failure mode. In the model there is only structural disorder present, which means that the failure thresholds have the same value for all the beams, however, the physical properties of beams are determined by the random configuration of polygons. The time evolution of the system is generated by molecular dynamics simulations solving the equation of motion of all particles for the translational rotational degrees of freedom. For more details of the model construction see [4-6].

Penetration of a projectile – crack patterns

In the simulation the sample construction starts with the Voronoi tessellation of a square-shape area from which we cut out a disc-shaped specimen with the radius of 100 polygons. To simulate the penetration of a cone the central element of the disk was removed and it was replaced by a regular polygon with 20 corners. The regular polygon represents the intersection of the penetrating cone with the disc. The starting diameter of this particle was set such that no overlap occurred with the particles of the disc. The cone was assumed to move perpendicular to the disc which can be captured in the two-dimensional model by increasing the radius of the regular polygon. It is an essential condition that the cone has a very small apex angle to avoid a bending of the disc caused by out-of-plane forces. As the size increased the cone-particle overlapped the nearby polygons of the disc, which provided compressive loading along the perimeter of the cone. The main control parameter of the system is the speed v of the increase of the projectile radius, which corresponds to the penetration speed. (The loading velocity is given in terms of the sound speed of the material throughout the manuscript.) The penetration process was stopped when the size of the projectile reached 3 times the average polygon diameter, which proved to be sufficient to achieve complete breakup of the body. To have a clear view on the structure of the crack pattern the initial sample was reconstructed by placing the particles back to their initial position.

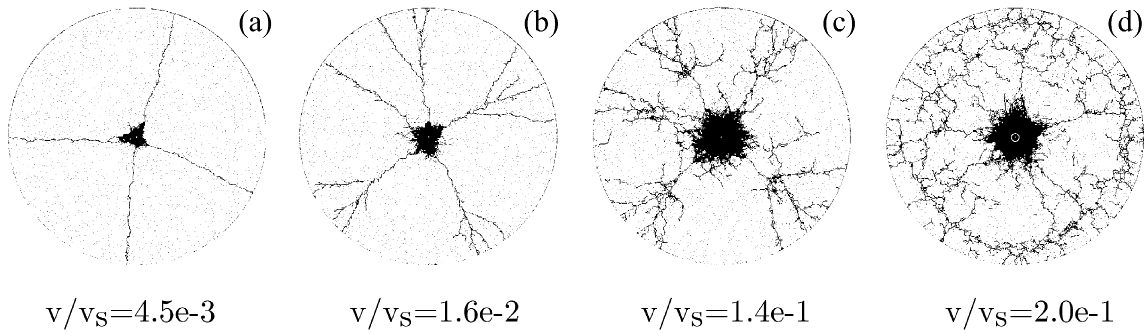

Fig. 1: Final reconstructed states of the disc-shaped sample at different loading velocities v/v_s = 0.0045, 0.016, 0.14, 0.2 from left to right, where v_s denotes the sound speed of the material. Cracks are defined as connected sets of broken beams.

Cracks were defined in the reconstructed discs as connected sets of broken beams. Figure 1 presents examples of the final reconstructed state of the system at different loading velocities where the cracks can clearly be observed. Note that in the middle of the sample in the vicinity of the loading cone a completely destroyed zone is formed where all beams are broken. The sample suffered some randomly scattered damage but an ordered crack pattern also emerges. We identify extended cracks with the condition that they start from the destroyed zone and reach to the external boundary. At low speed of loading the specimen breaks into two pieces due to two cracks which propagated opposite to each. As the loading gets faster the number of cracks gradually increases while the pattern remains regular. Fig. 1(a) presents the case where 4 segmentation cracks formed nearly perpendicular to each other. Our simulations revealed that not only the number of cracks increases but their geometrical structure also changes with increasing penetration speed: it can be observed in Fig. 1(b) that the main cracks develop sub-branches, which later on further branch building up a hierarchical tree-like crack structure (see Fig. 1(b,c)). For the quantitative characterization of the structural change of cracks with increasing loading speed we separated single segmentation cracks from each other presented in Fig. 2.

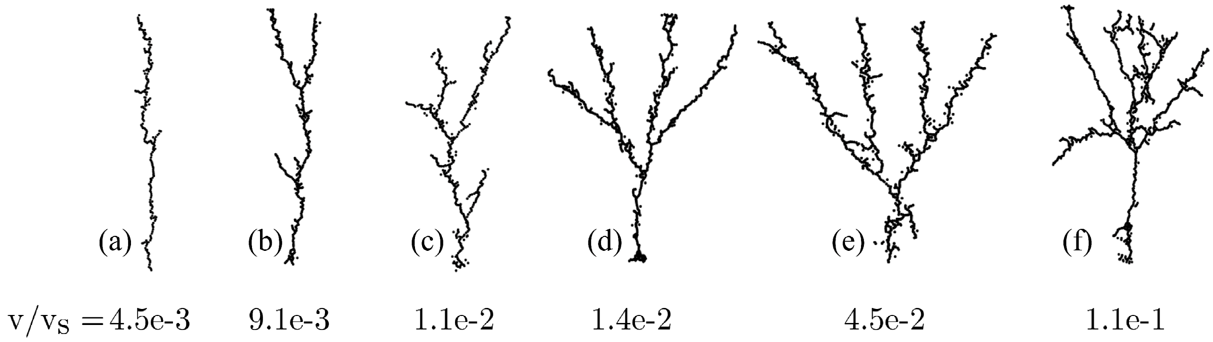

Fig. 2: Representative examples of single cracks for several loading velocities. At low speed the crack propagation is straight without branches. Increasing the velocity sub-branches and later on sub-sub-branches appear giving rise to higher energy dissipation.

