Topological Phases and Phase Transitions

Lecturer: Prof. Zsolt Gulácsi,
University of Debrecen, Department of Theoretical Physics.

Ordered Phases in Condensed Matter Lecture Series

— Shenyang *March 2019 —*

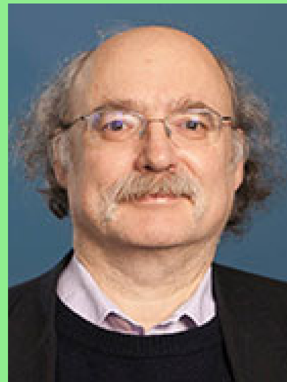
Nobel Prize in Physics - 2016

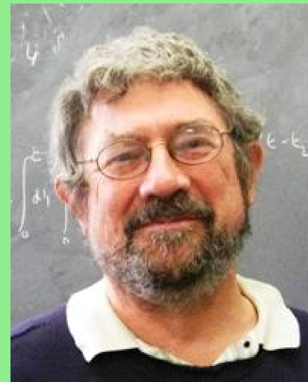
Nobel's legacy: "To be awarded for Outstanding contributions for Humanity"

Alfred Nobel (1833-1896)

Prof. Göran K. Hansson, president of the Swedish Royal Academy announced on 4 Oct. 2016: Physics Nobel Prize 2016: "For the theoretical discovery of topological transitions and topological phases of matter"

The Physics Nobel Prize winners in 2016





David Thouless	Duncan Haldane	M. Kosterlitz
B:1934(Bearsden)	1951(London)	1942(Aberdeen)
PhD:1958(Cornell)	1978(Cambridge)	1969(Oxford)
Age: 82	65	74
Poz:EmeritusProf.	E.Higgins.Prof.	E.Farnsworth.Prof.
Were: Seattle(USA)	Princeton(USA)	Providence(USA)
Univ.of Washington	Princeton Univ.	Brown Univ.
Prize: 1/2	1/4	1/4

Multi-generational Nobel Prize Laureates

David Thouless | Duncan Haldane | Michael Kosterlitz

Hans Bethe
1906-2005/1967

Phil Anderson
1923-/1977

David Thouless
1934-/2016

Arnold Sommerfeld
1868-1951

J.H. van Vleck
1899-1980/1977

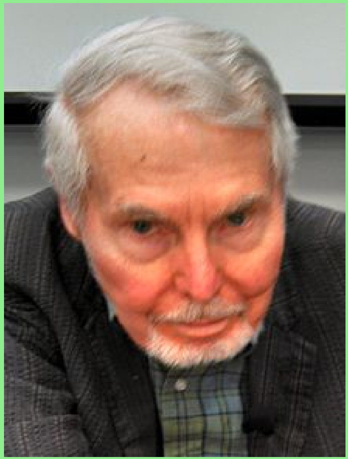
Hans Bethe
1906-2005/1967

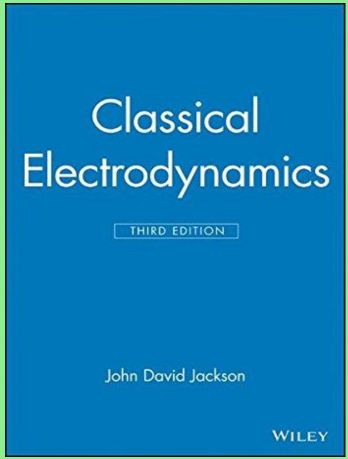
Göttingen


(E.K)


(A.S)

About Sommerfeld





John David Jackson (1925-2016)

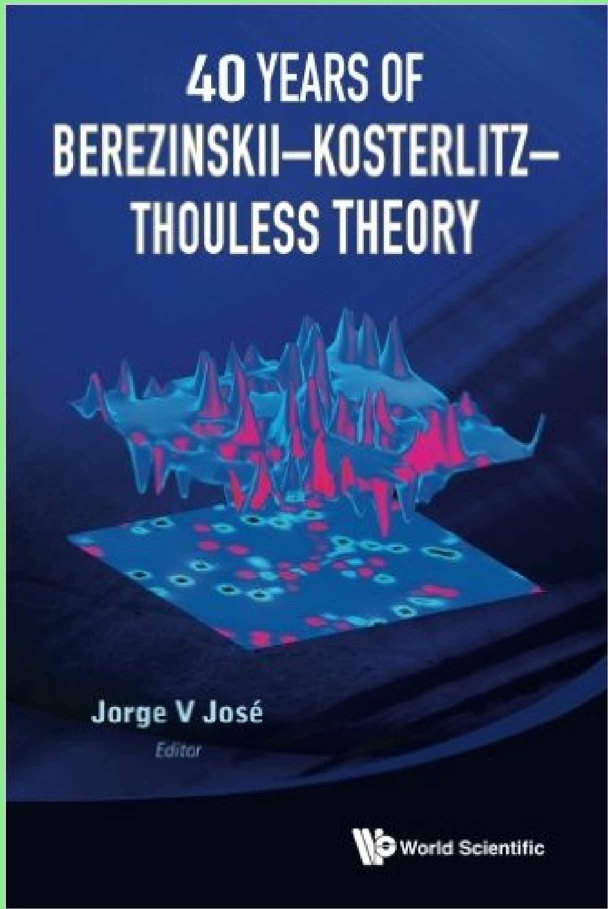
Arnold Sommerfeld (1868-1951)

Sommerfeld's disciples:

Nobel winners: Heisenberg, Pauli, Debye, Bethe, Pauling, Rabi, von Laue, Meissner

No Nobel winners: Peierls, Fröhlich, Ewald, Lenz, Brillouin, Heitler, Landé, Eckart, Morse, Condon, Hopf, Edwin Crawford Kemble: (van Vleck, Mulliken, Slater, Zener, Oppenheimer, Feenberg)

Jorge V. Jose, University Indiana, USA, 2013



World Scientific, 2013, ISBN: 978-981-4417-62-4

Vadim L. Berezinskii

Vadim L'vovich Berezinskiĭ (Obituary)

A. A. Abrikosov, L. P. Gor'kov, I. E. Dzyaloshinskiĭ, A. I. Larkin, A. B. Migdal, L. P. Pitaevskiĭ, and I. M. Khalatnikov

Usp. Fiz. Nauk 133, 553–554 (March 1981)

PACS numbers: 01.60. + q

VADIM L'VOVICH BEREZINSKIĬ (1935-1980)

Born: Kiev, 15 July 1935; MsC (1959) State University Moskow, Institute of Physics; PhD (1963): same institution; Textile Institute Moscow (1963-1968); Thermal Institute Moscow (1968-1977); Chernogolovka: L.D. Landau Institute (1977-1980). Died: Moscow, at the age: 44, 23 July 1980.

Mermin-Wagner-Hohenberg-Coleman low

David Mermin (Cornell)

B: 1935, New Haven(US)

Herbert Wagner (Munich)

1935, Essen

Piere Hohenberg(NewYork)

B: 1934,Neuilly-sur-Seine

Sidney Coleman(Harward)

1937, Chicago (D:2007)

Mermin-Wagner-Hohenberg-Coleman low

"No-Go" Theorem:

Continuous symmetry cannot be spontaneously broken in $D \leq 2$ many body systems at $T > 0$, thermodynamic limit and in the presence of short-range inter-particle interactions.

Furthermore:

In $D = 2$ ($D = 1$) many body systems at $T > 0$ in the thermodynamic limit in the presence of short-range inter-particle interactions, and if the number of components n of the characteristic dynamical variable is $n > 1$ (n is arbitrary) spontaneously long-range order cannot occur.

Is it no phase transition (quality change) in $D \leq 2$?

The Le Chatelier - Braun Principle

Henry Louis Le Chatelier
(1850-1936, Sorbonne)

Ferdinand Braun
(1850-1918, Strassburg)
Nobel Prize in Physics 1909
(with G.Marconi)

When an external action appears on a macroscopic system in equilibrium, the response of the system is to defend himself by the attempt to eliminate (nulify) the external action. (1885, French Academy of Sciences).

The Le Chatelier - Braun Principle in Nature

Chemistry: Introduced by Le Chatelier (1885);

Biology: Claude Bernard: homeostasis (1865);

Biosphere: Introduced by James Lovelock (1991);

Physiology: Introduced by Bradford Canon (1926);

Pharmaceutics: Known as Ligand Binding Mechanism;

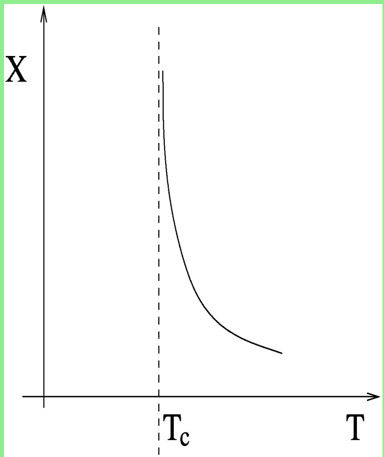
Sociology: Introduced by Jean-Francois Lyotard (1979);

Economics: Intr.by Paul Samuelson (1947), Nobel:1970;

Engineering: Known as negative feedback mechanism, described by James Watt in 1788;

Physics: Le Chatelier-Braun principle: e.g. gradients compensated by currents in transport processes: Ohm's law, Fourier's law, Seebeck effect, etc.; Lenz's law in electrodynamics, etc.

