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The effect of network topologies on the spreading of technological developments

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Abstract. We study an agent-based model, as a special type of opinion dynamics, of the spreading of innovations in socio-economic systems varying the topology of agents' social contacts. The agents are organized on a square lattice where the connections are rewired with a certain probability. We show that the degree polydispersity and long range connections of agents can facilitate, but can also hinder the spreading of new technologies, depending on the amount of advantages provided by the innovation. We determine the critical fraction of innovative agents required to initiate spreading and to obtain a significant technological progress. As the fraction of innovative agents approaches the critical value, the spreading process slows down analogously to the critical slowing down observed at continuous phase transitions. The characteristic timescale at the critical point proved to have the same scaling as the average shortest path of the underlying social network. The model captures some relevant features of the spreading of innovations in telecommunication technologies.

Keywords: interacting agent models, socio-economic networks

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Contents

1. Introduction

Opinion dynamics of social systems leading to consensus has been intensively studied in recent years. Various types of models have been introduced which capture the role of social validation [1]–[6], bounded confidence [2, 7, 8], the presence of extreme opinions [9, 10, 8, 13], the formation of spatial structures of different opinions (parties) [8], and the transition from unity to discord [8]. Opinion dynamic models have also been studied on different network topologies of social contacts giving rise to a broad spectrum of novel results [17, 19, 20].

It has long been recognized that opinion dynamics and consensus formation play also a crucial role in the spreading of certain technological developments in socio-economic systems [14, 15]. The technological evolution has two major components: (i) *innovation*: new products, ideas, paradigms emerge as a result of innovations which are then tested by the market; (ii) *spreading*: under certain conditions the new technologies get adopted and spread in the socio-economic system resulting in an overall technological progress [14]. The spreading of innovations is a very complex phenomenon which also involves decision making and consensus formation for groups of individuals, on the practical value of the technology. Recently, we have introduced an agent-based model for the spreading of technological developments in socio-economic systems where the technology is mainly used for the communication/interaction of agents [16]. In the model, agents use products of different technologies to communicate with each other which induce costs proportional to the difference of technological levels. The system evolves in such a way that agents change their technological level by adopting technologies of the interacting partners in order to reduce their costs. We studied the system by means of analytical calculations and computer simulations on a regular square lattice of agents and showed that the adoption–rejection decision, driven by the cost minimization, results in microscopic rearrangements of agents' technologies and can also lead to technological progress. Since costs are induced by the incompatibility of technologies, the cost minimizing adoption process gives rise to a 'consensus formation' of technological levels similar to opinion dynamics models [1]–[6], [11, 12]. However, the properties of the final homogeneous 'consensus' state are determined by the amount of advantages provided by the more advanced technologies [16].

In the present paper we study the effect of network topologies of agents' social contacts on the competition of products of different technological levels and on the spreading of new innovations. To make the model more realistic we consider networks of agents obtained by rewiring a square lattice. Analytic calculations and computer simulations revealed a broad spectrum of novel behaviors when the topology of social contacts was varied via the rewiring probability. We show that the degree polydispersity and long range connections of agents can facilitate, but can also hinder the spreading of new innovations, depending on the amount of advantages provided by the innovation. We determine the critical fraction of innovative agents required to initiate spreading and to obtain significant technological progress. As the fraction of innovative agents approaches the critical value, there is a slowing down of the spreading process analogous to the critical slowing down of continuous phase transitions. The relaxation time of the system at the critical point proved to have the same scaling as the average shortest path of the social network when changing the system size. The critical fraction of innovative agents of our model is analogous to the so-called *critical mass* effect which is a characteristic feature of those technologies whose practical value strongly depends on the number of agents already using it, like in the case of telecommunication technologies.

As a very important outcome of the work, we show that in the presence of solely two different technologies (opinions), our model behaves as a special type of consensus models with majority rules: in our case adoption occurs when the number of interacting partners, already using the advanced technology, exceeds a certain fraction of the entire community. The excess fraction is determined by the amount of advantages of the innovation.

