

## Part IV.

Ferromagnetism created by conduction electrons (itinerant ferromagnetism).

# 1.) Excitonic ferromagnetism

Was presented before as CDW + SDW coexistence.

# 2.) Stoner ferromagnetism

If we analyze the two particle interaction, we observe that in the case of the density waves, the two particle interaction strength has its major contribution from the momentum domain placed at the  $q = Q \neq 0$  transferred  $k$  value, where  $Q$  is the nesting vector. In a such situation the scattering amplitude  $f$  proportional to the vertex function  $\Gamma$  diverges at  $q \rightarrow Q$  at the transition to the density wave phase.

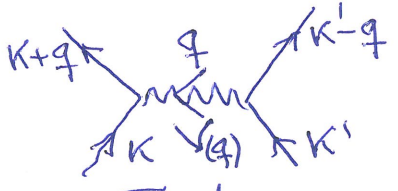


Fig 1

The transferred momentum at 2-particles interaction.

But it happens that the interaction potential  $V(q)$  collects its major contribution from the  $q \rightarrow 0$  region.

In this situation  $f$  and  $\Gamma$  diverges at  $q \rightarrow 0$ , and, from the point of view of the scattering, the forward scattering becomes important. A such case signals the instability of the Fermi liquid (only one-particle excitations are present) to ferromagnetism. This ferromagnetism is again created by itinerant electrons, but it emerges in the case when the  $q \rightarrow 0$  region dominates  $V(q)$ . However is not enough to have  $V(q)$  peaked at  $q \rightarrow 0$ . It is necessary to have many electrons present at Fermi level. In other words, the density of states must be high at  $E_F$ . For this usually a narrow band around  $E_F$  is needed and this often is provided by the 3d band (see Fig 2). Besides this, a repulsive interaction between  $\uparrow$  and  $\downarrow$  spin electrons is needed to be present. This can be e.g. an exchange interaction ( $-J \vec{S}_i \vec{S}_j$ ) or the local part of the Coulomb interaction (called Hubbard term:  $U n_{\uparrow} n_{\downarrow}$ ; given by the Pauli principle, only electrons with opposite spin can be on the same place) This last point (the exchange interaction) was mentioned because Stoner in its original paper invoked it. The  $U$  term appears in strongly correlated systems and has the same effect as  $J$  from the point of view of the repulsion introduced between opposite spin projections.

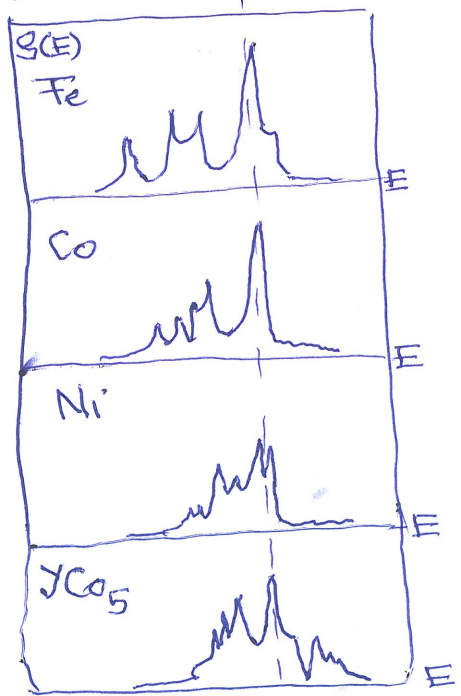


Fig 2. Density of state for some ferromagnetic metals.

But what is the connection between  $V(q \rightarrow 0)$  and  $J$ ? First of all  $V$  is the Coulomb repulsion between electrons, so particles with the same charge.

But these particles are fermions so behave quantum mechanically. Hence exchange of particles must lead to a sign change in the wave function, which leads to the exchange interaction. But during particle exchange we not transfer momentum. Hence the exchange interaction is in fact at quantum mechanical level a main contribution in  $V(q \rightarrow 0)$ . But given by Pauli principle (which is a consequence of the sign change in  $\psi$  at the particle exchange) the exchange interaction gains a spin-dependent form, which will provide ferromagnetism. Please note, that this ferromagnetism is provided by the Coulomb repulsion + Pauli principle.

Now the question is: how this type of ferromagnetism appears? First: in order the interaction to be effective (no  $\kappa$  transfer is present), a high number of electrons with the same  $\kappa = \kappa_F$  must be present (please note that the close neighborhood of  $E_F = \frac{\hbar^2 \kappa_F^2}{2m}$  matters). For this reason is necessary to have high density of states. Second: The repulsion between opposite spin projection increases the interaction energy, hence the total energy of the system. But we can decrease  $E_{int}$  (interaction energy) if we decrease the number of electrons holding a given spin projection (say  $\downarrow$ ). Indeed, if  $N_{\downarrow} = 0$ , there is no exchange interaction, so is not present increase in  $E_{int}$  given by  $J$ . But because  $N = N_{\uparrow} + N_{\downarrow}$  must remain constant, if  $N_{\downarrow}$  decreases, this is possible only if  $N_{\uparrow}$  increases, hence  $N_{\uparrow} > N_{\downarrow} \Rightarrow m = g\mu_B(N_{\uparrow} - N_{\downarrow}) \neq 0$ , hence ferromagnetism appears. But the problem is that if  $N_{\uparrow}$  increases, the kinetic energy increases since  $\uparrow$  states are occupied till  $E_F$ , empty states are present only above  $E_F$ .