Divergences in Physical Quantities at T_c



X: could be e.g. $\chi = \frac{\partial M}{\partial H} \to \infty$:

If $\delta M >> \delta H$, an infinitesimally small external field variation (δH) causes an uncontrollable high magnetisation variation (δM) inside the system.

X: e.g. vertex function $\Gamma \sim 1/\epsilon \to \infty$:

if the permittivity (dielectric constant) vanishes, the screening capacity of the system disappears.

The reason of quality changes in many-body systems:

The system cannot defend himself agains external actions. In order to be able to do that (Le Chatelier-Braun principle) changes its quality. This is true in any D.

If for $D \leq 2$ long range order cannot appear, the system produces other ordering in order to defend himself.

The KTB transition: Starting publications

Preliminaries:

B.J.Alder, T.A.Wainwright, Phys.Rev.127,359(1962):
2D disc, numerical study, shows a phase transition which is interpreted as solid-gas phase transition.

H.E.Stanley, T.A.Kaplan, Phys.Rev.Lett.17,913(1966):
$T >> 1$ expansion for the 2D XY spin model: phase transition is obtained at which χ diverges.

Main publications:

V.L. Berezinskii, Zh.Eksp.Teor.Fiz. 59, 907 (1970):
2D XY spin model in the presence of $\vec{H}$: At $T > T_c$ one has $M \sim H$, while for $T < T_c$, one obtains $M \sim H^{\lambda}$, $0 < \lambda < 1$. Sharp transition at T_c.

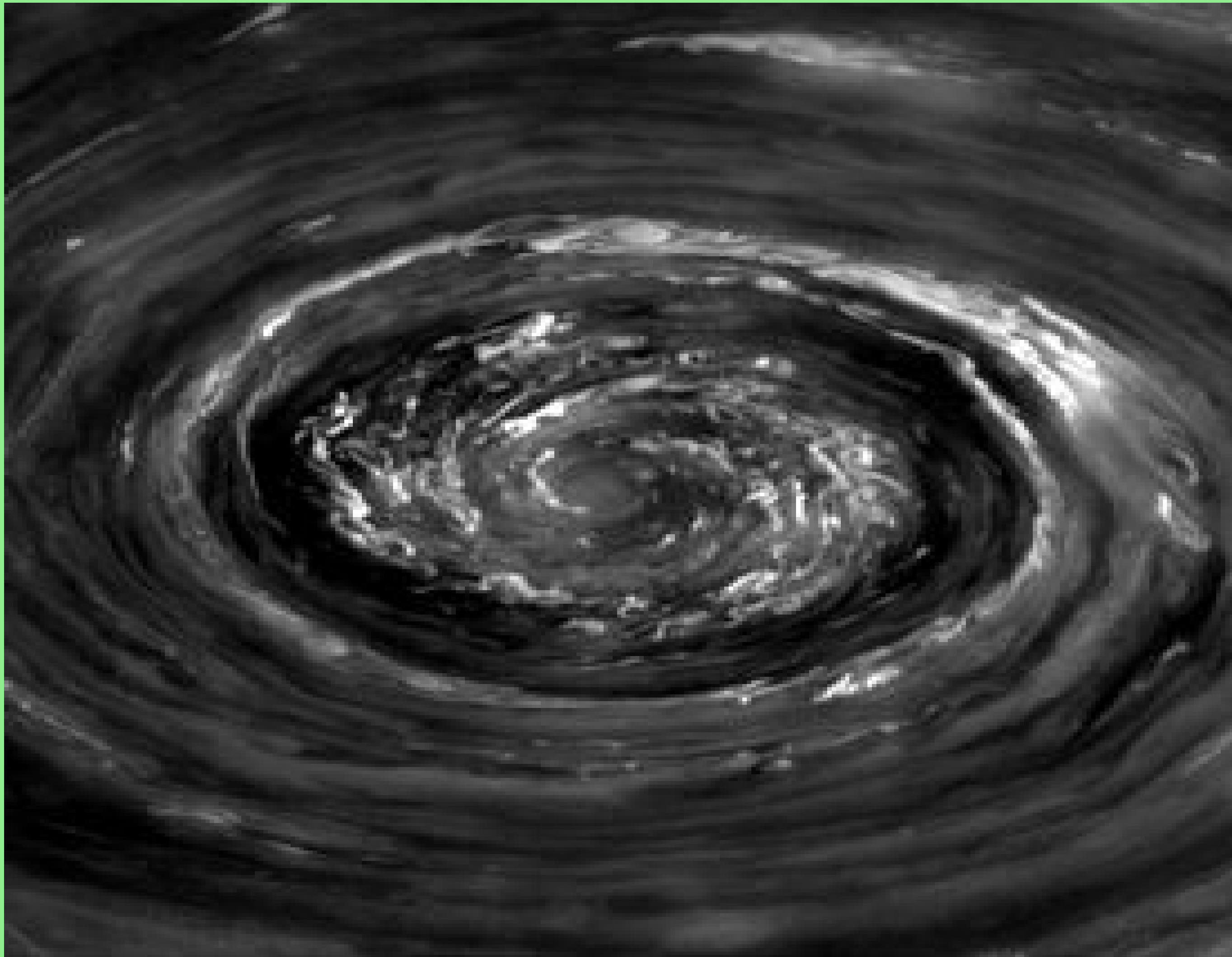
V.L. Berezinskii, Zh.Eksp.Teor.Fiz. 61, 1144 (1971): At $T < T_c$: cvasi-molecules whose global vorticity is zero; at $T > T_c$: gas built up from independent vortices.

J. M. Kosterlitz, D. J. Thouless, Jour. Phys. C: Solid State Physics 5, L124 (1972): Treatements of 2D melting. For the first time was written: “topological ordering”.

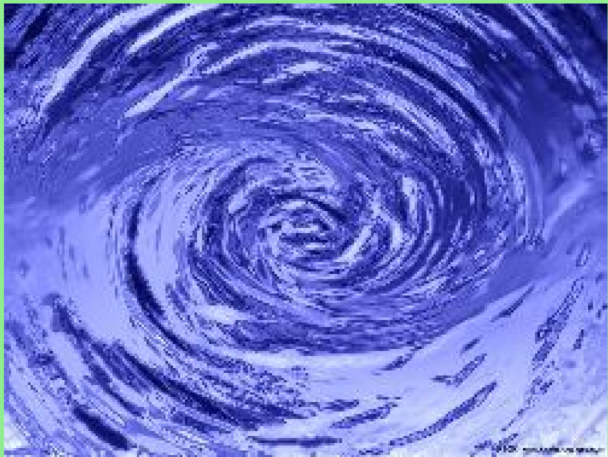
J. M. Kosterlitz, D. J. Thouless, Jour. Phys. C: Solid State Physics 6, 1181 (1973): The description of the KTB transition as we know it today.

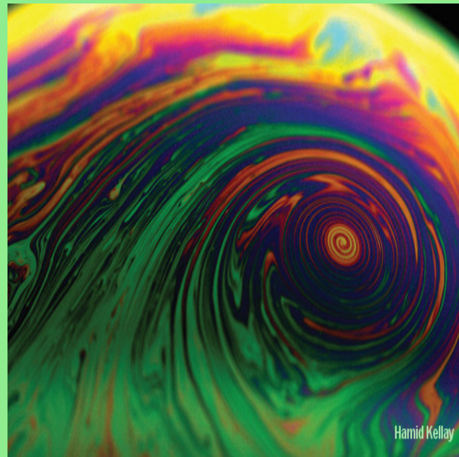
J. M. Kosterlitz, Jour. Phys. C: Solid State Physics 7, 1046 (1974): RG proof of the existence of the phase transition. Deduction of the critical behavior, critical exponents, proof of the infinite order nature of the phase transition in the Ehrenfest’s scheme.