2. Model

In the model we represent the socio-economic system by a set of agents which possess products of different technological levels and use it to cooperate with each other. The technological level of the product an agent has (the technological level of the device that the agent uses for communication) is described by a real variable τ such that a larger value of τ stands for more advanced technologies. The technology is used by the agents to cooperate/communicate with their social partners which is easiest if the partners have products of the same technological level. Using technologies of different levels can induce difficulties which may be realized by additional costs. It is reasonable to assume that the cost C induced by the communication of agents i and j is a monotonic function of the difference of the technological levels $|\tau_i - \tau_j|$. For the purpose of the explicit mathematical analysis we consider the simplest functional form and cast the cost of cooperation into the following form:

$$C(i \rightarrow j) = a|\tau_i - \tau_j|. \tag{1}$$

The equation expresses that being at different technological levels (having different τ values) incurs cost—the greater the difference in τ, the higher the costs—while being at the same technological level is cost-free. This crude assumption models a socio-economic system which favors the local communities being at the same technological

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level. In order to take into account that it is favorable for agents to use products of higher technological level than those of their interacting partners, we assume that the value of the multiplication factor a depends on the relative technological level of interacting agents as

$$a = \begin{cases} a_1, & \text{if } \tau_i > \tau_j \\ a_2, & \text{if } \tau_i < \tau_j \end{cases} \qquad \text{where } a_1 < a_2. \tag{2}$$

The condition $a_1 < a_2$ implies that using more advanced technologies than the surroundings, $\tau_i > \tau_j$, can lower the costs compared to the opposite case. Note that due to the condition of equation (2), the cost function is not symmetric with respect to agents i and j, which is expressed by the arrow $\rightarrow$ in the argument of C.

If agent i has n collaborating partners with technological levels $\tau_1, \tau_2, \ldots, \tau_n$, the total cost of its collaboration can be obtained by summing up the cost function, equation (1), over all connections:

$$C(i) = \sum_{j=1}^{n} C(i \rightarrow j). \tag{3}$$

The agents are assumed to be able to change their technologies in order to reduce their costs, which gives rise to a non-trivial time evolution of the system. We assume that agents do not invent new technologies; instead they can adopt/copy the technology of one of their interacting partners, the one which provides the minimization of the cost function $C(i)$. It is important to emphasize that there is no cost associated with the change of technological levels, i.e. the adoption of a new technology can freely be performed by agents if it provides future cost reduction. The rejection–adoption mechanism based on the local cost minimization results in the spreading of the adopted technologies while the rejected ones disappear from the system. Our model emphasizes that the crucial component of the spreading of innovations is the copying with the aim of ensuring compatibility and, hence, reduction of the difficulties (cost) of communication. Note that the cost is not associated with the usage of a specific technology; instead it characterizes the interaction of different level technologies since the technologies are used for communication. Hence, in the model there is no intrinsic advantage of using more advanced technologies; the cost is the same independently of the technological level when consensus has been reached. The time evolution of the model has been studied in [16] by means of cellular automata simulations where the agents were organized on a square lattice. The presence of providers through which technologies reach the agents can also be taken into account in the framework of the model [16].

The adoption of certain technologies and rejection of others can also be considered as a special case of opinion spreading, where decision making is not based on a simple majority of agents of the same opinion (technology), but involves a minimization of a real function. In the following we demonstrate that the underlying network topology of agents' social contacts has a substantial effect on the dynamics of the system.

3. Agents on complex networks

In order to obtain the complex network topology of agents' social contacts, we apply the Watts–Strogatz rewiring algorithm [18]. Starting from a square lattice of agents with

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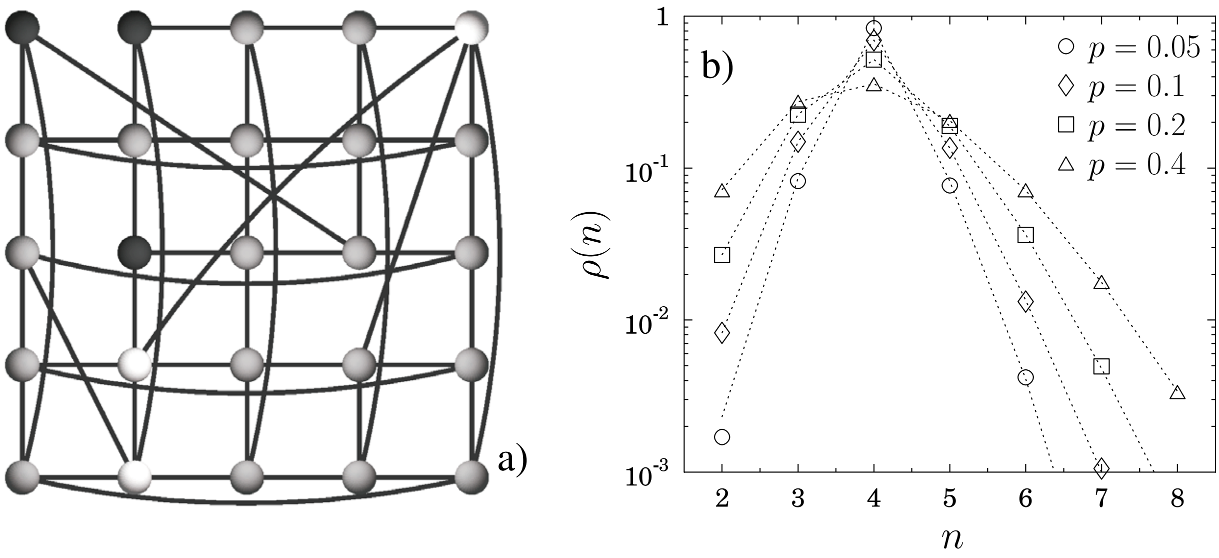