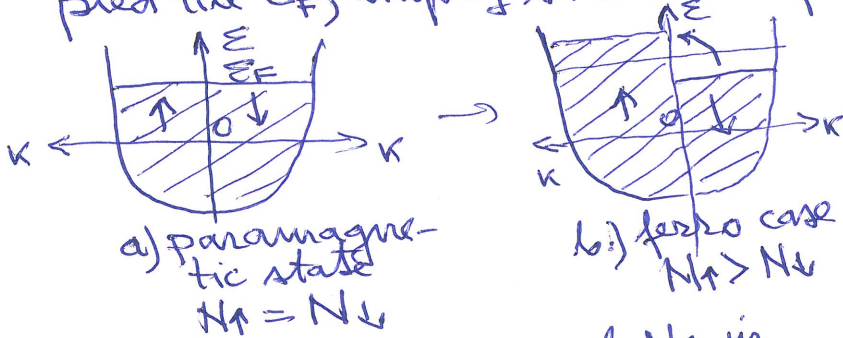


Fig 3: increase of  $N_{\uparrow}$  in detriment of  $N_{\downarrow}$

Fig 3. Note that the decrease in the kinetic energy for  $N_{\downarrow}$  is smaller than the increase for  $N_{\uparrow}$  because we are placed on different parts of the  $\epsilon = \epsilon(\kappa)$  parabola. Consequently, in the para state

$$E_P = E_{kin,P} + E_{int,P}$$

and  $N_{\uparrow} = N_{\downarrow}$ , but in the

ferro state  $E_F = E_{kin,F} + E_{int,F}$ , and  $N_{\uparrow} > N_{\downarrow}$ , but  $E_{kin,F} > E_{kin,P}$ ,  $E_{int,F} < E_{int,P}$ . In these conditions, the ferromagnetic state appears only for  $E_F < E_P$  (at  $T=0$ ). In conclusion, the ferromagnetic state only appears if the increase in the kinetic energy is less than the decrease in the interaction energy, i.e.:

$$E_{kin,F} - E_{kin,P} < E_{int,P} - E_{int,F}$$

Now this inequality can be analysed in different approximations

For rigid bands, and mean-field approximation one finds -3-  
ferromagnetism for

$$V(q=0) S(\epsilon_F) > 1, \text{ where } S(\epsilon_F) \text{ is the density of states at Fermi level.}$$

The obtained condition is named "Stoner criterion. For stronger (better) approximations one finds  $V(q=0) S(\epsilon_F) > \frac{3}{2} > 1$ .  $T_c$  in many cases is around 100K, but Ni ( $T_c = 628K$ ), and Fe ( $1044K = T_c$ ).

Concerning the excitations: the Stoner excitations look as follows: The original band will be exchange split in majority and minority bands (Fig 1a)

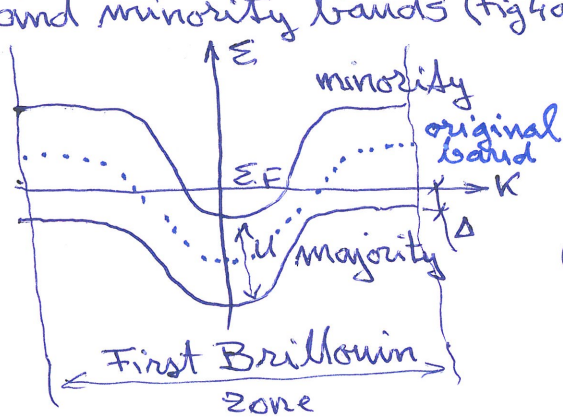


Fig 1a: exchange split bands

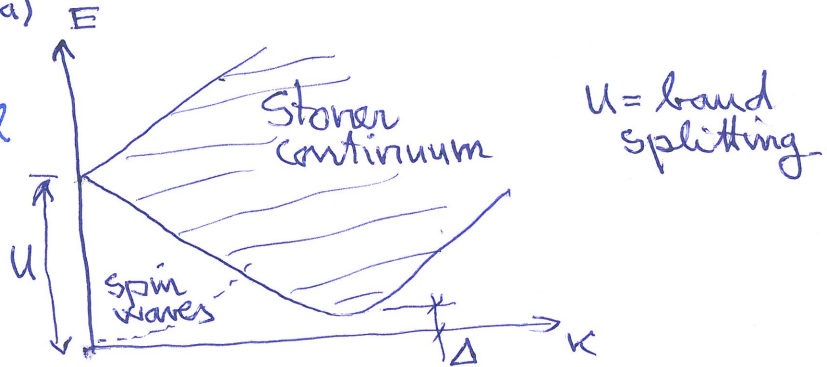


Fig 1b: The Stoner excitations

Now the Stoner excitations appear if one electron from the majority band (completely below  $\epsilon_F$ ) is excited in an empty state of the minority band. This excitation spectrum is continuous, and has gap ( $\Delta$ ).  $\Delta$  is the minimum distance between the majority band and the Fermi energy  $\epsilon_F$ .