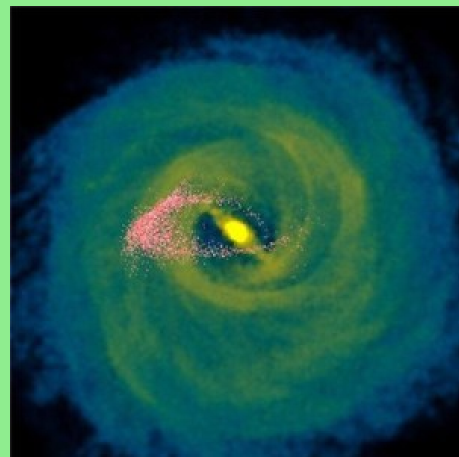
The vortex notion: cloud on Saturn



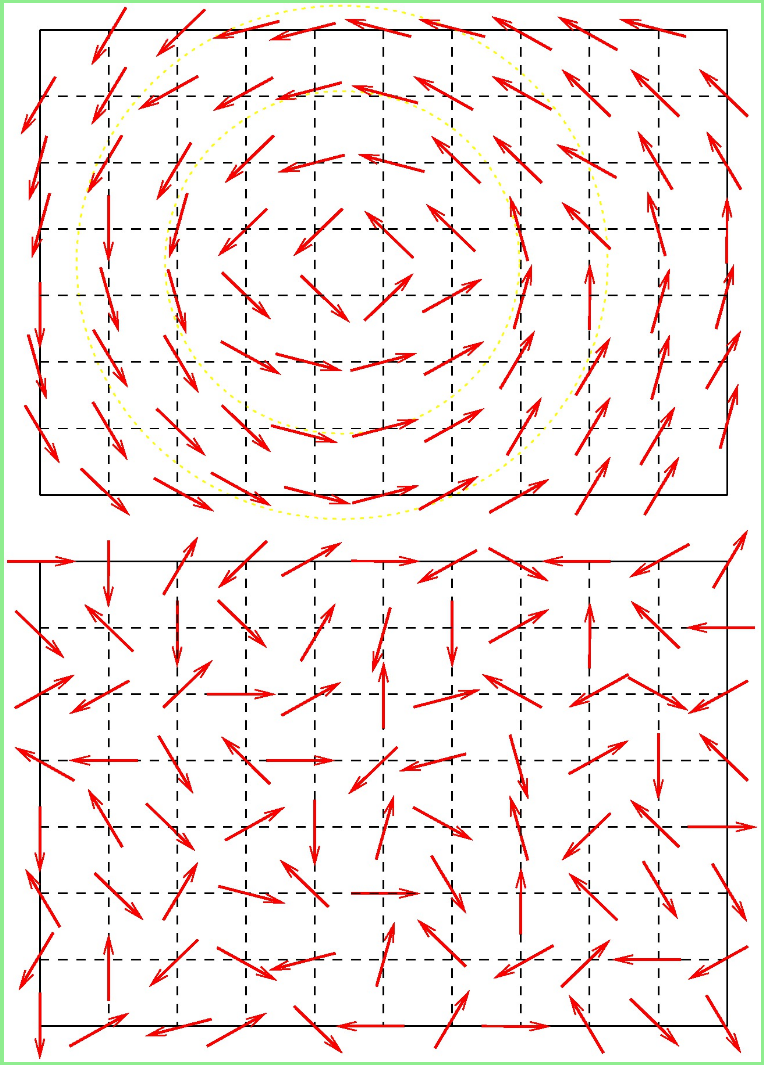
Vortex: is not a rare phenomenon



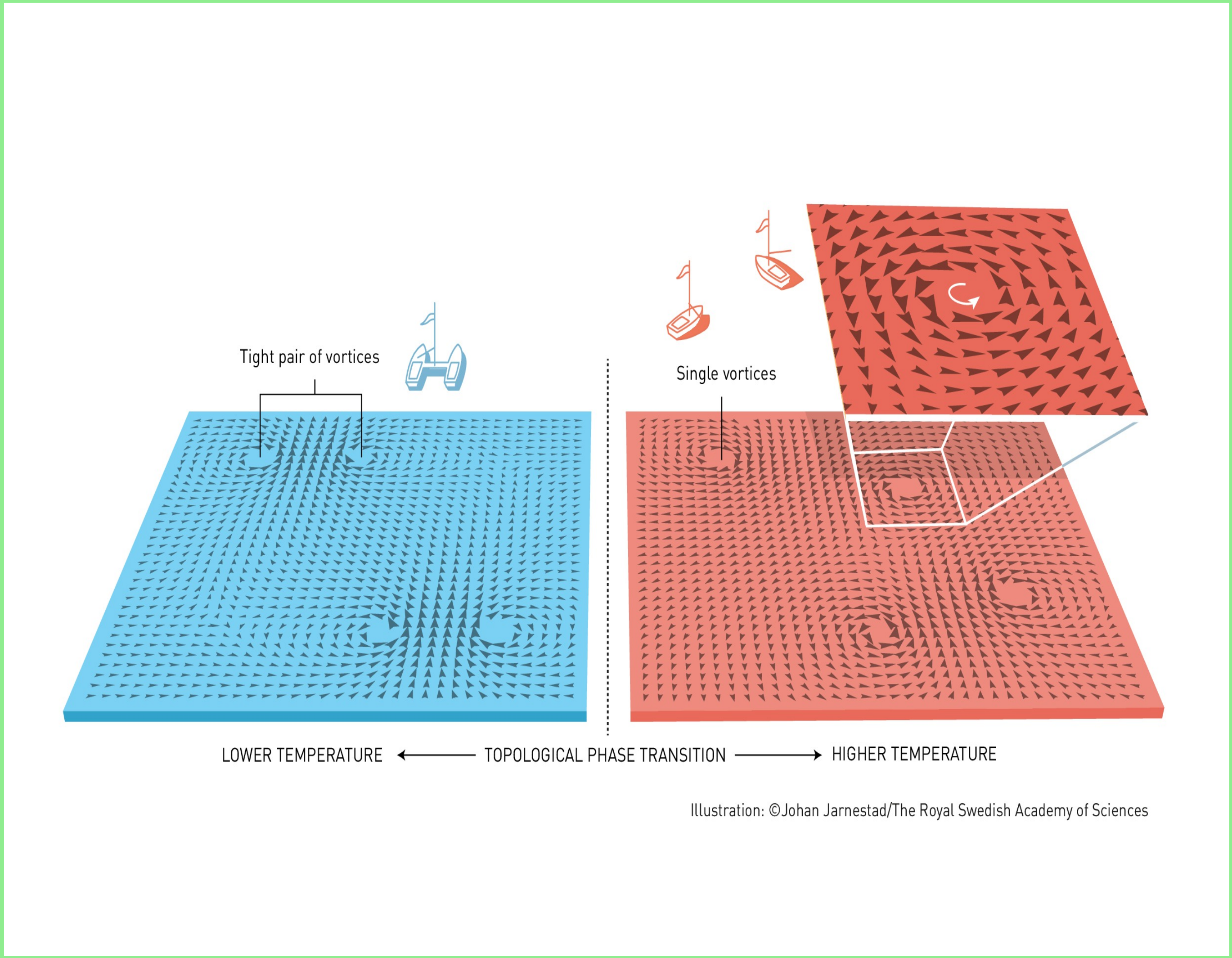




Vortex notion: a Spin vortex

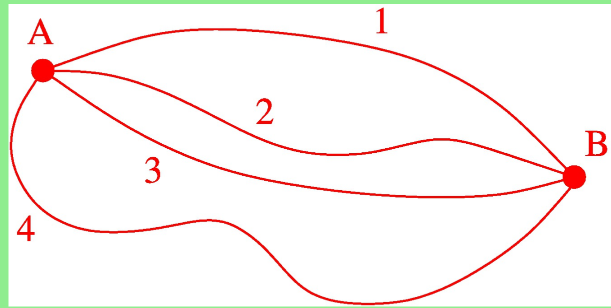


KTB transition: a topological phase transition



Vortex: a topological object

What is a topological object ?



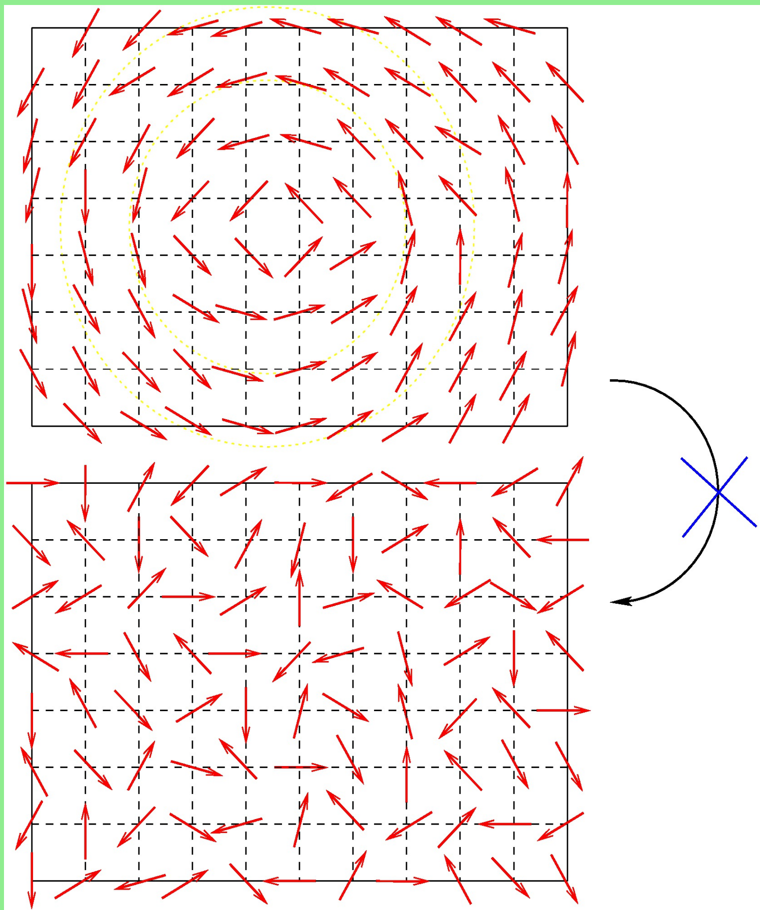
Homotopy transformations:

Continuous deformation transformations
E.g. in the figure: $1 \to 2 \to 3 \to 4$.