Figure 1. (a) Rewiring the square lattice. The grayscale of the nodes corresponds to the number of their connections (lighter gray stands for a higher degree). (b) Degree distribution $\rho(n)$ of the rewired square lattices for different rewiring probabilities p.

periodic boundary conditions, each node of the lattice is removed with a probability p and the connection is re-established between two agents selected randomly with a uniform distribution. The rewiring procedure is illustrated by figure 1(a). As a consequence of rewiring, degrees different from 4 occur in the social network with a certain probability and the topology of the system is changed from short ranged ($p = 0$) to random regular graphs ($p \to 1$) with long range connections [18]. The distribution ρ of the degree of agents n, i.e. the number of connections of the agents, can be determined analytically as the convolution of a binomial and a Poissonian distribution [17]:

$$\rho(n) = \sum_{s=0}^{\min(n-k,k)} \binom{k}{s} (1-p)^s p^{k-s} \frac{(pk)^{n-k-s}}{(n-k-s)!} \mathrm{e}^{-pk}, \tag{4}$$

where k denotes half of the average degree and n is the observed degree. The weights of the binomial and Poissonian components are in a linear relation to the rewiring probability p. Degree distributions of rewired lattices generated with the Watts–Strogatz method are illustrated in figure 1(b). It can be observed that on increasing the rewiring probability p, the distribution $\rho(n)$ gets broader, increasing the polydispersity of social contacts while the average degree remains constant, $\langle n \rangle = 4$.

Recently, we have shown that if an agent has n interacting partners with technological levels $\tau_1 < \tau_2 < \cdots < \tau_n$ the decision that it makes depends solely on the amount of advantages $r = a_2/a_1$ that more advanced technologies provide with respect to the lower level ones, i.e. the specific values of the parameters a_1 and a_2 of the cost function equation (3) do not have any significance [16]. Namely, the ith-highest technological level

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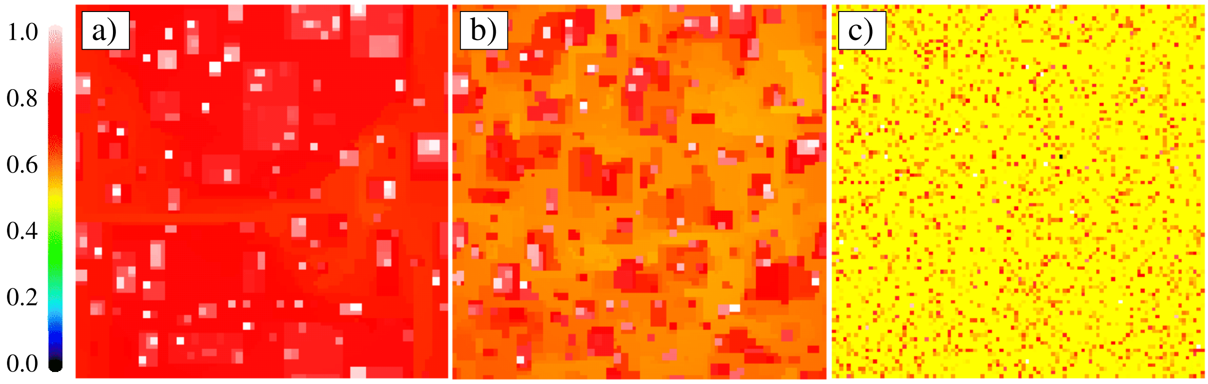

Figure 2. Final states of the system for different rewiring probabilities; (a) $p = 0$, regular square lattice, (b) $p = 0.05$, small world network, (c) $p = 1$, random regular graph. The color code shows that the average technological level of the system decreased with increasing connectivity. The value of the cost factor r was fixed to $r = 1.4$; compare also to figure 3.

is adopted by the agent when r falls in the interval

$$\begin{aligned} &\frac{i-1}{n-i+1} < r < \frac{i}{n-i} \qquad \text{for } 1 \leq i < n, \\ &n-1 < r \qquad \text{for } i = n. \end{aligned} \tag{5}$$

It can be seen that in the above equations the value of r does not determine the precise value of the selected technology, just its rank. Hence, when agents have randomly distributed technological levels in the initial state, even in a short range system ($p = 0$) a complex time evolution occurs through the adoption–rejection mechanism. It is interesting to note that at a given value of r, the rank of the adopted technology in equation (5) depends on the size of the community n. Agents having different numbers of interacting partners on a social network can choose technological levels with different ranks, which then has important consequences even on the macroscopic time evolution of the system.