But the Goldstone mode (acoustic spin waves: dotted line in Fig 1b) exists, this is gapless, but this is not able to destroy the ordered phase (it acts only in majority band).

### 3 Flat band ferromagnetism:

Let us have a completely flat band which is half filled, and which satisfies the connectivity condition. This situation produces a saturated itinerant ferromagnetism. The reason for this is as follows: If the flat band is half filled, the number of electrons is equal to the number of lattice sites, and equal to the number of  $k$  states in the first Brillouin zone (containing all possible  $k$  states). Now the Coulomb interaction and Pauli principle acts between the electrons which produces the Hubbard interaction (local repulsion between electrons with opposite spins).

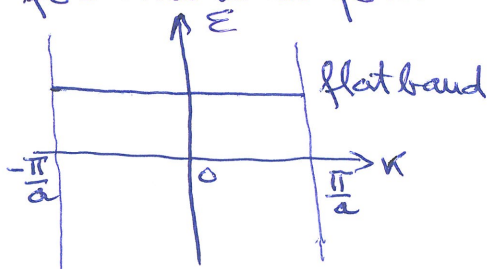


Fig 5: Flat band.

Now the Coulomb interaction and Pauli principle acts between the electrons which produces the Hubbard interaction (local repulsion between electrons with opposite spins).

Since the number of electrons it is equal to the number of sites, it is possible in principle to have on each site 1 electron with arbitrary spin (Fig 6a). But it is also possible to have

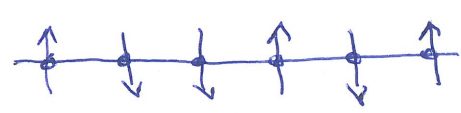


Fig 6a: One electron on each site

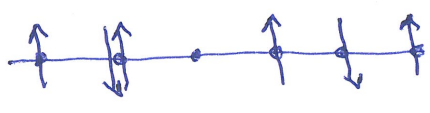


Fig 6b: Presence of double occupancy and empty site

double occupancy, and than empty sites present (see Fig 6b.).

But if double occupancy appears, the Hubbard inte-

raction, namely  $+U n_i n_{i+1}$ , increases the interaction energy. Hence the system moves electrons from double occupied sites to empty sites in order to decrease the interaction energy. This movement for a given electron in a given  $k$  state, means a movement in another state  $k'$ , but the kinetic energy is not changed because the band is flat. Consequently, in an arbitrary moment  $t$ , the situation from Fig. 6a will be preferred, energetically. Now, the following question arises: How we get ferromagnetism from the configuration presented in Fig 6a? The answer to this condition is simple: by the connectivity condition.

The connectivity condition means that the Wannier states of the neighboring electrons superpose (have common sites). Let me give an example taking a pentagon chain (polyaminotriazole),

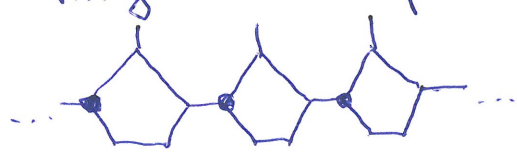


Fig 7a: Pentagon chain. Big dots are the lattice sites



Fig 7b: The Wannier state of an electron present at the indicated lattice site

see Fig 7a, and the Wannier state of one electron in this system, see Fig 7b. Consequently, if we put two electrons in

nearest neighbor lattice sites position, their Wannier states have common sites, and on these sites, if the spins are opposite, the Hubbard interaction will act increasing the interaction energy. This mechanism will align the nearest neighbor electron spins in the same direction, hence ferromagnetism arises.

(See e.g.  $\text{Sn}_2\text{Nb}_2\text{O}_7$  in Phys. Rev. Lett 120, 196401 (2018)) This Material is pyrochlore oxide, whose magnetic moment comes from Sn-s and O-p orbitals which are usually non-magnetic.

The flat band ferromagnetism gives the possibility to find ferromagnetism from completely non-magnetic elements. In this manner we can have ferromagnetism even in polymers, organic materials build up from H,O,C,N (see e.g. the review paper: Int. Jour. Mod. Phys. B27(4), 1330009 (2013), 84 pages.)

Note that flat band ferromagnetism appear even at  $U \rightarrow 0$  (but  $U > 0$ , arbitrary small). So interaction strenght threshold is not present for flat band ferromagnetism. Similar situation one has for density waves and also superconductivity, but given by the Stoner condition, not in the Stoner ferromagnetism case.