Homotopy classes:

Two objects (e.g. functions) are not contained in the same homotopy class if by a homotopy transformation (continuous deformation transformation), these objects (functions) cannot be transformed each in another (are not interconvertible).

Vortex: a topological object



- There is no “homotopy” in between a completely disordered spin state and a vortex state, i.e.:

- The completely disordered spin state cannot be transformed in a vortex state by a homotopy transformation.

- These two states are in different homotopy classes, i.e are topologically different.

Vortex: a topological object

Topological transition:

A transition for which the difference between the ordered and disordered state can be done only by topological concepts.

The aim of the Topology:

The topology analyses properties that remain unchanged after homotopic transformations (continuous deformation transformations).

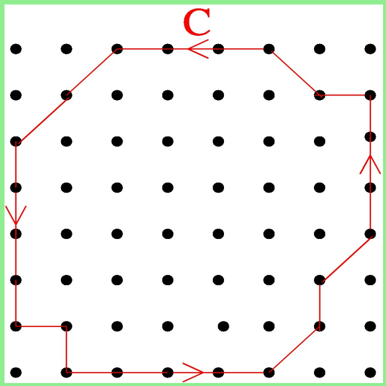
Importance for Physics:

The topological characteristics and properties since remain unchanged after arbitrary continuous transformations (deformations) are extremely stable (even impurities do not affect them). These are:
<u>"topologically protected properties"</u>.

Topological properties: integer numbers

Properties invariant under homotopic transformations (i.e. topological properties) can be always expressed with integer numbers.

E.g. at the KTB transition in 2D XY model this property is the "vorticity" v:



$$v = \frac{1}{2\pi} \sum_C \delta\theta(\vec{r})$$

$$v = \frac{1}{2\pi} \oint_C \nabla\theta(\vec{r})\, d\vec{r}$$

In the present case one has $|v| = 0, 1, 2, 3, \ldots$.

About Topology

Etimology (origin of the word, name):

Topology: topos = place, logos = study (Greek)

G.W. von Leibniz (1646-1716)

Leonhard Euler (1707-1783)

J.Benedict Listing (1808-1882)

The 2D KTB transition is not rare. Examples:

a) 2D XY spin system: pairs of spin vortices with opposite vorticity build up the ordered phase.

b) melting in 2D: pairs of dislocations with opposite Burger vector are present in the ordered phase.

c) 2D neutral plasma: electrical dipols formed from oppositely charged particles build up the ordered phase.

d) 2D He4 superfluid: below T_c vortex-antivortex pairs are present in the ordered phase.

e) 2D lattice of Josephson junctions (nano Pb discs in a triangular lattice): pairs of Abrikosov vortices form the ordered phase.

f) 2D superconductor-insulator transition: Cooper pairs linked to impurities form the “pairs” below T_c.

A simple explanation for the KTB transition

The energy of one vortex:

$$H_{XY} = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j),\ J > 0,$$

$$H_{XY} = \frac{J}{2} \int d^2 r (\nabla \theta(\vec{r}))^2,\ |\nabla \theta(\vec{r})| = \frac{1}{r},$$

$$E = \frac{J}{2} \int_a^L d^2 r\, (\frac{1}{r})^2 = \pi J\, ln \frac{L}{a}.$$

One vortex increases the enery. Consequently, at $T = 0$ one vortex (individual vortices) cannot be present.

Free energy in the presence of one vortex:

$$F_{1v} = E - TS = \pi J ln \frac{L}{a} - k_B T\, ln(\frac{L}{a})^2$$

$$T_{KTB} = \frac{\pi J}{2 k_B}$$

But at $T > 0$ the free energy matters. If F decreases, the new state is preferred.

One has $\delta F = F_{1v} < 0$ at $T > T_c$: Individual vortices are present above T_c, consequently, vortex pairs at $T < T_c$.

Characteristics of the KTB transition

1) Exponential divergences in $t = (T - T_c)/T_c$:

$\xi \sim exp(b/\sqrt{t}),\ b \simeq 1.5$

2) In the Ehrenfest's scheme: ∞ order transition:

$\Phi \sim \xi^{-2}$

3) Critical exponents only in "severe" sense:
(defined in function of ξ, not t: $\chi \sim \xi^{\bar{\gamma}}, C \sim \xi^{\bar{\alpha}}, G(r) \sim r^{-\eta}$)
$\bar{\alpha} = \alpha/\nu = -2,\ \bar{\gamma} = \gamma/\nu = \frac{7}{4},\ \delta = 15,\ \eta = \frac{1}{4}$, **($m \sim h^{1/\delta}$).**

4) Only "severe" scaling laws are present:

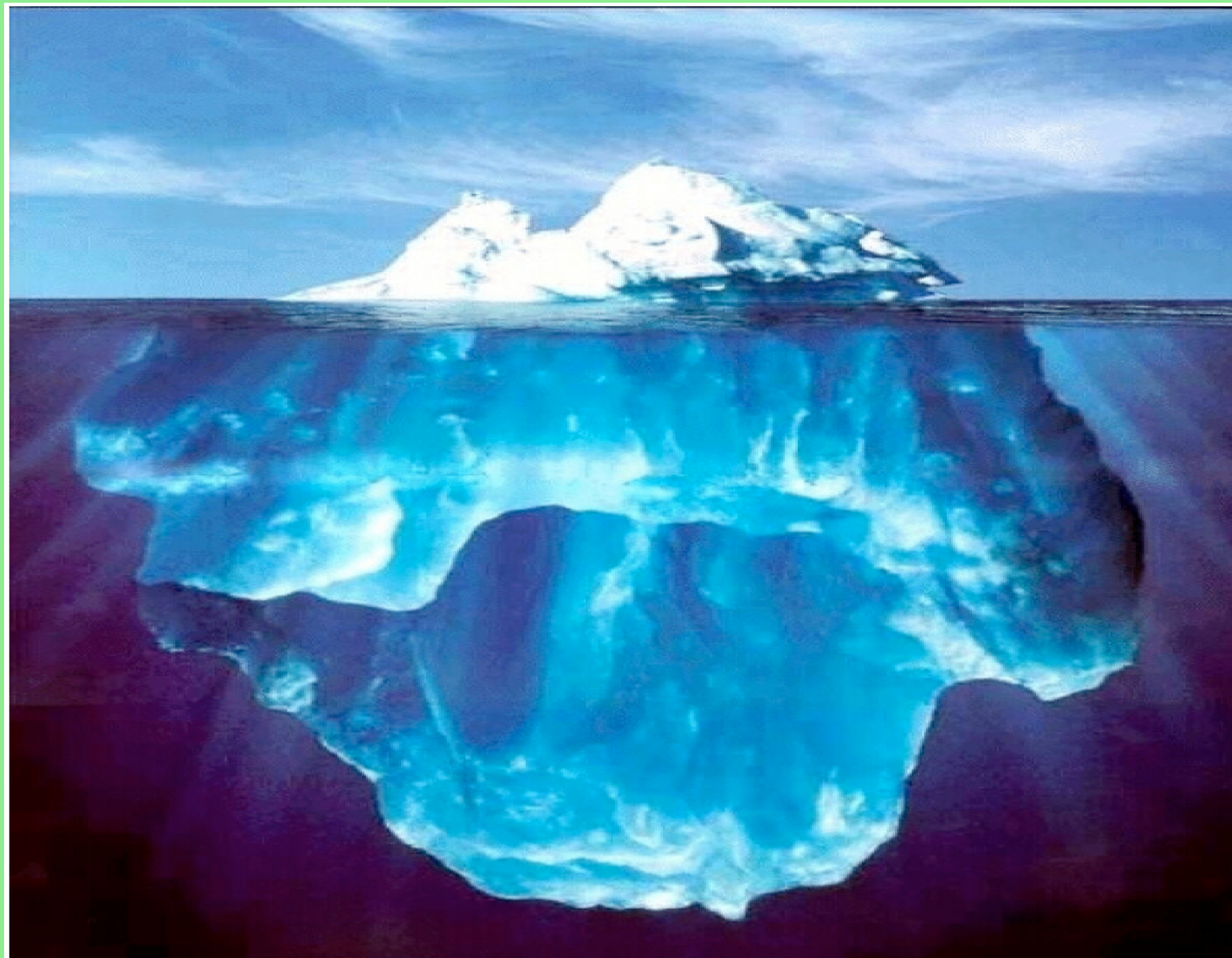
$\bar{\gamma} = 2 - \eta,\ \bar{\alpha} = -d,\ \delta = \frac{d+2-\eta}{d-2+\eta}$

5) In the ordered phase at $T < T_c$:

$\xi(t) = \chi(t) = \infty,\ for\ t < 0$

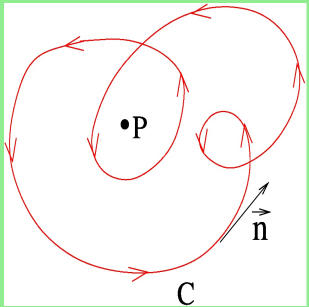
6) "Severe" is almost the 2D Ising transition ($\bar{\alpha} = 0$).