4. Continuous distribution of technological levels in the initial state

We analyzed the behavior of the model system by means of computer simulations starting from a uniform distribution of technological levels between 0 and 1 with the probability density and distribution function $p(\tau) = 1$ and $P(\tau) = \tau$, respectively. For the time evolution, asynchronous update was used such that single agents were randomly selected and updated according to the dynamic rules presented in section 2. The iteration step was incremented after N trials, where N denotes the total number of agents in the system. The macroscopic technological progress of the system can be characterized by monitoring the average technological level $\langle\tau\rangle$ of the system.

We carried out computer simulations on different topologies to determine the average technological level $\langle\tau\rangle$ in the final state of the time evolution when a frozen configuration is attained. Representative examples of the final states of the system are presented in figure 2 for three values of the rewiring probability p using the same cost factor $r = 1.4$.

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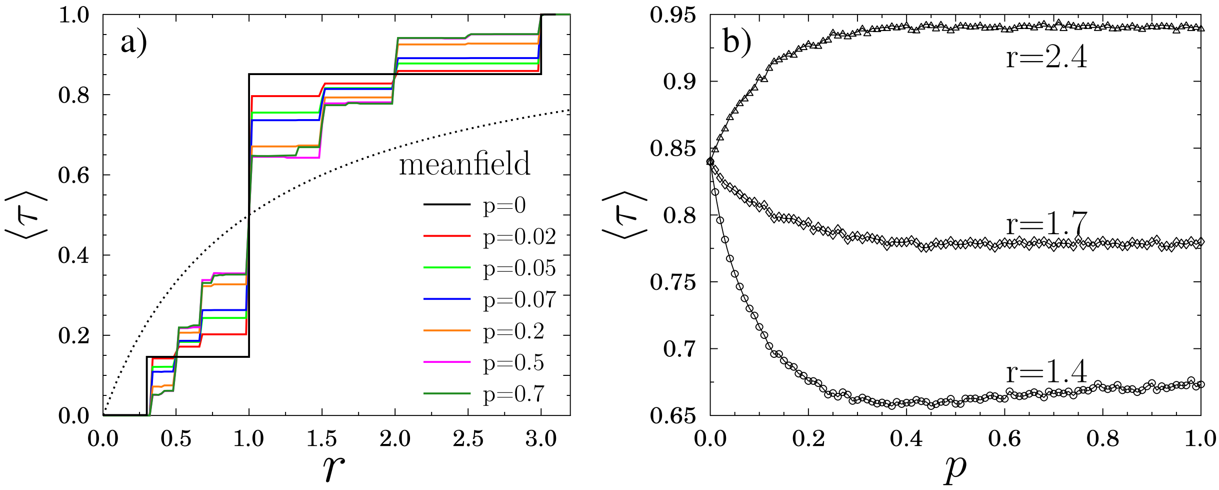

Figure 3. (a) Average technological level of the system obtained after long time evolution as a function of r. (b) $\langle\tau\rangle$ as a function of the rewiring probability p for three specific values of r. The presence of long range connections can increase and even decrease the average technological level of the system, depending on the cost factor r.

The color scale shows that a nearly homogeneous state is attained as a result of the adoption–rejection mechanism, where the technological level attained depends on the topology. The average technological level of the final state $\langle\tau\rangle$ is presented in figure 3(a) as a function of r for several different values of the rewiring probability p. It can be observed that for any value of the rewiring probability p the average technological level $\langle\tau\rangle(r)$ is a monotonically increasing function of r; furthermore, it is composed of distinct steps whose height and number are determined by the topology of the network. The steps are the consequence of the behavior described by equation (5), i.e. intervals of r exist inside which always technologies of the same rank i are selected by the agents:

$$\frac{rn}{1+r} \leq i \leq \frac{rn}{1+r} + 1. \tag{6}$$

Increasing the rewiring probability p, the distribution $\rho(n)$ gets broader, increasing the number of different degrees in the network which then results in a higher number of steps of $\langle\tau\rangle(r)$. It can be seen in figure 1(b) that for $p = 0.05$, the possible degrees are $n = 2, 3, 4, 5, 6$. Using equation (5) one can determine the interval limits of r for each n value, from which the overall r limits of the entire network can be obtained as $1/5, 1/4, 1/3, 1/2, 2/3, 1, 3/2, 2, 3, 4, 5$. In the special case of $p = 0$ (regular square lattice) the interval limits are $1/3, 1, 3$, as can be observed in figure 3(a). For comparison, in figure 3(a) we also present the mean field solution of the model, i.e. in the fully connected case, when all agents are connected with all others, the average technological level $\langle\tau\rangle_{\rm mf}$ in the final state of the system can be obtained analytically as

$$\langle\tau\rangle_{\rm mf} = 1/\left(1 + 1/r\right). \tag{7}$$

A very important outcome of the above calculations is that the degree polydispersity of agents' social contacts makes the socio-economic system more sensitive to the details of the novel technology (i.e. to the specific value of the cost factor r). It can be observed in figure 3(a) that increasing connectivity of the system, the presence of long range connections can increase but can also decrease the average technological level attained in the final state, depending on the value of the cost factor r. For high enough cost factor r the long range contacts facilitate the spreading of advanced technologies, while for lower r values the opposite effect occurs, i.e. the dominance of low level technologies enhanced also by the long range contacts prevents technological advancement. Figure 3(b) provides some quantitative insight into this effect, where we present $\langle\tau\rangle$ as a function of the rewiring probability p for three different values of r. All the curves start from the same point at $p = 0$, since on a regular square lattice always the third highest technology is selected when r falls in the interval $1 < r < 3$. For increasing p the curves converge to r dependent asymptotic values.

Thinking in terms of opinion dynamics, the steps of $\langle\tau\rangle$ in figure 3(a) represent possible consensus states of the system, which are determined by the cost factor r and by the network topology. Further insight into the formation of the consensus state is provided by the next section where the competition of two technologies is considered.

5. Spreading of new innovations

In the above analysis we considered a random initial distribution of technological levels with a continuous distribution and studied the evolution of the socio-economic system with the microscopic rejection–adoption mechanism. In the model a new innovation can be incorporated by increasing the technological level of a randomly selected agent by a certain amount. Then the question arises of whether this innovation can spread in the system or whether it gets lost due to the overwhelming dominance of low level technologies. It can easily be seen from the decision mechanism of agents that in a homogeneous state the innovation of a single individual rapidly disappears from the system. However, increasing the fraction of innovative agents ϕ, spreading can start when ϕ exceeds a critical value. This is a manifestation of the so-called *critical mass* effect [14], which is a characteristic feature, for instance, of telecommunication technologies whose practical value depends on the number of agents already using it.

In the mean field limit the critical mass, i.e. the critical fraction of innovative agents, can be determined analytically. Let us assume that the system has attained a homogeneous state where all N agents have products of technological level τ_1. A fraction ϕ of agents decide to purchase a product of higher quality with technological level τ_2 where $\tau_2 > \tau_1$. In terms of telecommunication technologies this situation would mean, for instance, that τ_1 represents mobile phones with SMS capability, while τ_2 stands for the more advanced phones with MMS features. Since all the agents interact with all others, the cost C of agents can easily be determined: the total cost of agents with τ_1 and τ_2 technological levels can be written in the forms

$$C(\tau_2) = a_2(\tau_2 - \tau_1)\phi N, \qquad \text{and} \qquad C(\tau_1) = a_1(\tau_2 - \tau_1)(1 - \phi)N, \tag{8}$$

respectively. Agents of τ_1 will upgrade to τ_2 if the change results in a cost reduction, i.e. $C(\tau_2) < C(\tau_1)$ must hold. On the basis of this condition, the critical fraction ϕ_c of

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innovative agents can be obtained as

$$\phi_c = \frac{1}{1+r}. \tag{9}$$

It is interesting to note that ϕ_c does not depend on the difference of technological levels $\Delta\tau = \tau_2 - \tau_1$, it is a unique function of the amount of advantages provided by the more advanced technology r. In the parameter regime $r \geq 1$ of practical relevance, the critical fraction falls in the interval $0 \leq \phi_c \leq 1/2$. This analysis implies that the system has two phases: when the fraction of innovative agents is small $\phi \leq \phi_c$, the advanced technology rapidly disappears and the system attains a homogeneous state at the lower technological level τ_1. However, for $\phi > \phi_c$ the advanced technology spreads and a homogeneous state is attained again, but at the higher technological level τ_2. Using again the example of mobile phones, below ϕ_c it is better to get rid of the more advanced MMS phones since they do not give any benefit; however, above the critical point, the weight of the communication with other individuals having the same higher level phone exceeds the weight of the low level technology.