In conclusion the flat-band ferromagnetism is given by the itinerant electron system in a half filled flat band which satisfies the connectivity condition and has an arbitrary small Coulomb repulsion. The system manifests quantum mechanically, such that the local part of the Coulomb repulsion, given by the presence of the Pauli principle give rise to the Hubbard interaction.

One must notes that the flat-band is not a Fermi liquid. This is because in  $k$ -space the Fermi surface cannot be defined (but the Fermi energy can be), consequently, the Luttinger theorem (according to which, the volume enclosed by the Fermi surface remains the same when the interaction is turned on) not holds. So the Stoner ferro-magnetism appears only in Fermi liquids, but the flat-band ferro-magnetism emerges outside of the Fermi liquids, and as was mentio-ned previously, not needs magnetic atoms.

Today an intensive research exists for the manufacturing of flat bands. For example by doping Pyrochlore oxides (PRL 120, 198401 (2018)), introduction of side groups in organic conducting polymers (PRL 88, 127202 (2002)) job which performed also at the le-vel of computer simulations (PR B68, 174419 (2003)). One notes that effective flat bands can be produced also by interaction (Eur. Phys. Jour B87, 143 (2014)), dynamic band structure engineering (arXiv:1902.10287).

#### 4.) Half-metal ferromagnetism:

The half-metal phase is a special phase in some materials which has the property that the subband with a given spin projection (say  $\downarrow$ ) is completely localized, and the subband with the opposite spin projection ( $\uparrow$ ) is conducting. A possible spin-subband picture of the half-metal is presented in Fig. 8. Such materials have a special importance today in spintronics because they produce also spin-currents (Spintronics aims to produce electronic devices that are based on spin currents instead of charge currents (see e.g. PRL 122, 077203 (2019))). As seen in Fig 8, one has a gap at the Fermi level for spin  $\downarrow$ , but the spin  $\uparrow$  band has no gap, hence is conducting.

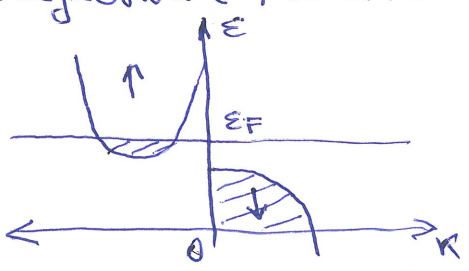


Fig 8: The spin-subband picture of the half-metal.

A classical example for this is  $\text{CrO}_2$  (PRL 121, 257201 (2018)), the first observed material with such properties is the Heusler alloy system (PRL 50, 2024 (1983)), and today an intensive research activity exists in the attempt to discover new half-metallic systems and study their properties: as examples I mention:

Twisted bilayer graphene (arXiv: 1902.10287), the spin gapless semi-conductors (Phys. Met. Metallography 19, 1321 (2018)), hole doped  $\text{TeS}_2$  layers (arXiv: 1901.09147), metals presenting Dirac points as graphene e.g.  $\text{EuCd}_2\text{As}_2$  (arXiv: 1901.08234), or  $\text{Cr}_2\text{CoAl}$  (PR Materials 3, 021201 (2019)), materials holding topological properties as  $\text{Li}_3(\text{FeO}_3)_2$  (arXiv: 1811.05254), Mn-V alloys (arXiv: 1811.12600) as for example  $\text{Mn}_2\text{VAl}$ , thin films as  $\text{Mn}_2\text{Ru}_x\text{Ga}$  (PRB 98, 220406 (2018)), rare-earth compounds as  $\text{Pr}_{0.95}\text{Mn}_{0.05}\text{O}_3$  (arXiv: 1809.10350), ribbons as H-saturated  $\text{InSe}$  (PR Materials 2, 114001 (2018)), monolayers with high mechanical flexibility (arXiv: 1805.07538), etc.

It is important to mention to not confound the half-metal notion with semi-metal notion. Semimetals as Bi have 2 active bands at the Fermi level as shown in Fig 9, bands 1 and 2, but both bands contain

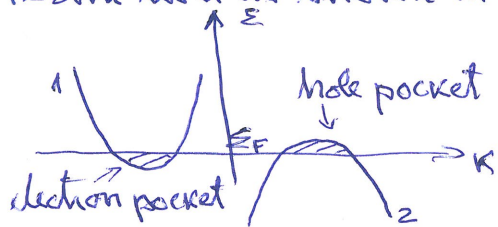


Fig 9. Band structure of the semi-metal

both spin projection. The characteristic of semi-metals is that they present in the same time both electron and hole type of excitations. Other semimetals are As, or Sb (Antimony) have a temperature independent carrier density below the room temperature. In Bi this is true only at very low temperatures. It must be noted that the bands 1 and 2 intersect the Fermi level at different  $k$  values, and because of this reason often is said that semimetals have a "negative indirect bandgap". Indirect band gap often appears e.g. in  $\text{AlSb}$  or  $\text{Si}$ ,  $\text{Ge}$  where the maximum of the completely filled valence band and the minimum of the completely empty conduction band (these are  $T=0$  data) are placed at different  $k$  values (see Fig 10), and the transition between them is going on phonon-assisted (since at the transition the electron must change its momentum which is given usually by the emission of a phonon).