The Topology in Physics in the year 1975



New topological notion: the winding number

The winding number Q [Henri Poincare (1885)]:



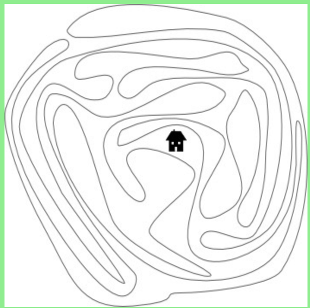
Q=2

H.Poincare

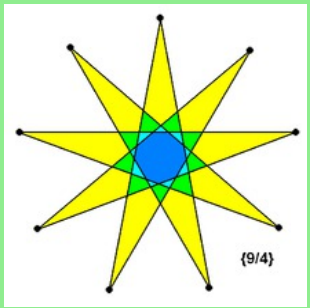
Topological notion: It shows how many times an unit vector $\vec{n}$ is pivoted around a point P while it moves on a closed curve.

$$Q = \frac{1}{4\pi}\int_C d^2x\, \vec{n}\cdot(\partial_1\vec{n}\times\partial_2\vec{n}),\ x=(x_1,x_2)$$

Mathematical Applications:



a)

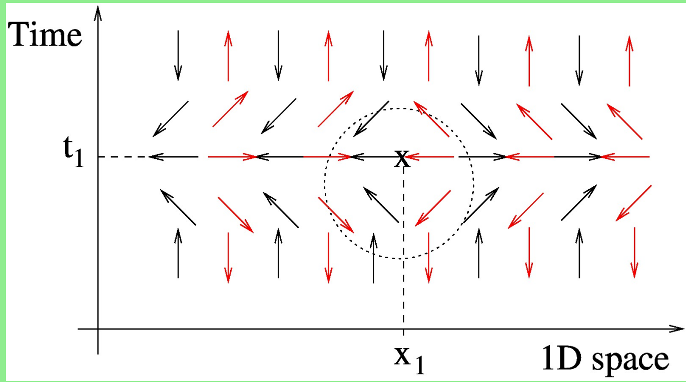


b)

a) Encloses the wall the hous ? No: Q=0.

b) Star polygon (9 peaks, Q=4) contrary to convex polygon (Q=1).

Haldane gap: S=1, 1D AF Heisenberg chain



Duncan Haldane: "Vortex" in space-time = winding number

F.D.M. Haldane, Phys. Lett. A93, 464 (1983)

F.D.M. Haldane, Phys. Rev. Lett. 50, 1153 (1983)

Gap if $e^{2\pi iQS} = 1$ [Q deduced in (it, x) variables in 1D]. Hence for $S = 1$ gap is present. Contrary to this, in $S = 1/2$ cas the spectrum is gapless [known from Bethe ansatz]. Experimentally proved in 2002 [$CsNiCl_3$].

The Euler characteristic

Leonhard Euler (1750: Topology as Science): [1707(Basel)-1783(St.Petersburg)]

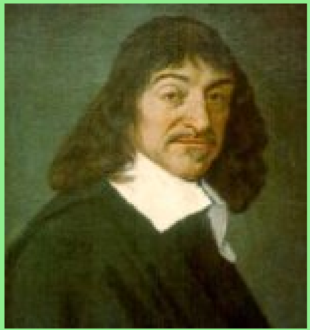
χ = V - E + F

V = number of vertices,

E = number of edges,

F = number of faces.

Complete Mathematical Proof: 1811 (Cauchy).



Rene Descartes (1596-1650)

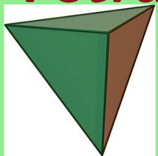
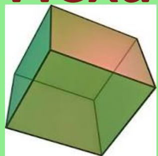
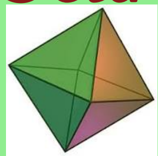
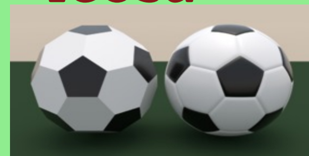
A.M.Legendre (1752-1833)

Augustin L.Cauchy (1789-1857)

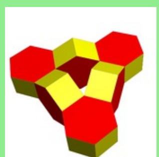
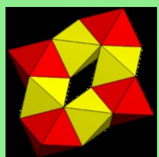
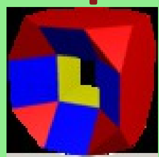
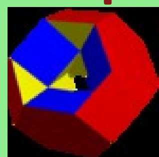
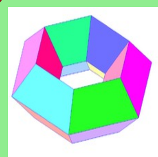
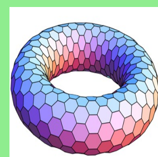
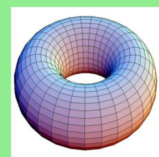
Euler characteristic = Surface characteristic

Numbers in order: V,E,F ($\chi = V - E + F$):

Hedrons(convex polyhedra): $\chi = 2$

Tetra	Hexa	Octa	Dodeca	Icosa	Snub-Dodeca	Truncated-Icosa
						
4,6,4	8,12,6	6,12,8	20,30,12	12,30,20	60,150,92	60,90,32

Polyhedra with one hole: $\chi = 0$

6HexPri.	8Octa.	4Cup8t.	6Cup6py.	HexTor.		Torus
						$\rightarrow$ 
48,84,36	24,72,48	32,64,32	30,60,30	24,48,24		

In Conclusion: if p=number of polygons on the surface

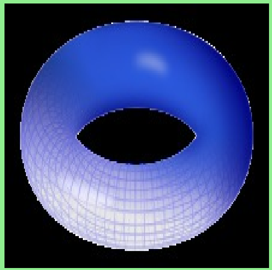
It results: $lim_{p \to \infty} \chi$ = a characteristic of the surface

The “genus” notion: topological invariant

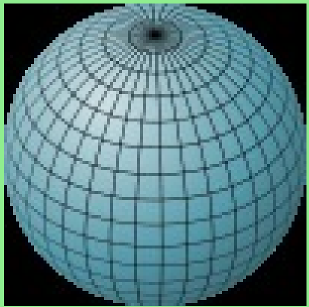
L. Euler
1707-1783


,

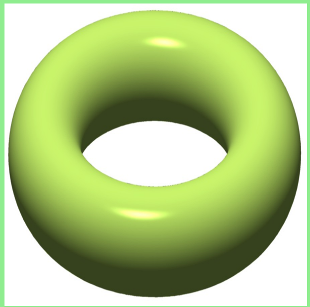
Topological
equivalent


,

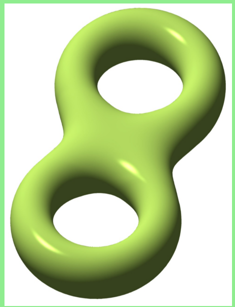
Topological
equivalent



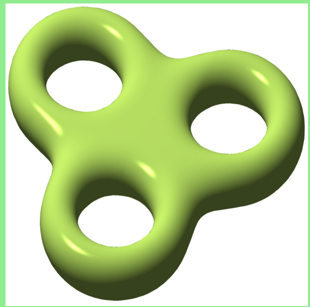
g=0



g=1



g=2



g=3

Euler characteristic of the S surface: $\chi(S) = 2 - 2g$

Gauss-Bonnet formula (1828)

Carl Friedrich Gauss

(1777-1855)

Pierre Ossian Bonnet

(1819-1892)

"Theorema Egregium (Latin)" (remarkable theorem)

If the S closed surface has the Gaussian curvature K_G, [$K_G = k_1 k_2$, where $k_1 = 1/R_1, k_2 = 1/R_2$ are the principal radii], then

$$\int_S K_G dS = 2\pi\chi(S) = 4\pi(1-g)$$

E.g.: for Sphere, $K_G = 1/R^2$, $\chi = 2$, $g = 0$.