In the mean field limit the system does not have time evolution, i.e. all agents make the same decision and the final state is reached with a single update. However, when the interaction of agents is localized to their nearest neighbors, a complex time evolution occurs. We carried out computer simulations starting from a homogeneous initial state of the complex network of N agents where all the agents had the same technological level τ_1. In the initial state a fraction ϕ of agents, randomly scattered over the network, takes an elevated technological level τ_2 with $\tau_2 > \tau_1$. The time evolution of the system was followed by computer simulations until a steady state was attained using asynchronous updates. In the mean field limit a sharp transition occurs at the critical point when increasing the fraction of innovative agents ϕ. However, when agents interact with their nearest neighbors on their social network, the spreading of more advanced technologies can appear at any finite ϕ with a certain probability P_s. We determined numerically the spreading probability P_s as a function of the fraction of innovative agents ϕ by counting the number of samples where only the higher technology remained in the final state. Figure 4(a) presents the spreading probability P_s of the system as a function of ϕ for a small world network of agents $p = 0.05$. It can be seen that P_s is a monotonically increasing function of ϕ with two clearly separated regimes: there exists a critical fraction of innovative agents ϕ_c which has to be exceeded to obtain the spreading of the new innovation with probability 1. Below the critical point, $\phi < \phi_c$, the new innovation practically dies out with a high probability and the system does not have any overall technological progress. However, when the critical fraction is exceeded, $\phi > \phi_c$, the new innovation diffuses over the entire system, all agents adopt the innovation and the socio-economic system attains a state with a high technological level. The critical fraction ϕ_c was determined numerically as the location of the inflexion point of $P_s(\phi)$.

Figure 4(b) demonstrates that the presence of long range connections in the network of agents, introduced by the rewiring, has a substantial effect on the spreading process which is also influenced by the value of the cost factor r: when the new technology provides enough advantages (large enough value of r; $r = 2.9$ in the figure), the long range connections facilitate the spreading of the new technology; hence, the critical fraction ϕ_c is a decreasing function of the rewiring probability p. However, when the advantages are not high enough (lower r value but still larger than 1; $r = 1.4$ in the figure), the topology

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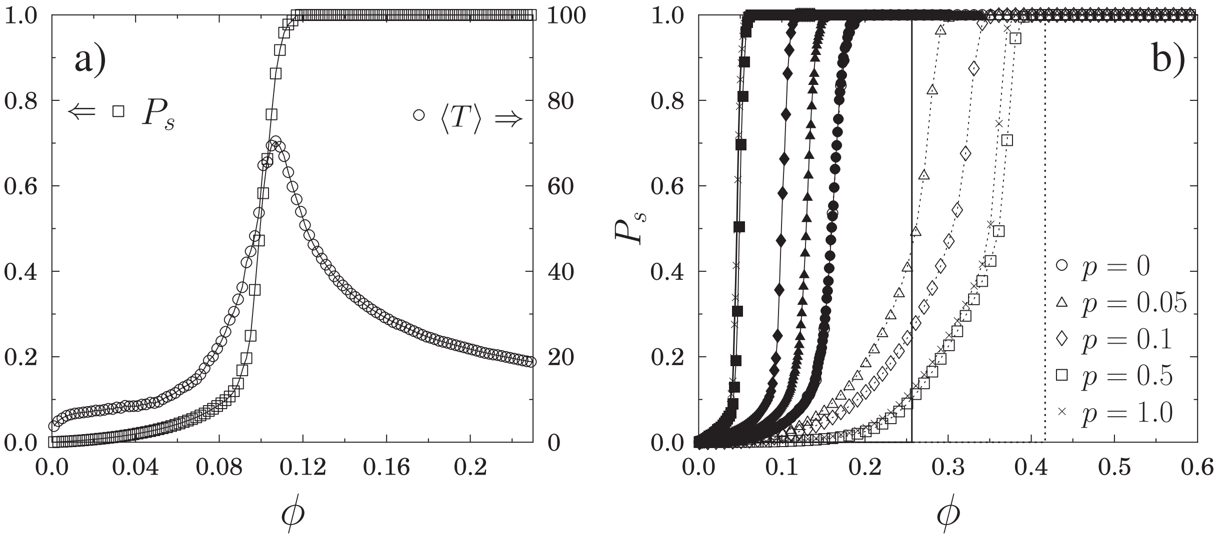

Figure 4. (a) Probability of spreading P_s as a function of the fraction of innovative agents ϕ together with the average relaxation time $\langle T\rangle$ for $p = 0.05$ and $r = 2.9$. (b) P_s as a function of ϕ for different topologies and two different values of the cost factor $r = 2.9$ (filled symbols) and $r = 1.4$ (open symbols). The vertical straight lines indicate the mean field values of ϕ_c for $r = 1.4$ (dotted line) and $r = 2.9$ (continuous line). The system size was set to $N = 10\,201$.

has a counter-effect, i.e. long range connections force the agents to retain their low level technology. Hence, a higher fraction of innovative agents is required to obtain spreading so that ϕ_c becomes an increasing function of p. The dependence of the critical fraction ϕ_c on the rewiring probability p is illustrated by figure 5 for the two r values used in figure 4(b). It can be observed that for both cases, $\phi_\mathrm{c}(p)$ converges to some asymptotic values for large p.