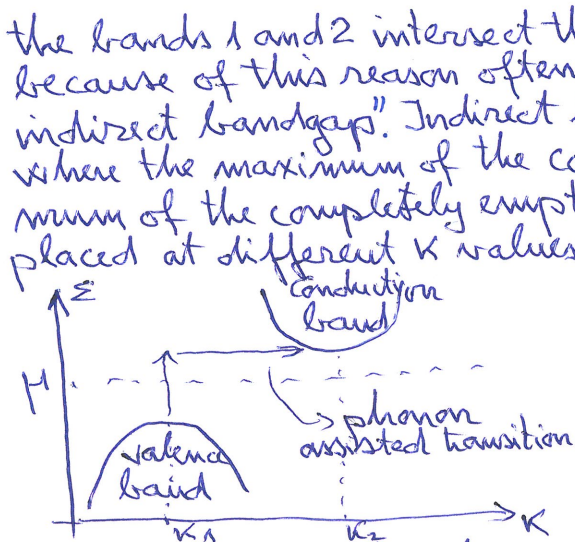


Fig 10: indirect band gap  
Note that at  $T=0$ ,  $\mu = \epsilon_F$

Direct bandgap ( $k_1 = k_2$  in Fig 10) appears e.g. in  $\text{InAs}$  or  $\text{GaAs}$ . If a direct bandgap is crossed by an excited electron it emits a photon (at deexcitation).

### How can one reach half-metal properties?

1) It is possible to obtain half-metal properties by doping above half filling a flat-band which had connectivity condition, hence gave rise at half filling to ferromagnetism. Such band has usually localized electrons at each site, which are spin-polarized (↑). It happens that the supplementary introduced electrons become itinerant, and in order to maintain the minimum energy state their spin projection becomes opposite (↓). This is necessary, because given by the Pauli principle, there are not present states with spin ↑. See PRL 105, 266403 (2010).

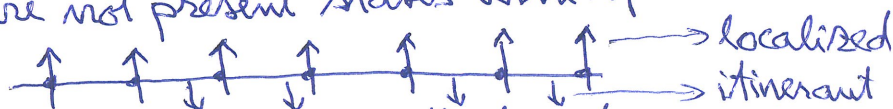


Fig 11: Half-metal with doped flat band

2) Another possibility is to take a semimetal and to dope it. -7-  
 The rigid band model not always hold, by doping it happens that the negative indirect bandgap becomes direct bandgap.

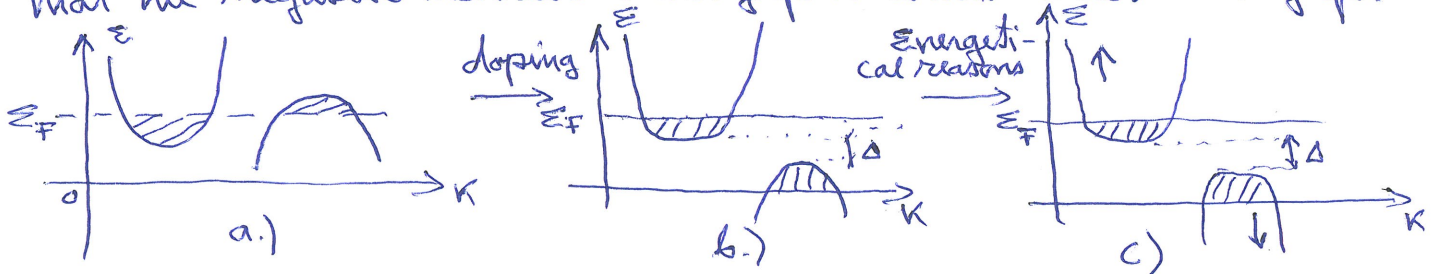


Fig 12: Doping a semimetal: a) the semimetal, b) doped semimetal leading to indirect bandgap, c) half-metal obtained by energetic reasons.

Furthermore it happens that from energetical reason, in order to minimize the energy, the two obtained bands with positive indirect bandgap become spin-polarized, providing in this case ferromagnetism. Often, in this last process, the many-body spin-orbit interaction plays a role since it breaks the double spin-projection degeneracy of each band (see e.g. PRB 97, 075118 (2018); PR Materials 2, 114001 (2018)).

3) Other techniques: from the point 2) we have seen that in order to obtain half-metal properties a) we have to have a procedure which produces a relative movement of the bands near the Fermi level and b) we have to have a procedure which acts differentially on different spin-projection species: e.g. to couple in many-body spin orbit interaction. As presented in arXiv: 1902.10287 for the case of the Bilayer Graphene, for a) a twisting of the system is necessary and for b) an application of an external electric field perpendicular to the graphene layers is necessary which produces the potential gradient at the surface which couples in the spin-orbit many-body coupling ( $\sim \vec{\sigma} \cdot (\nabla \times \vec{k})$ ). As mentioned in the mentioned manuscript, this procedure leads to half-metallic state in bilayer-graphene.