From Euclidean to Riemann (curved) space

J. Bolyai (1802-1860) **N. Lobachevsky (1792-1856)** **B. Riemann (1826-1866)**

Riemann curvature:

$$R^{\alpha}_{\beta,\mu,\nu} = \frac{\partial}{\partial x^{\mu}}\Gamma^{\alpha}_{\beta,\nu} - \frac{\partial}{\partial x^{\nu}}\Gamma^{\alpha}_{\beta,\mu} + \Gamma^{\alpha}_{\mu,\gamma}\Gamma^{\gamma}_{\beta,\nu} - \Gamma^{\alpha}_{\nu,\sigma}\Gamma^{\sigma}_{\beta,\mu},\ R_{\alpha,\beta,\mu,\nu} = g_{\alpha,\gamma}R^{\gamma}_{\beta,\mu,\nu},$$

$$\Gamma^{\alpha}_{\mu,\nu} = \frac{g^{\alpha,\sigma}}{2}\left(\frac{\partial g_{\nu,\sigma}}{\partial x^{\mu}} + \frac{\partial g_{\sigma,\mu}}{\partial x^{\nu}} - \frac{\partial g_{\mu,\nu}}{\partial x^{\sigma}}\right),\ ds^2 = g_{\alpha,\beta}dx^{\alpha}dx^{\beta},\ g^{\alpha,\beta} = (g^{-1})_{\alpha,\beta}$$

$$R_{\alpha,\beta,\mu,\nu}\cdot x_{\beta} = (\nabla_{\mu}\Gamma^{\alpha}_{\nu,\beta} - \nabla_{\nu}\Gamma^{\alpha}_{\mu,\beta})\cdot x_{\alpha},\ \nabla_i V^m = \frac{\partial}{\partial x^i}V^m + \Gamma^m_{k,i}V^k.$$

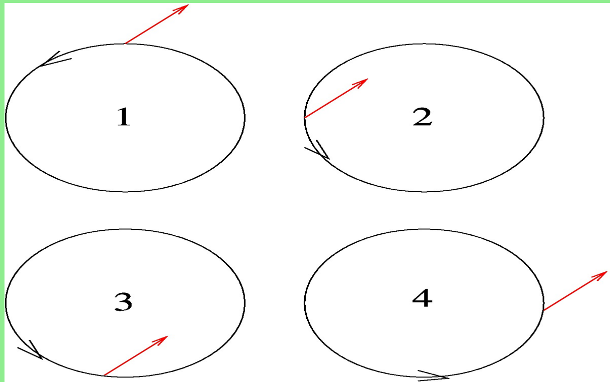
First application in physics: General Relativity:

Matter = Restrictions on Riemann curvature

Ricci: $R_{a,b} = R^c_{a,c,b}, R = g^{a,b}R_{a,b}, G_{a,b} = R_{a,b} - Rg_{a,b}/2,\ R_{a,b} = Ricci\ tensor,\ K_G = R/2,\ G_{a,b} = Einstein\ tensor$

Einstein: $G_{a,b} + \lambda g_{a,b} = (8\pi k/c^4)T_{a,b},\ T_{a,b} = stress - energy\ tensor,\ \lambda = cosmological\ constant$

Parallel transport in Riemann space



Parallel transport

T.Levi-Civita (1829-1900)

E.B.Christoffel (1873-1941)

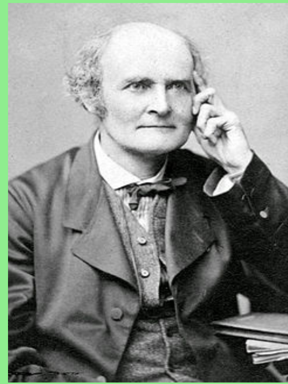
- The space is curved: only if the parallel transport on a closed curve at return modifies the vector.

- Levi-Civita connection: realizes the parallel transport

- Christoffel simbols: $\Gamma^{\alpha}_{\beta,\gamma}$ are the components of the Levi-Civita connection ($\nabla_i V^m = \frac{\partial}{\partial x^i} V^m + \Gamma^m_{k,i} V^k$).

- The curveture is in fact: Rotor ($\nabla\times$) from connection ($\Gamma^{\alpha}_{\beta,\gamma}$), see [$R_{\alpha,\beta,\mu,\nu} \cdot x_\beta = (\nabla_\mu \Gamma^{\alpha}_{\nu,\beta} - \nabla_\nu \Gamma^{\alpha}_{\mu,\beta}) \cdot x_\alpha$].

Chern extension of the Euler characteristic



Shiing Shen Chern (1911-2004) **Heinz Hopf (1894-1971)** **Artur Cayley (1821-1895)**

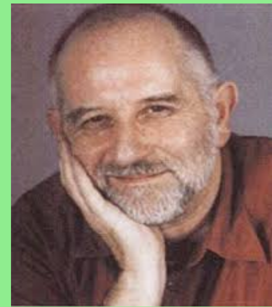
Chern-Gauss-Bonnet theorem: Hopf (1925), Chern (1944):

$$\int_M Pf(\Omega)\, dM = (2\pi)^m \chi(M)$$

M **is a** $2m$ **dimensional Riemann manifold,** $\Omega =$ **curvature of M given via the Levi-Civita connection,** $Pf(\Omega)$**: the Pfaffian of** Ω **[**$Pf(\Omega)$**: introduced by Cayley (1852),** $Pf(\Omega) = (det(\Omega))^{1/2}$**],** $\chi(M)$ **: the generalised Euler characteristic = Chern number for the class m.**

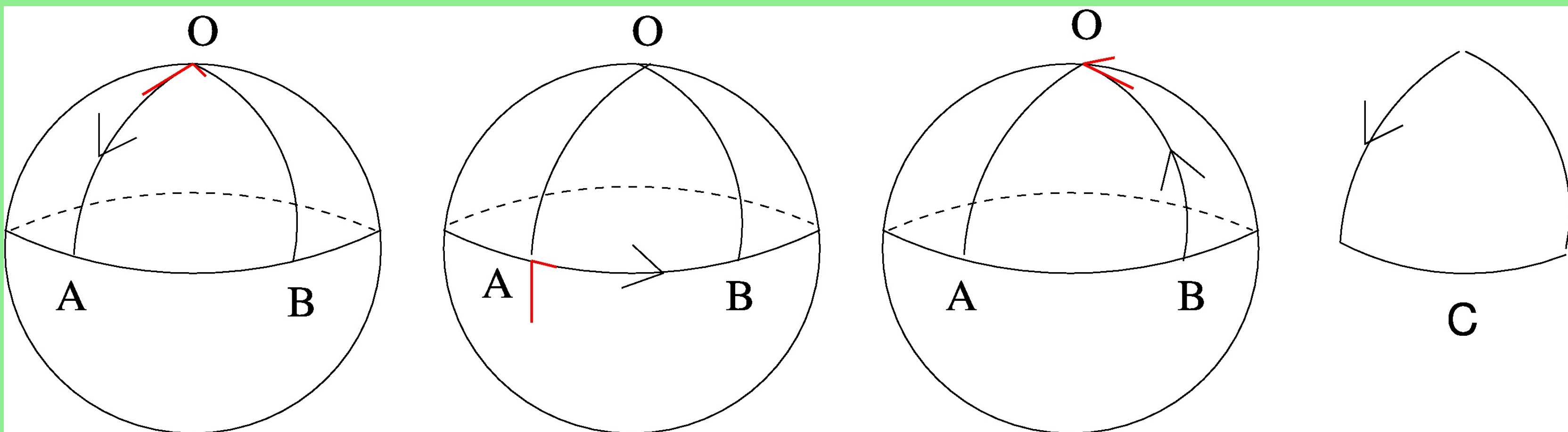
Berry-Pancharatnam phase (1984)

S. Pancharatnam



M. Berry

Berry phase: described in optics (1956) by Shivaramakrishnan Pancharatnam (1934-1969). Sir Michael Berry (Born: 1941) "rediscovered" it after $\sim$ 30 years (1984).



A Berry curvature

Berry phase factor $e^{i\gamma}$:

$$\hat{H} = \hat{H}(\mathbf{r}(t)),\ |\Psi_n\rangle = e^{i\gamma_n(t)} e^{-\frac{i}{\hbar}\int_0^t \epsilon_n(\mathbf{r}(\tau))d\tau} |\psi_n(\mathbf{r}(t))\rangle,$$

$$\gamma_n = i\oint_C \langle\psi_n(\mathbf{r})|\nabla_{\mathrm{r}}|\psi_n(\mathbf{r})\rangle \cdot d\mathbf{r} = \oint_C \mathbf{A}_B \cdot d\mathbf{r}$$

Peirls phase: $\phi_P = \oint_C \mathbf{A} \cdot d\mathbf{r}$, $\mathrm{B} = \nabla \times \mathbf{A}$

Michael Berry
(B: 1941)

R.E. Peierls
1907-1995

Consequences:

$\mathbf{A}_{B,n} = i\langle\psi_n(\mathbf{r})|\nabla_{\mathrm{r}}|\psi_n(\mathbf{r})\rangle$ **Berry connection.**

$\gamma_n = \oint_C \mathbf{A}_{B,n} \cdot d\mathbf{r} = \int_S (\nabla_{\mathrm{r}} \times \mathbf{A}_{\mathrm{B,n}}(\mathbf{r})) \cdot d\mathrm{S}$ **(Stokes).**

$\mathrm{K}_{B,n}(\mathbf{r}) = \nabla_{\mathrm{r}} \times \mathbf{A}_{B,n}(\mathbf{r})$ **Berry curvature:**

corresponds to $Pf(\Omega)$ in Chern-Gauss-Bonet

formula at m=1 ($D = 2$).

(Chern number of the first Chern class).