The susceptibility of the socio-economic system to the perturbation exerted by the appearance of the new innovation can be characterized by the average relaxation time $\langle T\rangle$, i.e. the number of iteration steps required to reach the final homogeneous state irrespective of the achieved technological level. In figure 4(a) we also plotted $\langle T\rangle$ as a function of ϕ along with the spreading probability P_s for a small world network. It can be observed that as the fraction of innovative agents approaches the critical point, the relaxation time $\langle T\rangle$ of the system has a relatively sharp maximum. This behavior implies that in the vicinity of the critical fraction, the rejection–adoption process slows down, i.e. it takes a significantly longer time to reach the consensus state. It is important to note that the position of the maximum coincides with the critical point ϕ_c. This phenomenon is analogous to the critical slowing down observed at continuous phase transitions. Figure 6(a) demonstrates that increasing the system size N, the maximum of the average relaxation time $\langle T\rangle$ as a function of ϕ gets sharper. In order to determine the size scaling of $\langle T\rangle_\mathrm{max}$, simulations were carried out, varying N in a broad range for several different values of p. It can be seen in figure 6(b) that $\langle T\rangle_\mathrm{max}$ has the same scaling behavior as the average shortest path $\bar{l}(N)$ of agents' network, i.e. $\langle T\rangle_\mathrm{max} \sim N^{1/d}$ is obtained for $p = 0$, while for finite values of the rewiring probabilities a logarithmic dependence arises $\langle T\rangle_\mathrm{max} \sim \ln N$. (Here $d = 2$ is

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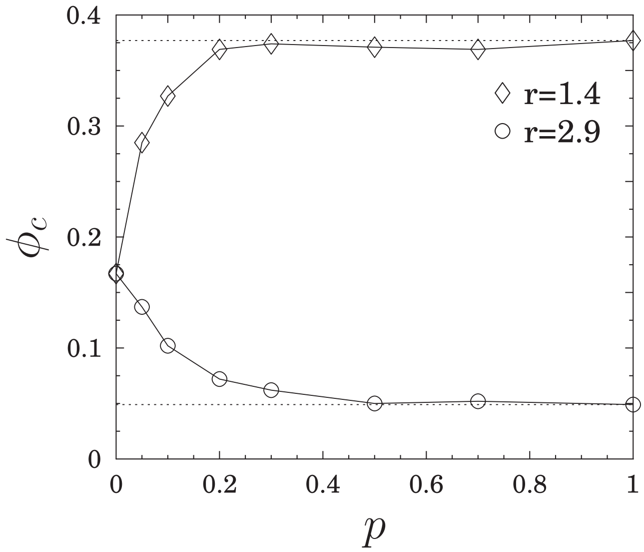

Figure 5. Critical fraction ϕ_c of innovative agents required by spreading of technologies as a function of the rewiring probability p for the two cost factors r considered in figure 4(b). The two curves start from the same point because, on the regular square lattice, agents make the same decision for r values in the interval $1 < r < 3$.