### 5) Ferromagnetism from spin gapless semiconductors.

According to the band structure theory till now the following phases were mentioned:

A) If only one band is active around  $E_F$ : metal (Fig 13a) and insulator (Fig 13b).

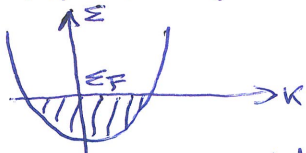
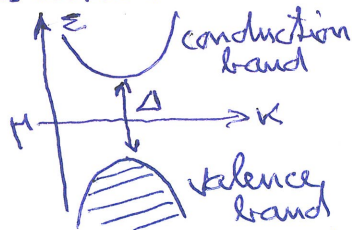


Fig 13: a.) Metal

Band structure phases with one active band



b.) Insulator ( $\Delta \neq 0$ )  
 called semiconductor if  $\Delta < 4\text{eV}$

This two phases differ by  $\Delta$  at  $E_F$ :  $\Delta = 0$  for metals, and  $\Delta \neq 0$  for insulators (note that semiconductor is an insulator for small  $\Delta < 4\text{eV}$  which behaves as an insulator at  $T=0$ ).

B) If two bands are active around  $E_F$  two more phases are mentioned, namely semimetal (when the two bands hold both spin species) and half-metal (when the two bands are spin subbands) See Fig 14.



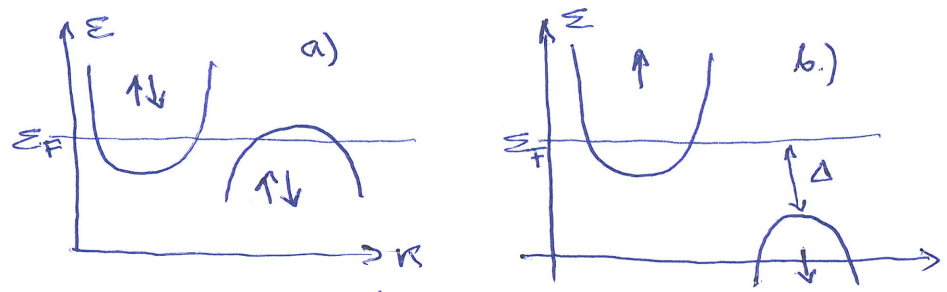


Fig 14: Phases for two active bands at  $E_F$   
 a) Semimetal, b) Half metal

Note that for Fig 14a both bands hold both spin-projection species, while for Fig 14b the two bands hold two different spin projection species. But, it turns out that for two active bands

around  $E_F$  another possible phase exists, namely the spin-gapless semiconductor. It has been theoretically suggested by X.L. Wang (Australia) in 2008. It looks as follows: First let us introduce the notion of gapless non-metallic system (Fig 15) (PRL 100, 156404 (2008)). This is a system for which the valence band and the conduction band touch each other in an unique point at Fermi level. Such systems are known from nineties (90'), and they cannot be called metals (even if  $\Delta=0$ ) because they have zero density of states at

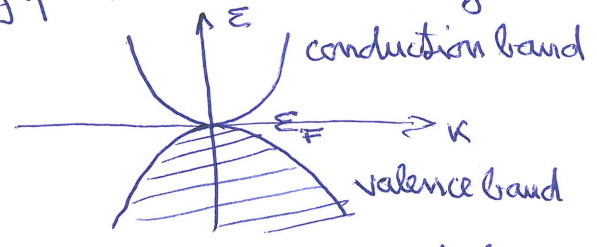


Fig 15: Gapless non-metal

the Fermi level. But at  $T > 0$  by excitation, electrons appear in the conduction band, hence their behavior resembles to the behavior of semiconductors, consequently were named gapless semiconductors. But in these materials is not present threshold energy to move electrons to the conduction band, so they have 2 to 4 orders of magnitude higher mobility than the mobility of classical semiconductors and, given by their band structure are extremely sensitive to external influences e.g. pressure, or magnetic field. Such compounds have been known to be rare, in the past have been only found in Hg based compounds like  $HgCdTe$ ,  $HgCdSe$ ,  $HgZnSe$ , which are extremely toxic, and easily oxidize, hence their application was limited.