Topology of the Quantum State

Chern-Gauss-Bonnet formula with Berry curvature:

$$\int_S \mathbf{K}_B(\mathbf{r}) \cdot d\mathbf{S} = 2\pi C_1$$

$\mathbf{K}_B(\mathbf{r}) = \nabla_{\mathbf{r}} \times \mathbf{A}_{B,m=1,n=1}(\mathbf{r})$ is the Berry curvature, $\mathbf{A}_{B,m=1,n=1}(\mathbf{r}) = i\langle\psi_{n=1}(\mathbf{r})|\nabla_{\mathbf{r}}|\psi_{n=1}(\mathbf{r})\rangle$ is the Berry connection (field), (this provides the Berry phase along a closed curve C as $\gamma_1 = \oint_C \mathbf{A}_{B,m=1,n=1}(\mathbf{r}) \cdot d\mathbf{r}$). $C_{m=1} = C_1$ is the Chern number (for the first Chern class and the ground state).

Topological phase: a phase with non-trivial topology (has non-zero Chern number).

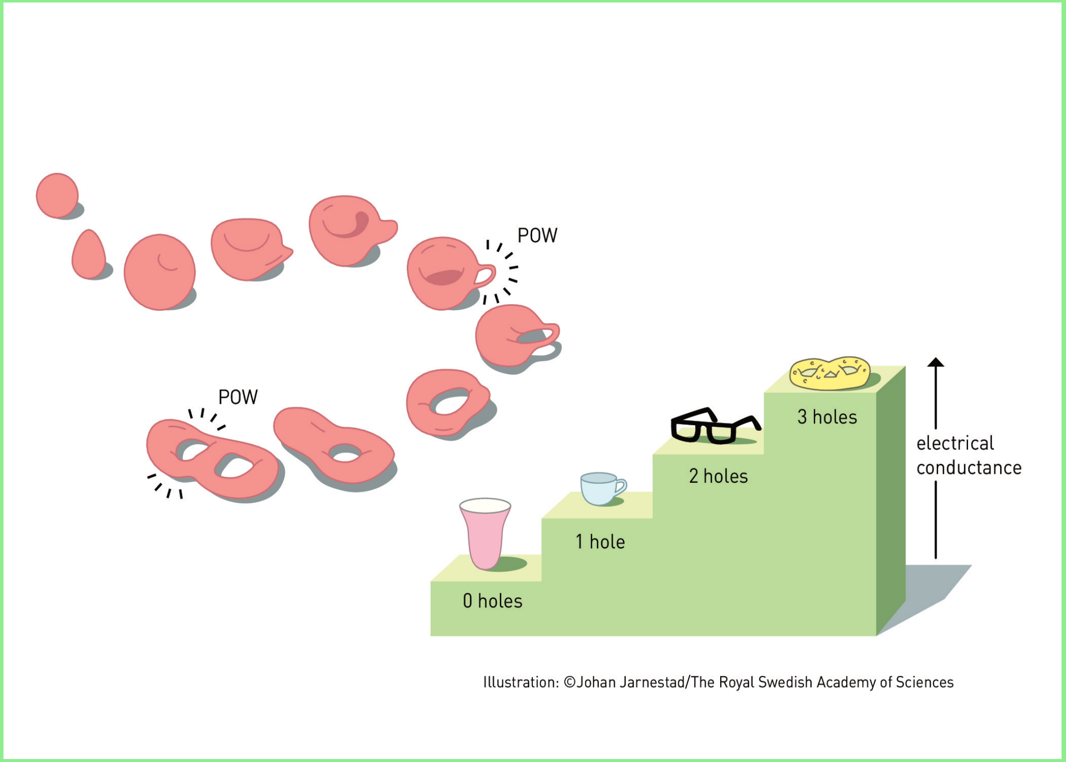
Importance: Besides special properties, a such a phase has topological protected properties (deformations, impurities do not modify them).

Topological protected property

Quantum Hall effect

D.J. Thouless, M. Kohmoto, M. P. Nightingale, M. den Nijs, Phys. Rev. Lett. 49, 405 (1982).

Quantised conductance: $\sigma_{xy} = \frac{e^2}{h} C_1$ (deduced in k-space)



The four authors

David Thouless(B: 1934)
University of Washington

Mahito Kohmoto(B: 1958)
University of Tokyo

Peter Nightingale (B: 1952)
University of Rhode Island

Marcel den Nijs (B: 1957)
University of Washington

Mathematical Proof

D.J. Thouless, M. Kohmoto, M. P. Nightingale, M. den Nijs, Phys. Rev. Lett. 49, 405 (1982).

Mahito Kohmoto
(B: 1958)

Barry Simon
(B: 1946)

J.E. Avron, R. Seiler, B. Simon, Phys. Rev. Lett. 51, 51 (1983)

M. Kohmoto, Ann. Phys. (Berlin) 160, 343 (1985)

Haldane: Kohmoto's role in the Laughlin case

F. D. M. Haldane, Phys. Rev. Lett. 51, 605 (1983).

Robert B. Laughlin
(B: 1950)

Duncan Haldane
(B: 1951)

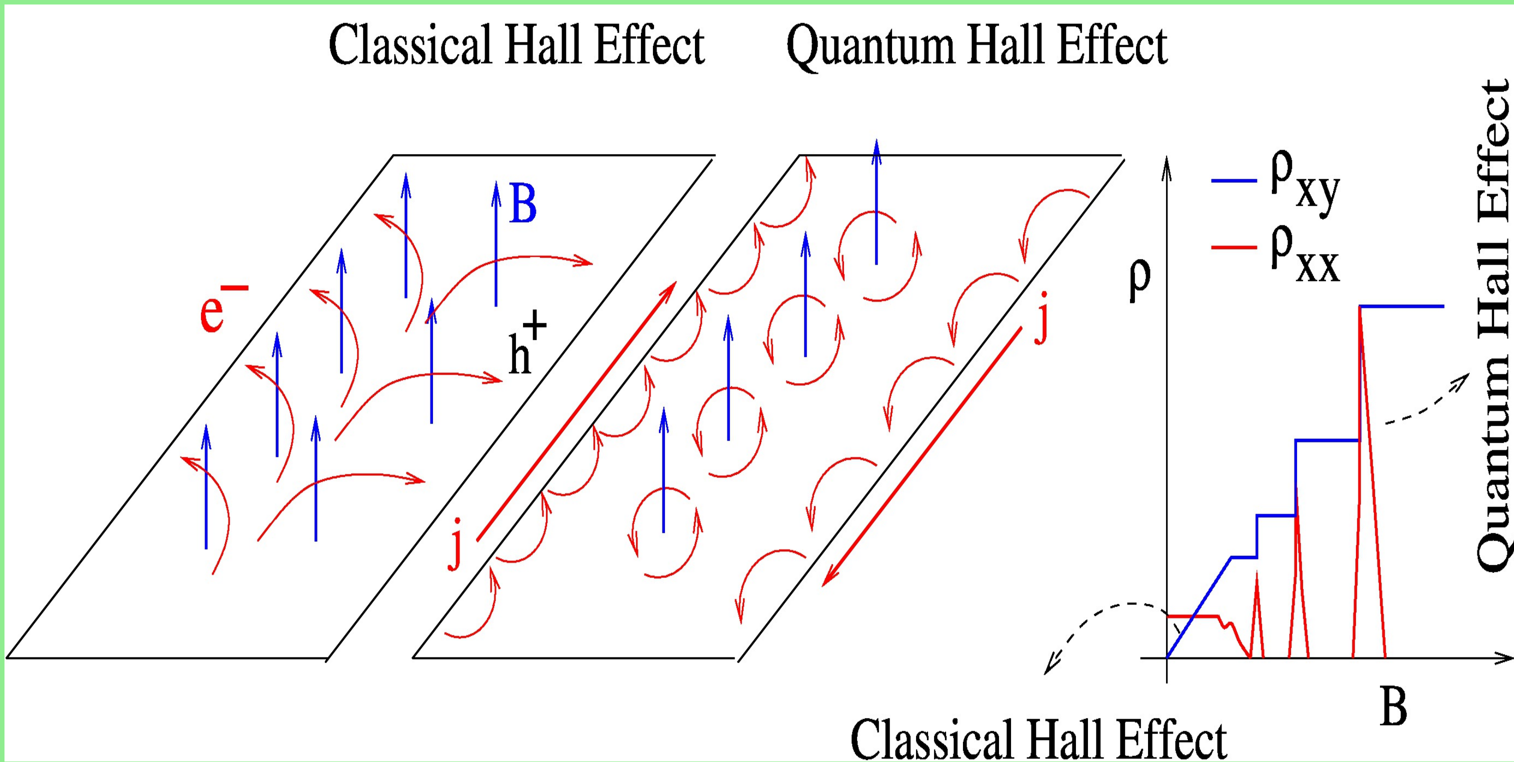
R. Bob Laughlin: Nobel prize 1998 (Fractional QHE)

The topological phase is interesting not only because of its topological protection: This is a new world, e.g.: fractional charge, fractional statistics, not boson nor fermion but "anyon" behavior, etc.

Topological Insulator

M.Z.Hassan, C.L.Kane, Rev.Mod.Phys. 82, 3045 (2010).