Using the sand-box method we pointed out that the crack trees of Fig. 2 have fractal structure, however, the value of the fractal dimension D_f increases with the loading speed: for straight cracks typical in the limit of slow loading $D_f \approx 1$ was obtained (see Fig. 2(a)), however, the fractal dimension increased up to $D_f \approx 1.65$ with increasing loading speed (see Fig. 2(e,f)). It is important to emphasize that the crack structure in Figs. 1 and 2 and the range of the fractal dimension is in a very good agreement with recent experimental findings [1,2].

Time evolution at high loading speed - transition from fracture to fragmentation

When the loading speed surpasses a threshold value v_c the crack pattern drastically changes. It can be observed in Fig. 1(d) that for $v > v_c$ a fully connected crack pattern emerges and the specimen falls apart into a large number of pieces. This first occurs when a ring of crack is formed in the vicinity of the external surface due to the interference pattern of elastic waves. Figure 3(a) presents the average number of cracks identified by our algorithm as a function of v. It can be observed that a characteristic speed v_c can be pinpointed where the average crack number drops down to nearly one due to the ring crack. Ones the ring is completely established we redefine the cracks to run between the destroyed zone and the ring. The number of these types of cracks is again an increasing function of the loading velocity (see Fig. 3(a)).

In order to characterize the final breaking scenario of the fracture process we determined the probability distribution of the mass of fragments p(m). Simulations revealed that the distribution p(m) has a power law behavior over a broad range of masses

$$p(m) \propto m^{-\tau}, \quad (2)$$

where the value of the exponent τ depends on the loading speed. Fig. 3(b) presents the value of τ as a function of v. It can be seen that at low speed of loading dominated by straight or branched segmentation cracks, the value of the exponent has high fluctuations; however, once the connected crack network emerges a unique value is obtained τ≈1.9 which is again in a good agreement with experimental results [1].

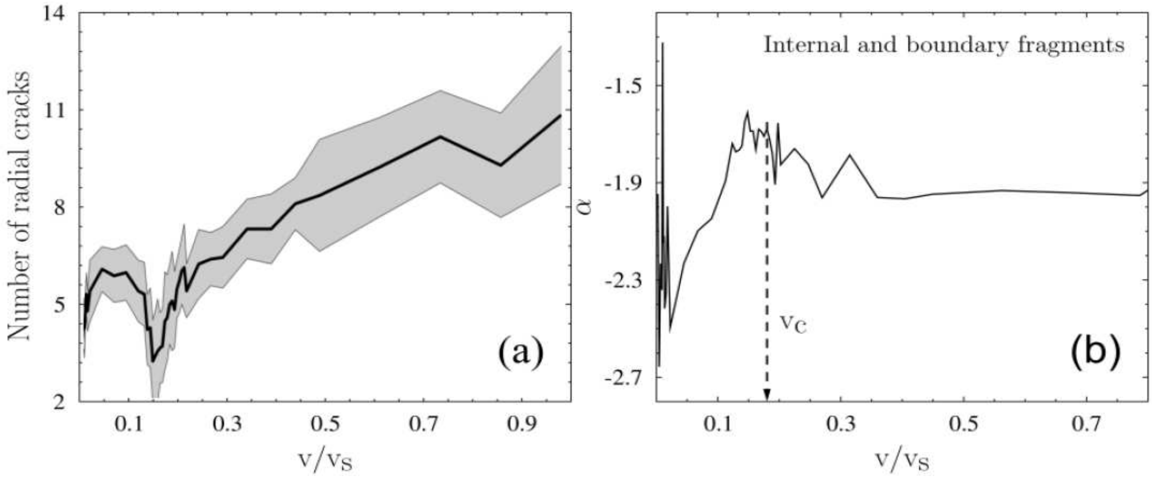

Fig. 3: (a) Evolution of the number of radial cracks as function of the loading speed. (b) The exponent of the mass distribution of fragments as a function of v/v_s.

Summary

We presented a numerical study of the evolution of crack patterns generated in a two-dimensional brittle material due to projectile penetration. Varying the penetration speed different phases were identified: at low speed two cracks are formed with nearly straight shape. Increasing the speed the number of cracks increases, furthermore, they develop side branches. When exceeding a threshold speed of loading a connected crack network emerges resulting in complete fragmentation of the solid. In order to characterize the geometrical structure of patterns, we determined the fractal dimension of single cracks. The analysis showed that the fractal dimension is an increasing function of the loading speed starting from 1.0 and saturating around 1.6. In the high speed limit fragmentation occurs and the sample breaks up into a large number of pieces. We determined the exponent of the mass distribution which proved to be constant when the loading speed becomes sufficiently high.

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