What X.L. Wang has done is that he suggested that starting from gapless semiconductors (here in the bands both spin projections are present), by acting on one spin projection only (say  $\downarrow$ ) we can introduce a bandgap only for spin  $\downarrow$ , e.g. see Fig 16. In the presented case in Fig 16, the density of states at  $E_F$  is still zero, but gap appears for spin  $\downarrow$  and the  $\uparrow$  spins remain gapless. From here results the name of these systems: spin-gapless semiconductors, underlining that gap is present only for one given spin-projection. A such system at  $T > 0$  (and small  $T$ , such that  $k_B T < \Delta_1$ ) presents only spin  $\uparrow$  currents, and for  $\Delta_1 < k_B T < \Delta$  besides the 100% spin polarized

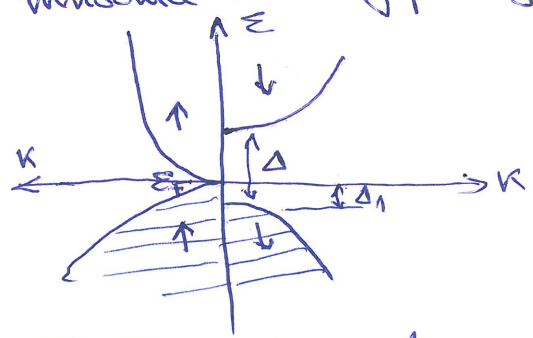


Fig 16: spin-gapless semiconductor

current becomes also ferromagnetic. (note that  $\Delta_1$  can also be zero). The material studied by X.L. Wang was  $PbPdO_2$ , and the action on one spin projection was given by doping with magnetic ions. Since then,

several other systems have been found with similar properties, -9-  
 e.g.  $Mn_2CoAl$ , (J. Phys. D: Appl. Phys. 41, 055010 (2008)) with  $T_c = 720K$  and  
 ground state magnetic moment per formula unit of  $2\mu_B$ ,  $Gd_3Al$  (Appl. Phys. Lett. 103,  
 232105 (2013))  
 It is interesting to note that the structure from Fig 15 has also graphene  
 but not with parabolic dispersion relation, but with  $E \propto k$  (situation at  
 half filling). Consequently also graphene-like structures are analysed  
 from this point of view.

## 6) Nanomagnetism.

Also nanomagnetism is a new chapter in the theory of magnetism which started to be observed when one reaches the nanoscale (the term "nanotechnology" has been introduced by N. Taniguchi in 1974, the notion has been popularised by K.E. Drexler in a book written in 1986, and the first big scientific recognition appears in 1996 when the Nobel Prize in Chemistry has been awarded for fullerenes ("buckyball",  $C_{60}$ ).

During these discoveries has been realized in fact that "what is small, is different", is governed by different laws and properties. From the point of view of the magnetism, at first view seems to mean that because of the nanoscale ( $10^{-9}m$ ) magnetic domains cannot be formed because there are higher than the sample itself. But in fact the reality is much deeper in essence. In order to realize this, let me present to you an example:

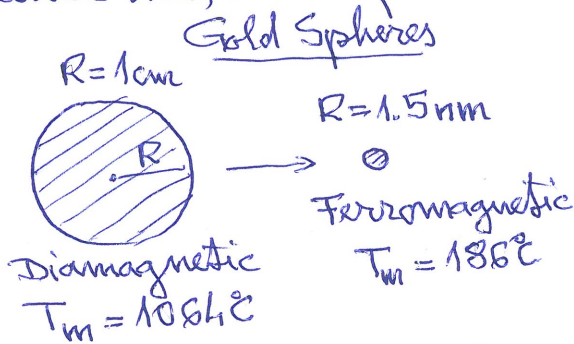


Fig 17: Gold (Au) Spheres

Let us take a gold sphere. As known from different tables presenting material properties, one has the melting temperature  $T_m = 1337K$  and from magnetic point of view is diamagnetic. Now if we start to decrease the diameter ( $D = 2R$ ) of the sphere, at  $D = 3nm$ , we have the melting temperature at  $T_m = 459K$  and the sphere is ferromagnetic.

(see PRL 93, 087204 (2004)). Similar properties have been observed in Pd which as a macroscopic metal ( $R = 10nm$ ) is paramagnetic, but at  $R = 1.2nm$  is ferromagnetic, with  $T_c$  (Curie temperature) above  $300K$ . (PRL 91, 237203 (2003)); The melting temperature at macroscopic size ( $R = 10nm$ ) is  $T_m = 1828K$ ; while for  $R = 1.2nm$ ,  $T_m \sim 900K$ .

The nature has been discovered much more before us that the nano world has different specific laws. E.g. bacteria have  $\mu m$  ( $10^{-6}m$ ) size, but they can be destroyed by the penicilline produced by fungi (some fungi). Viruses have  $nm$  size, and antibiotics (like penicilline) not act on them. Furthermore, the fingers of the Gecko animal have  $nm$  size wrinkles, and this is the reason why Gecko is able to walk on a vertical perfectly flat glass. Also the butterfly wings have a given nanostructure on their surface which provide that broad colour effect that the butterfly wings they have; etc. We humans, only now we start to discover and understand the nano world.