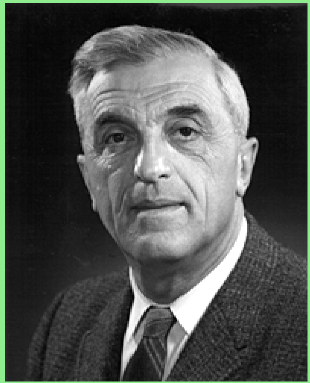
Topologically protected charge "edge" current:



Topological Bands

1928:

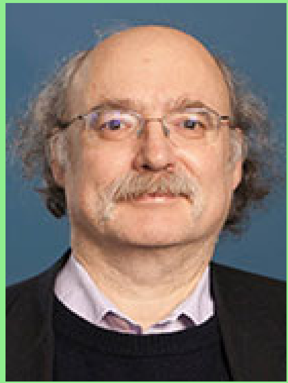
Band structure theory (PhD dissertation), 1952: Nobel Prize, First director of CERN (1954), Stanford University



Felix Bloch (1905-1983)

1983:

Novelties after 55 years: Topological Bands, Nobel Prize 2016, Princeton University



D.Haldane (B.1951)

The topology of the n-th band is given by $C_1(n)$:

The band: $\psi_n(\mathbf{k})$

Berry connection: $\mathbf{A}_{n,\mathbf{k}} = i\langle\psi_n(\mathbf{k})|\nabla_{\mathbf{k}}\psi_n(\mathbf{k})\rangle$

Berry curvature: $\mathbf{K}_{B,n} = \nabla_{\mathbf{k}} \times \mathbf{A}_{n,\mathbf{k}}$

Chern number: $\int_{B.Z.} \mathbf{K}_{B,n} \cdot d\mathbf{S}_{\mathbf{k}} = 2\pi C_1(n)$

Anomalous Quantum Hall Effect

Quantum Hall Effect without external magnetic field

F. D. M. Haldane, Phys. Rev. Lett. 61, 2015 (1983).

Chern insulator:

(Integer) QH Effect without external field (flux).

Time reversal symmetry must be broken, and we must have an active band with non-trivial Chern number. The Haldane's example: graphene with next nearest neighbor hopping and staggered magnetic field.

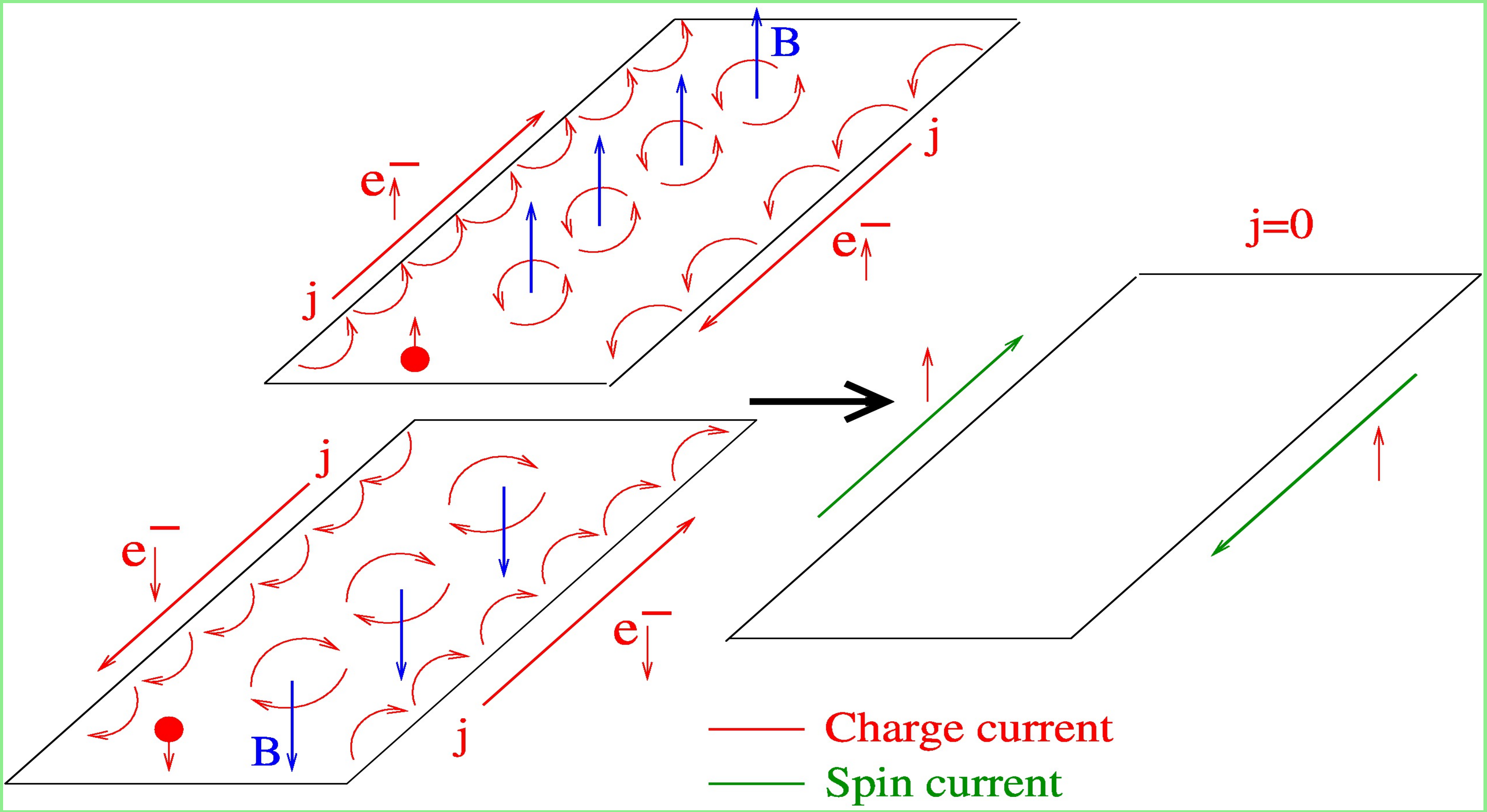
Experimental proof in 2013: $(Bi, Sb)_2Te_3$ containing Cr,

C. Z. Chang, J. Zhang, X. Feng et al., Science 340, 167 (2013).

Spin Quantum Hall Effect

C.L.Kane, E.J.Mele, Phys.Rev.Lett. 95, 226801 (2005).

Topological protected spin "edge" current:



The idea

F. D. M. Haldane, Phys. Rev. Lett. 61, 2015 (1988).

C. L. Kane
(B : 1963)

E. J. Mele
(B : 1957)

D. Haldane
(B : 1951)

C.L.Kane, E.J.Mele, Phys.Rev.Lett. 95, 226801 (2005).

Topology connected to the Hamilton operator

1985:

(α, β) arbitrary parameters, for which the system is periodic.

D.Thouless
(B: 1934)

1883:

Floquet theory: Periodically excited system: Bloch low for t.

G. Floquet
(1847-1920)

Topology on arbitrary (α, β) manifold

Q.Niu, D.J.Thouless, Y.S.Wu, Phys.Rev.B31, 3372 (1985)

If $\hat{H}$ is adiabatically moved along a closed curve of an arbitrary manifold, the same topology is reobtained.

Consequences: a) Topology connected to $\hat{H}$,
b) Topology e.g. on (x, t) surface.

Technological application possibilities:

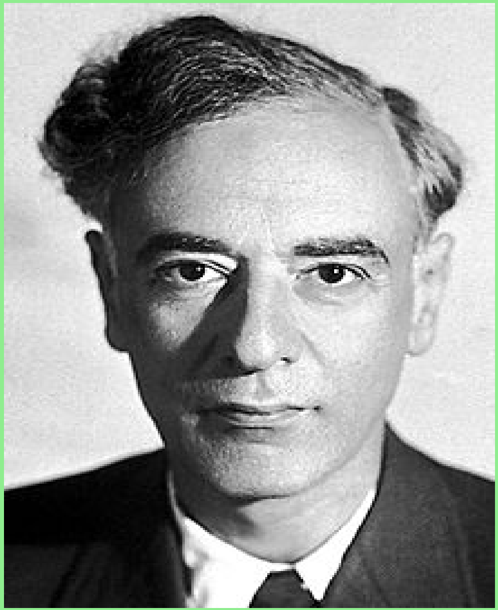
- Spintronics: topologically protected spin current

T. Jungwirth et al. “Spin Hall effect devices”, Nature Materials 11, 382 (2012)

- Quantum computation with topologically protected states

C.Nayak et al: “Non-abelian anyons and topological quantum computation”, Rev.Mod.Phys.80,1083 (2008)

Instead of Summary: The main message:



Lev Davidovich Landau
1908-1968, Nobel:1962

$\triangle$ = broken symmetry

We thought that in understanding qualitative changes in many-body systems the most important notion is $\triangle$. Now, we realize that first the topology matters, only afterwords the symmetry.

1. In case of broken symmetry the energy gap provide the defence. If this is not enough, then

2. Topological phase provide the defence: this is much stronger since is topologically protected.

University of Debrecen:

Acknowledgements

Important role in prepairing the slides:

Kertészné Molnár Zsuzsa

Gragya Józsefné

University of Debreceni:

Main Building Internal Yard

University Library

THANK YOU FOR YOUR ATTENTION !