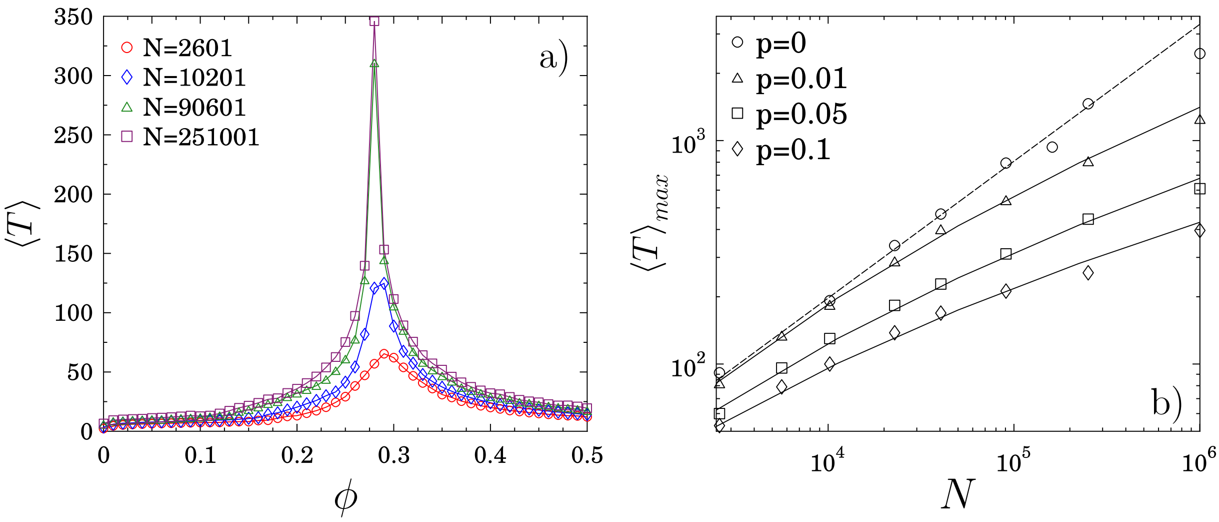

Figure 6. (a) Average relaxation time $\langle T \rangle$ as a function of the fraction of innovative agents ϕ for different system sizes N at the rewiring probability $p = 0.05$. (b) The maximum of the average relaxation time $\langle T \rangle_{\max}$ as a function of the system size N for different p values.

the dimension of the regular lattice.) This behavior clearly indicates that away from the critical point local decisions rapidly lead to the disappearance of one of the technologies, while in the vicinity of ϕ_c collective effects dominate.

6. Opinion dynamics with majority fraction

It follows from equation (8) that if an agent has n social contacts on the complex network, it adopts the higher level technology, when k out of n neighbors already have the technology, where $k > n/(1+r)$ holds. Hence, the cost factor of our model $r = a_2/a_1$ determines the fraction of agents

$$p^h = \frac{1}{1+r}, \tag{10}$$

with the higher level technology in the neighborhood, which is required to adopt it by the agent. In the case of $k/n < p^h$ the low level technology is copied. This dynamics always leads to a nearly homogeneous final state where almost all agents adopt the same technology, i.e. consensus is reached. Frustrated configurations where agents with different technological levels form coexisting islands can only occur in such a way that agents along the island boundaries repeatedly copy the same technology. This happens, for instance, on a regular square lattice when the island boundary is a straight line. Rewiring the square lattice prevents the occurrence of such pathological configurations.

It is important to emphasize that our model can be considered as a special type of opinion spreading model based on majority rules. In our model of the spreading of technologies it is not a simple majority that is required for spreading, but the threshold fraction of agents determines the adoption, which is then controlled by the cost factor of the technology. A similar type of opinion dynamics model has recently been introduced in [21] where agents' decisions are based on a majority fraction p^h. However, in that model the opposite state is adopted for $k/n < 1 - p^h$ which allows for a wide variety of frustrated configurations. In our model the cost minimization of the usage of the technology leads to a different microscopic decision mechanism and ensures consensus for any $r \neq 1$.

7. Discussion

We presented an agent-based model of the spreading of technological advancements, where the technology is used for the interaction/communication of agents. We demonstrated that the topology of agents' social contacts plays a crucial role in the spreading process leading to a broad spectrum of novel behaviors. The spreading process is driven by a cost minimization procedure based on which agents decide to adopt or reject a technology. Analytical calculations and computer simulations showed that long range connections on the social network can facilitate but can also hinder the diffusion of the advanced technology, depending on the amount of advantages more advanced technologies provide with respect to the low level ones. Starting from a continuous spectrum of technological levels, the decision making leads to a nearly homogeneous final state where a kind of 'consensus' of technologies is achieved. The technological level of the system attained in the final state, i.e. the success of new technologies and the overall technological progress of the system, have a complex dependence on the network topology.

We analyzed the appearance of new innovations in an initially homogeneous system. A critical fraction ϕ_c of innovative agents was identified which has to be exceeded in order to obtain spreading of the advanced technology. Below the critical point the new technology rapidly disappears from the system, while above it the new technology gets rapidly adopted leading to technological progress. Approaching ϕ_c from either side, a slowing down occurs whose characteristic time scales as the average shortest path of the network. The critical fraction depends both on the topology of the network and on the cost factor of the technology.

The model recovers the critical mass effect of technologies where so-called networking effects are dominant, like in telecommunication technologies. Our model can also be considered as a special type of opinion dynamics where it is not a simple majority that is required for spreading in a two-component system, but the majority fraction of agents of higher level technologies is determined.

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