The most important characteristics of the nano-world are connected -10- to: a.) The surface gains huge importance. If we take a constant volume, the surface is minimum in the case of the sphere. Even calculating with this minimum surface, the surface/volume ratio strongly increases with  $r \rightarrow 0$ . Hence, for a macroscopic body the volume (consequently the mass) is important. Contrary to this in nano-world (if  $r \sim 10^{-9}$ , one has  $\frac{S}{V} \sim 10^9$ ) the surface is important.

$$\frac{S}{V} = \frac{4\pi r^2}{\frac{4\pi r^3}{3}} = \frac{3}{r}$$

b.) Since the size is around  $nm = 10^{-9} m = 10 \text{ \AA}$ , the quantum mechanics becomes important in providing the main behavior. I note that the fact that for the future we must concentrate more on quantum mechanics is suggested also by the electronic industry (computers we need everywhere). Indeed, the Moore's law (observed in industry), originating from Gordon E. Moore in 1965, states that the number of transistors in a dense integrated circuit, doubles (becomes 2 times higher) in  $t_0 = 2$  years. And these dense integrated circuits are the computer chips in computers everywhere. It is true that initially was 18 months, and today often is claimed that  $t_0 \sim 2.5$  years, but in 1990 one transistor had the size of 800 nm, and today has a size of 14 nm in computer chips. Soon one reaches the atomic scale ( $\text{Å}$ ) where everything is governed by quantum mechanics (and one should think about quantum computing as well).

In conclusions: surface, and quantum mechanics become important for nano-magnetism. On this line I mention that a quantum treatment of small closed surface with itinerant electrons, and (local) Coulomb interaction was able to explain the ferromagnetism observed on nanoscale metallic spheres (IOP Conf. Series 47, 012048 (2013)).

Since one arrives to underline the importance of quantum mechanics e.g. on surfaces for nano-world, I mention that the same technique is important as well when the studied system, at least in one direction, has nanoscale extent, e.g. in the case of interfaces. In such situation also, extremely interesting phenomena occur. E.g., concentrating on ferromagnetism, it happens that two non-magnetic materials in contact provide a magnetic interface (e.g. Nature Materials 6, 493 (2007)). And more than this the two materials can be insulating, and the interface conducting and ferromagnetic (in the above presented paper as well). E.g. non-magnetic insulating perovskites  $SrTiO_3$  and  $LaAlO_3$  in contact provide such properties.

In explaining such a behavior one takes into account that on interfaces, perpendicular to the contact surface, strong potential gradients appear, which enhance the many-body spin-orbit interaction. Since this breaks the double spin projection degeneracy of each band, can produce ferromagnetism on an itinerant surface (Phil. Mag. 98, 1708 (2018)).

Concerning nanomagnetism, an intensive research is present in this direction: H adsorbed at Si surface (and nano size) PRB 98, 195124 (2018); nanomagnets use in computer technology: Nanotechnol. 44, 442001 (2018); surface effects in nanomagnets PR 98, 165444 (2018); Topologically protected superconductors generated by spin-ice nanomagnets Nanotechnol. 30, 244003 (2019); quasicrystalline nanomagnets PRB 98, 174408 (2018); spin-ladder quantum nanomagnets JMM Mater 469, 411 (2019); magnetostatic nanomagnets J. of Phys C30, 394001 (2018); Magnetic order and disorder in nanomagnets Sci. Rep. 8, 12374 (2018); Topological Nanowires in Si with nanomagnet arrays PR Applied Phys 10, 054071 (2018), etc.

### Atomic scale magnetism:

2016: individual Ho atoms presenting remanence Science 352, 318 (2016); Here the remanent magnetization remains present up to 30K. It appears because the non-magnetic substrate act such that the atom avoids spin relaxation. This case seems to be the ultimate size limit of magnetic information storage.

Before this event single-molecule magnets have been known as e.g. JACS Commun 127, 3650 (2005) where JACS = J. Amer. Chem. Soc. The molecule here studied was phthalocyanin (Pc) containing 1 Ho atom ( $(Pc)_2Ho$ ). The molecule presents hysteresis loop containing staircase like magnetization. Here the diminished relaxation of spins appear because a strong entanglement in between the electronic and nuclear spin systems. The staircase shape of the hysteresis down and upturns appears because resonant quantum tunneling in between the mentioned entangled states.

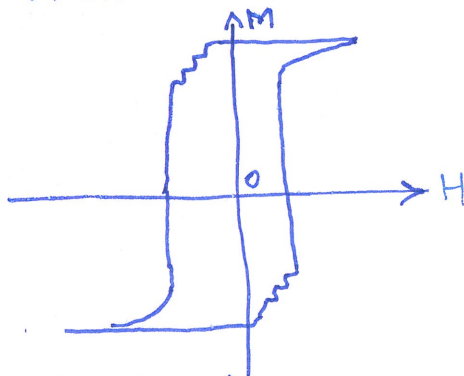


Fig 18: Staircase hysteresis

The single molecule magnetism has been discovered in 1993 (see the review MRS Bull. (Nov. 2000), pg. 66 (2000), vol 25. All members present staircase hysteresis (see Fig 18). The first described case has  $Mn_{12}O_{17}$  core with  $S=10$  on which an axial anisotropy is acting breaking the  $2S+1$  degeneracy of the  $S^z$  orientation hence providing a magnetic state. SMM = single molecule magnetism.